The event horizon of extreme mass-ratio mergers

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IGWM2017 Bilbao

funded by the European Research Council

Work with

Marina Martínez

CQG 33, 155003 (2016)
arXiv:1603.00712
IJMP D25, 1644015 (2016)

and with

Marina Martínez & Miguel Zilhão

to appear

Black Hole fusion

The most complex of all processes governed by $R_{\mu\nu} = 0$

Non-linearity at its most fiendish

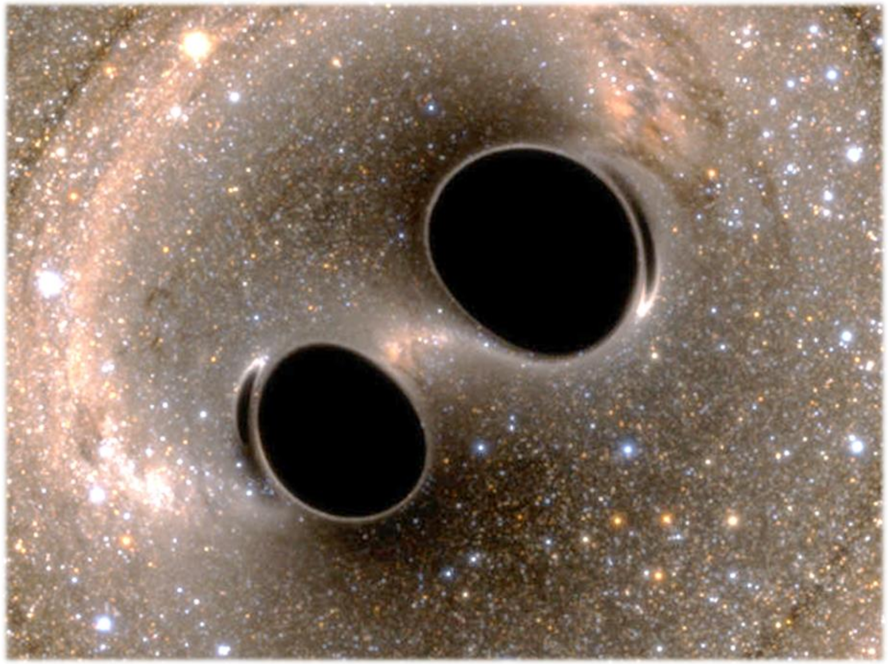
Black Hole fusion

The most complex of all processes governed by $R_{\mu\nu} = 0$

Non-linearity at its most fiendish

or maybe not—not always

This is what we'd *see* (lensing)



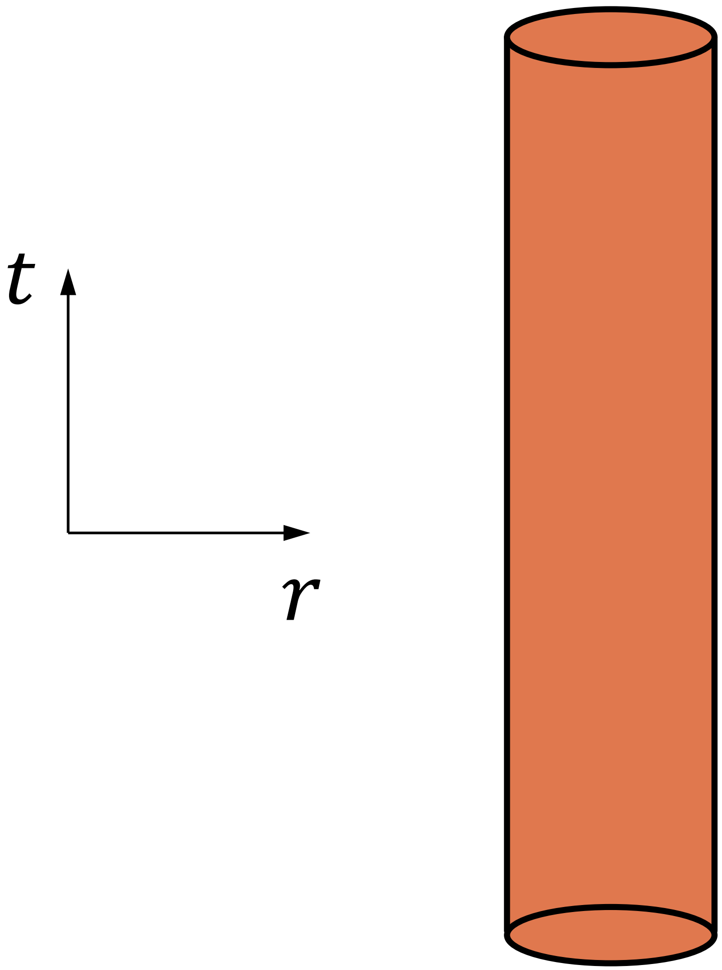
Not a black hole, but its *shadow*

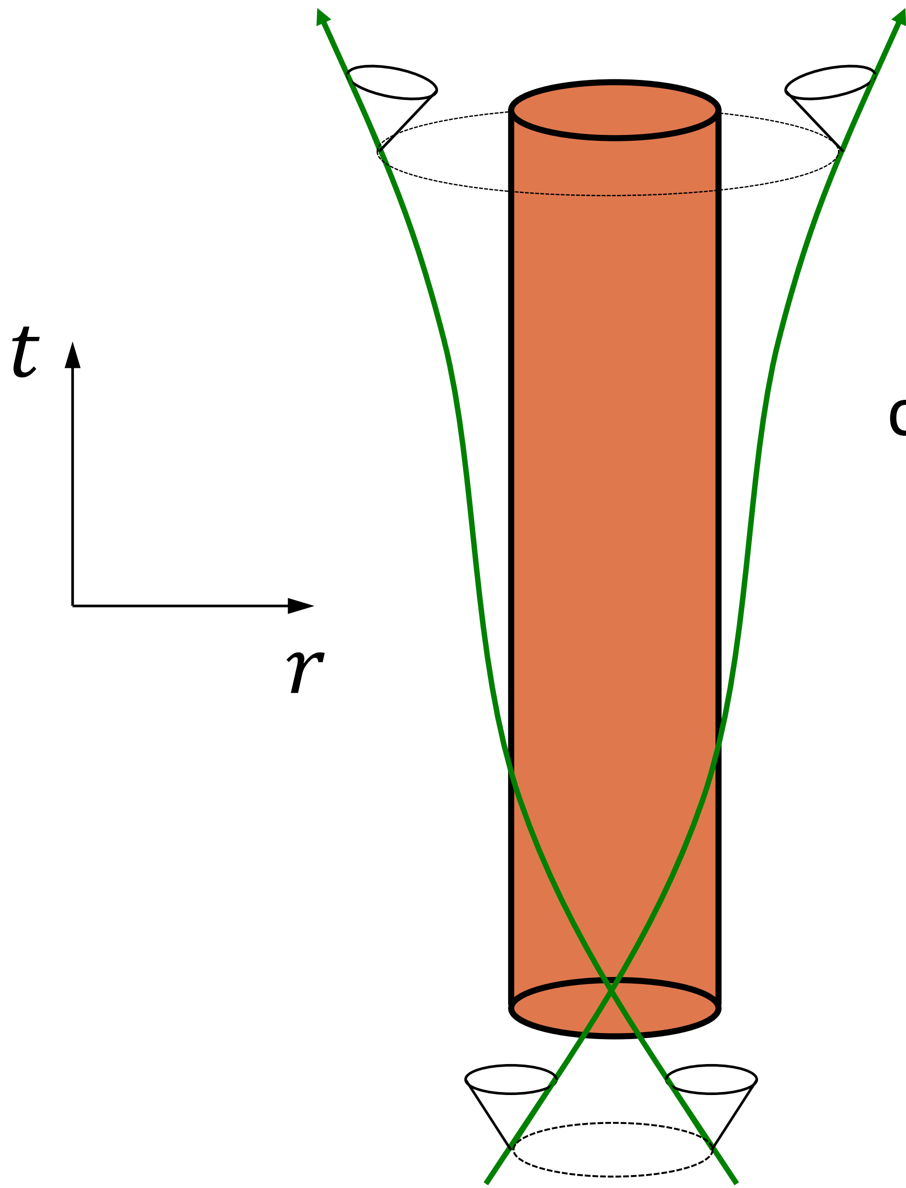
What is a black hole?

Spacetime region from which not even light can escape

Event Horizon

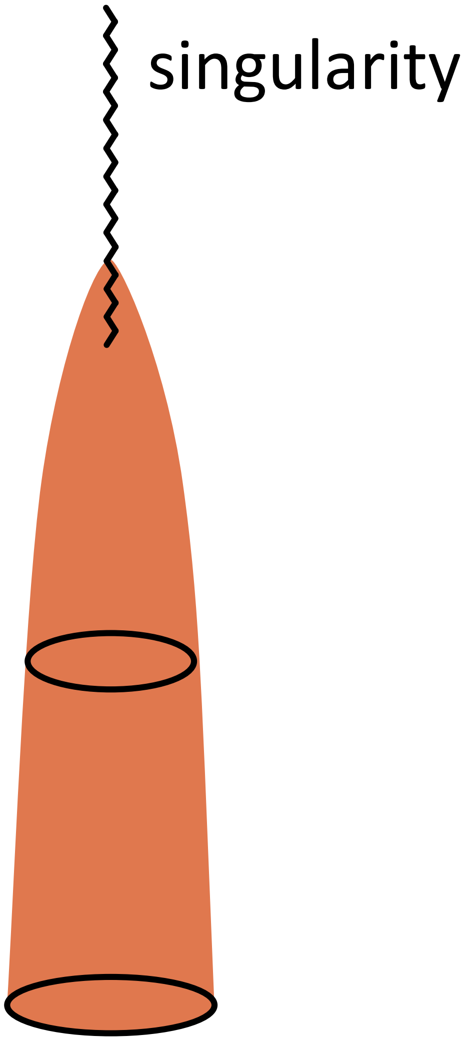
Star

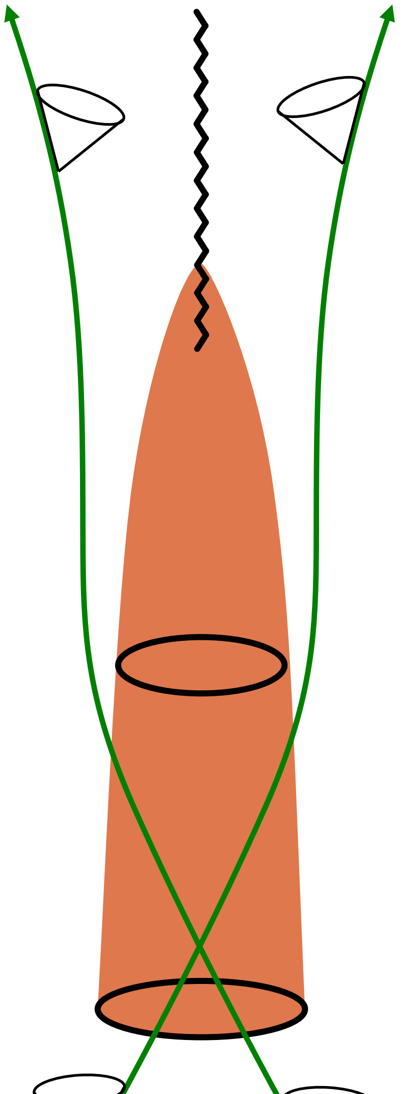


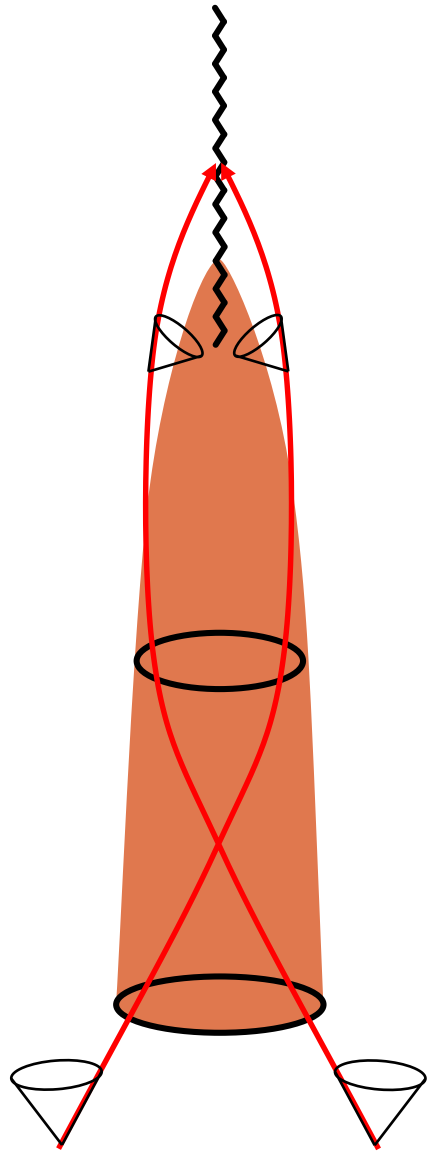


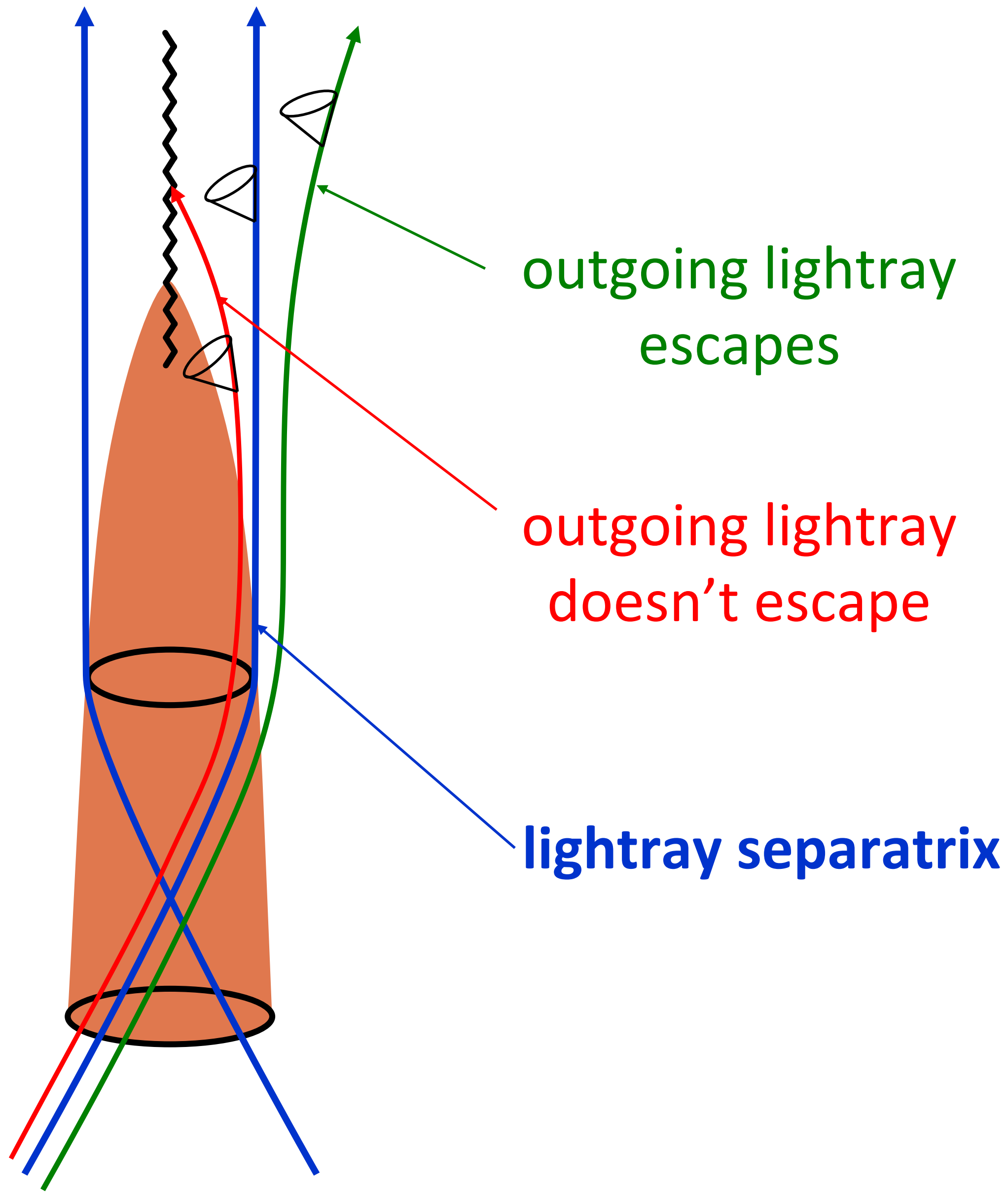
Spherical wavefronts contract, then expand

Collapsed Star

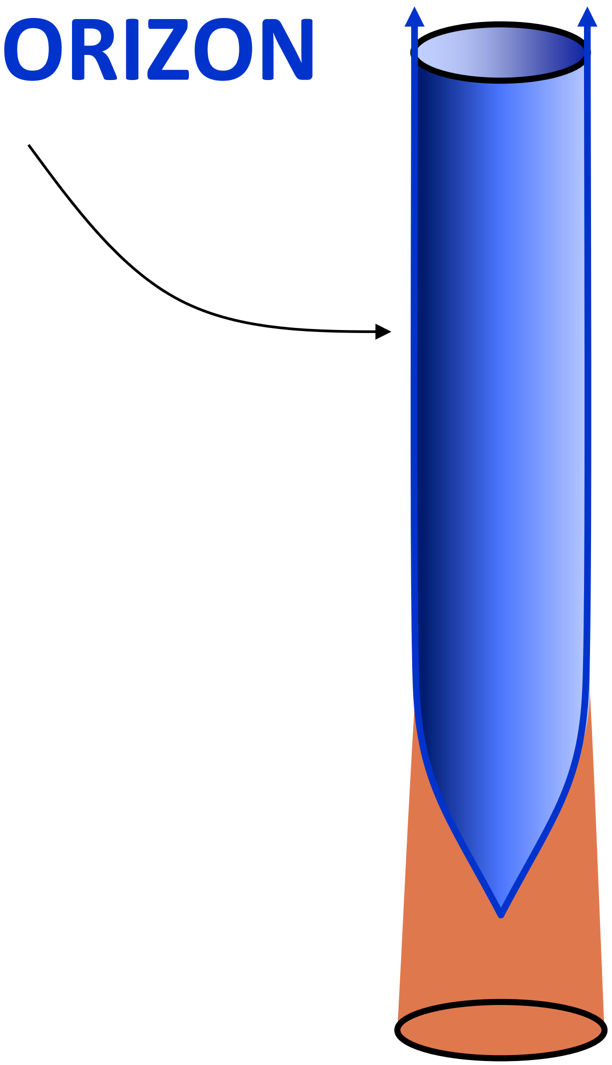






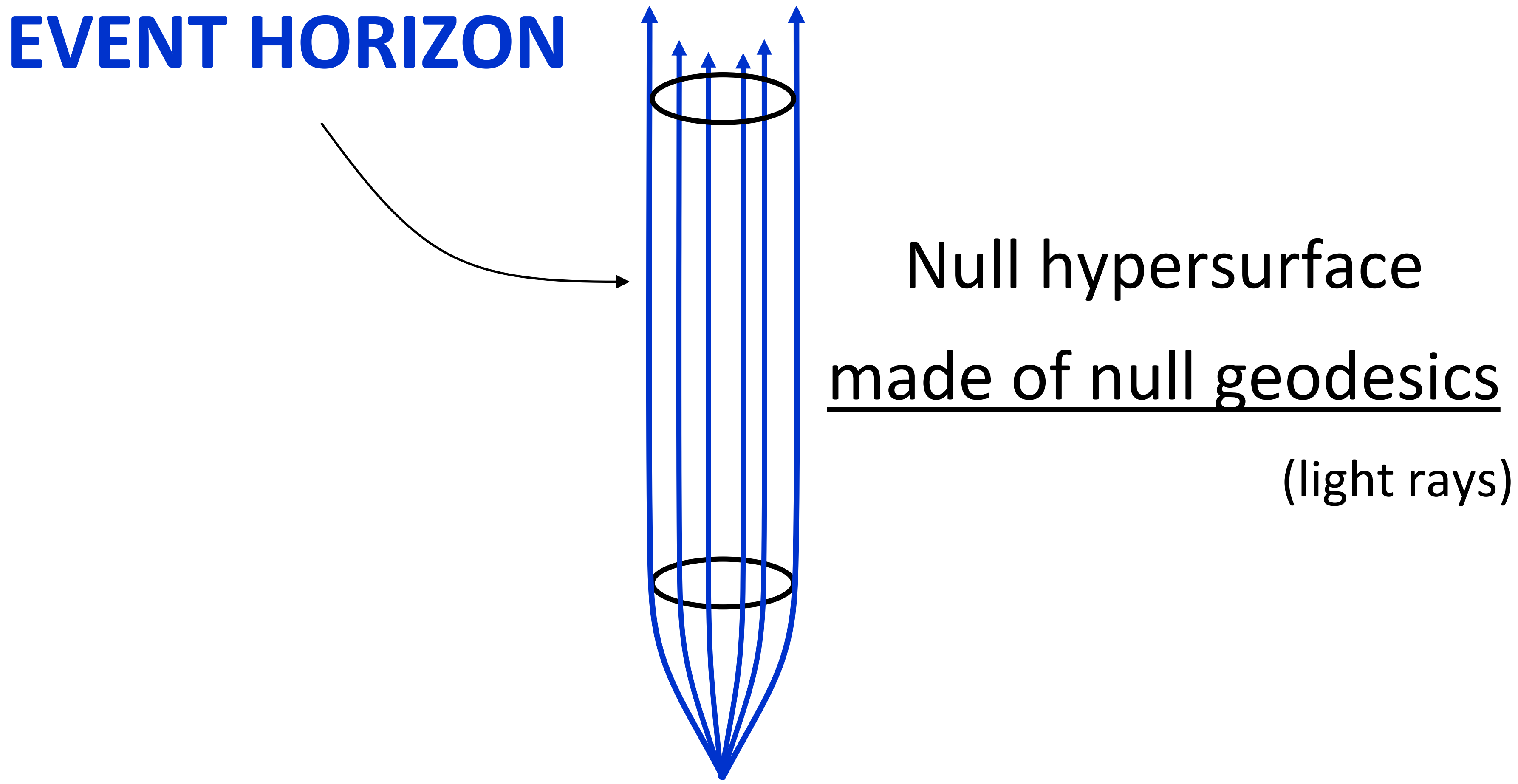

outgoing lightray escapes
outgoing lightray doesn't escape
lightray separatrix

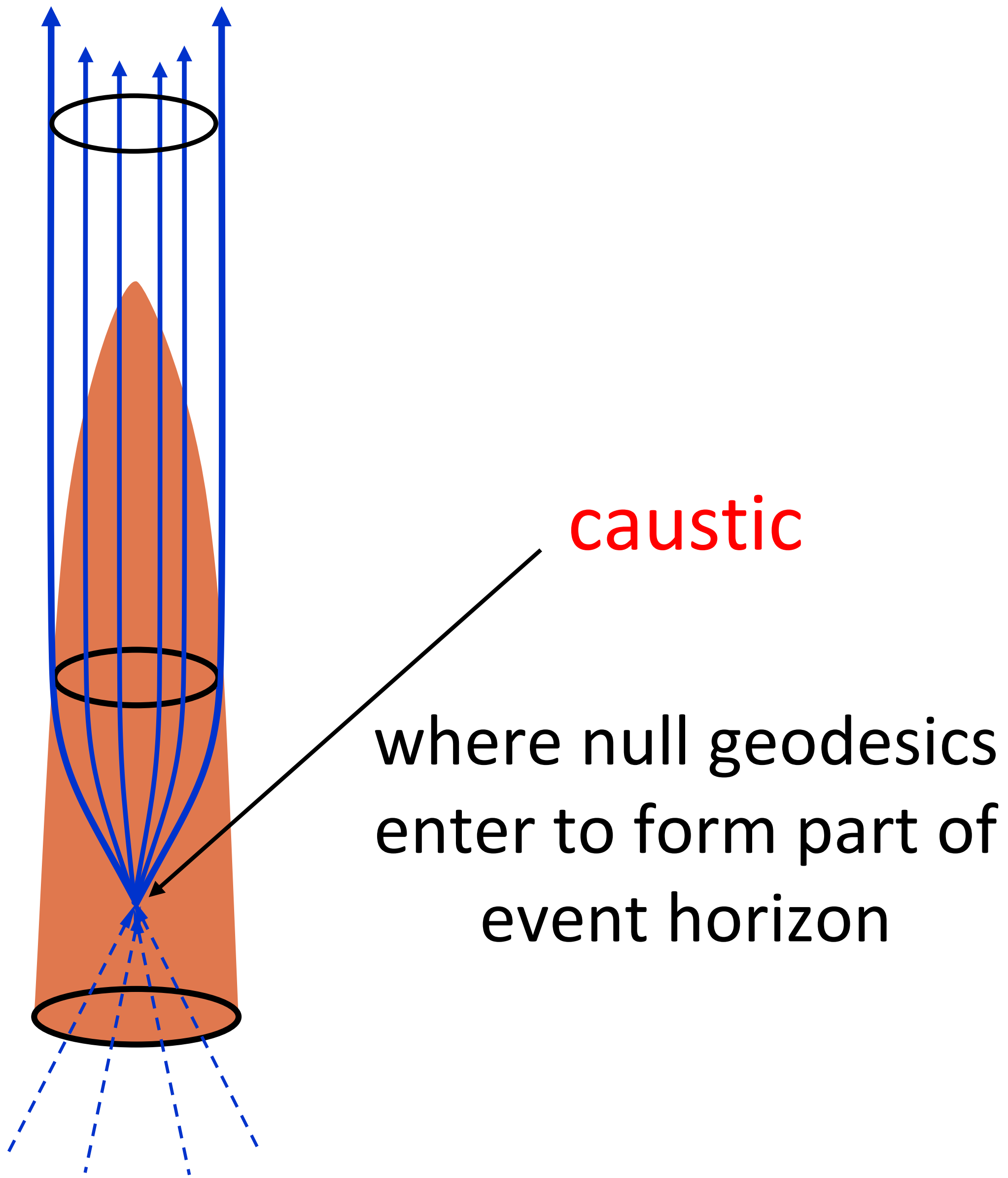
EVENT HORIZON



Null hypersurface

3-dimensional in
4-dimensional spacetime

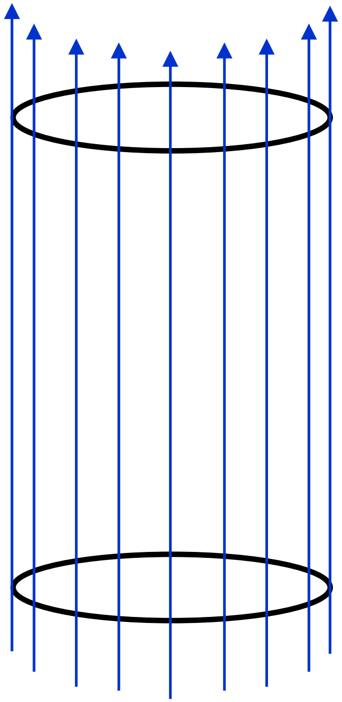

EVENT HORIZON
Null hypersurface
made of null geodesics
(light rays)


caustic
where null geodesics
enter to form part of
event horizon

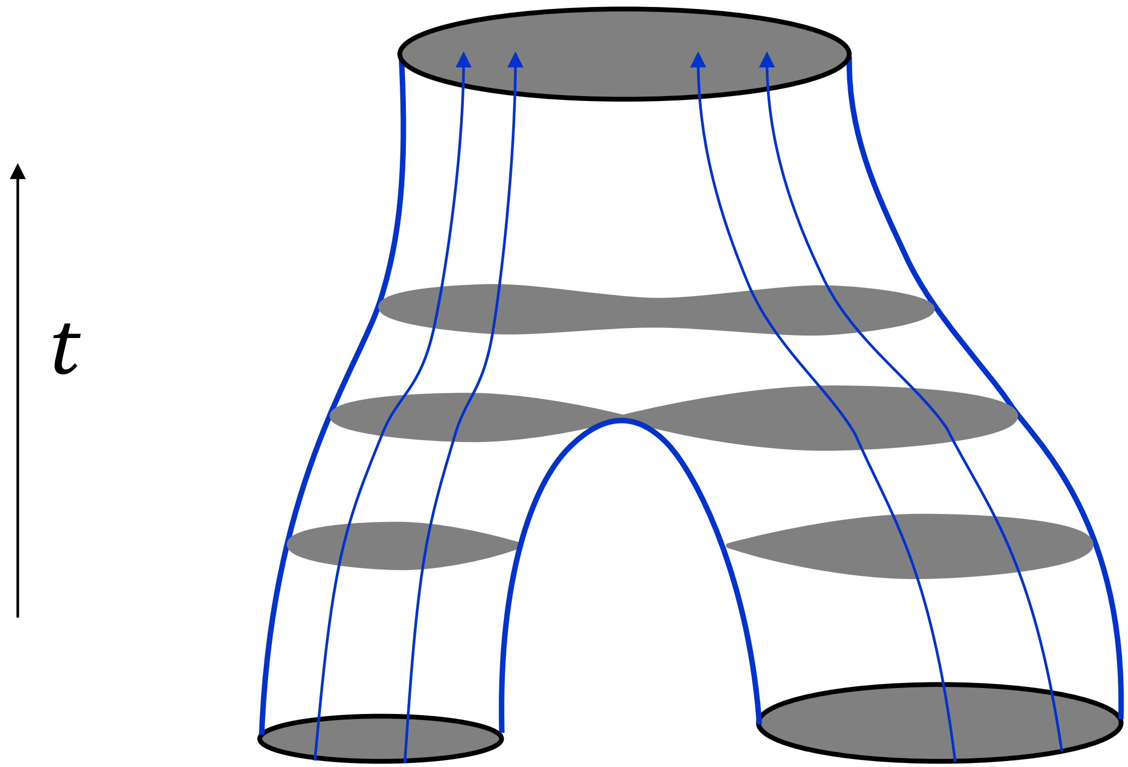
Event horizon

found by

tracing a family of light rays in a given spacetime

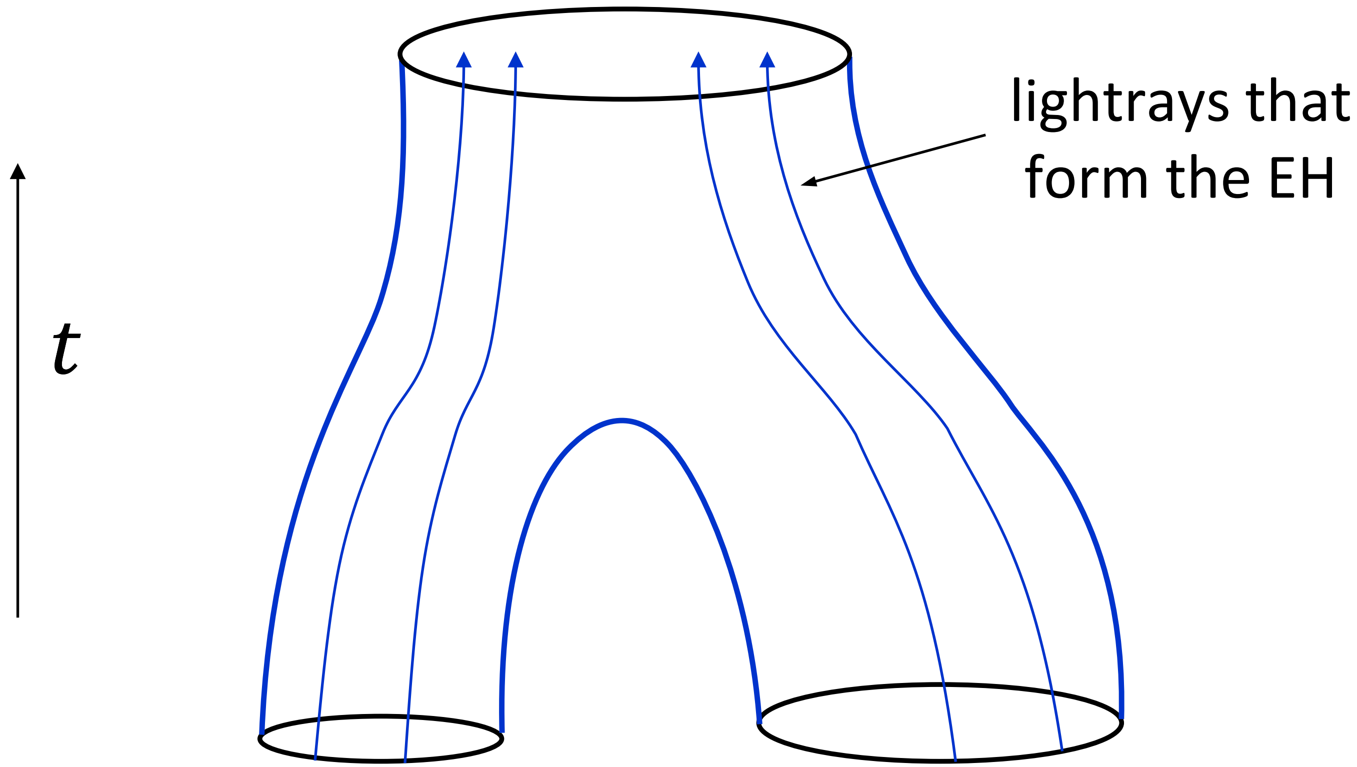


Event horizon of binary black hole fusion

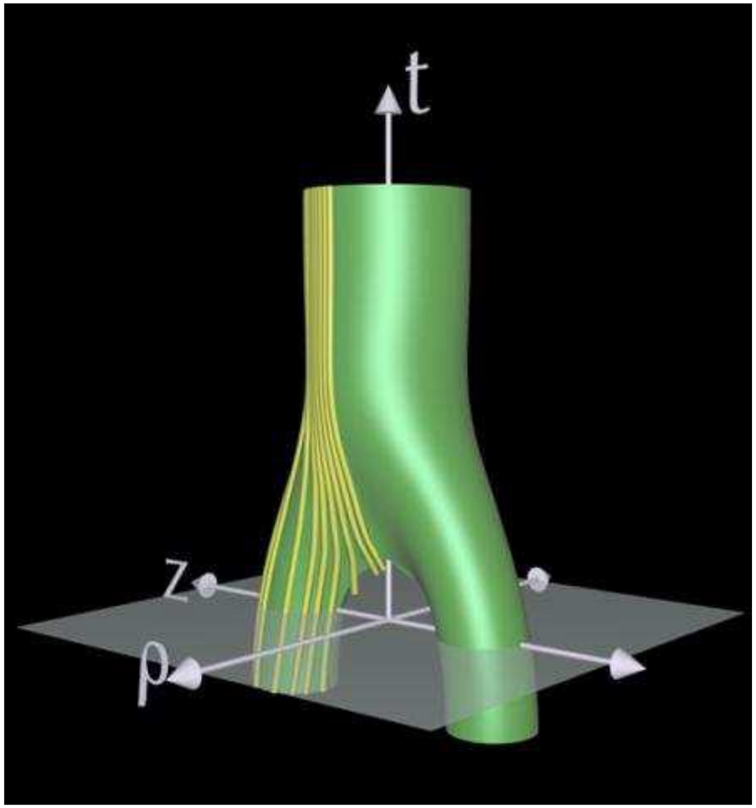


Event horizon of binary black hole fusion

"pants" surface



Event horizon of binary black hole fusion



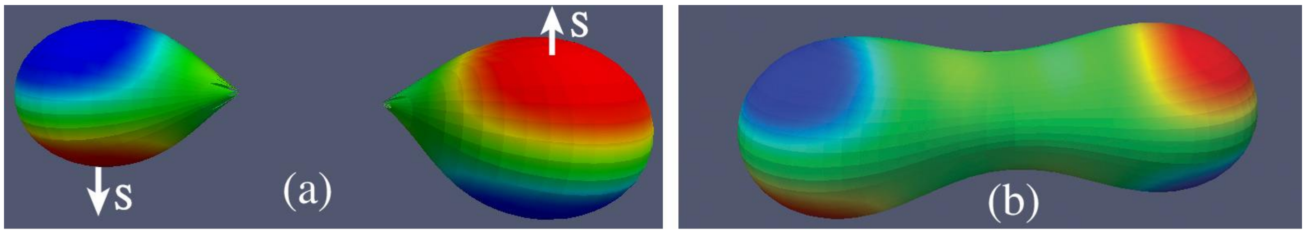
head-on
(axisymmetric)

equal masses

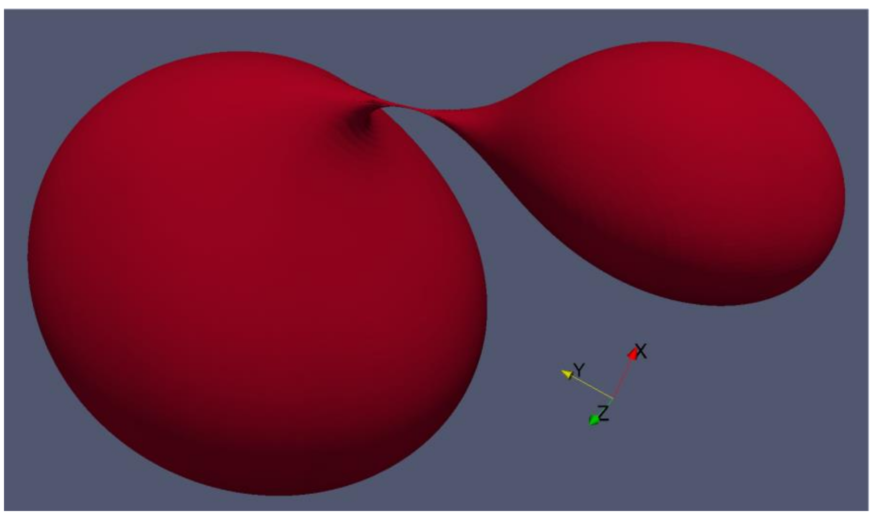
Cover of *Science,* November 10, 1995

Binary Black Hole ***Grand Challenge*** *Alliance* (Matzner et al)

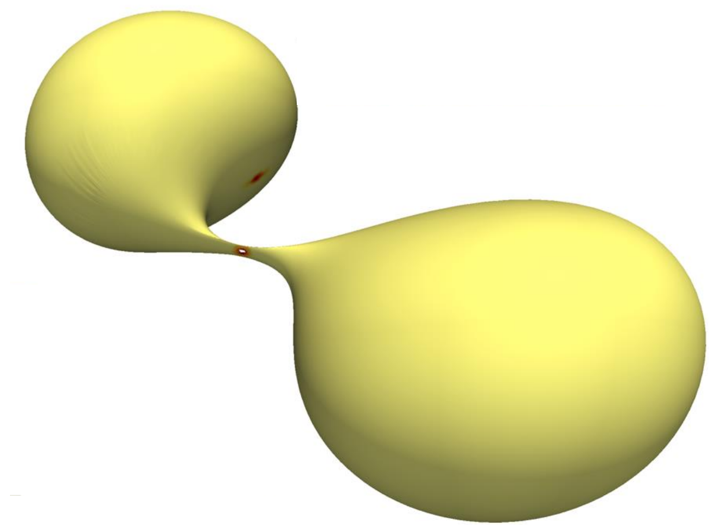
Spatial sections of event horizon of binary black hole fusion



Owen et al, Phys.Rev.Lett. 106 (2011) 151101



Cohen et al, Phys.Rev. D85 (2012) 024031



Bohn et al, Phys.Rev. D94 (2016) 064009

Surely the fusion of horizons can only be captured with supercomputers

Surely the fusion of horizons can only be captured with supercomputers

or so it’d seem

$\exists$ limiting (but realistic) instance where horizon fusion can be described **exactly**

It involves only elementary ideas and techniques

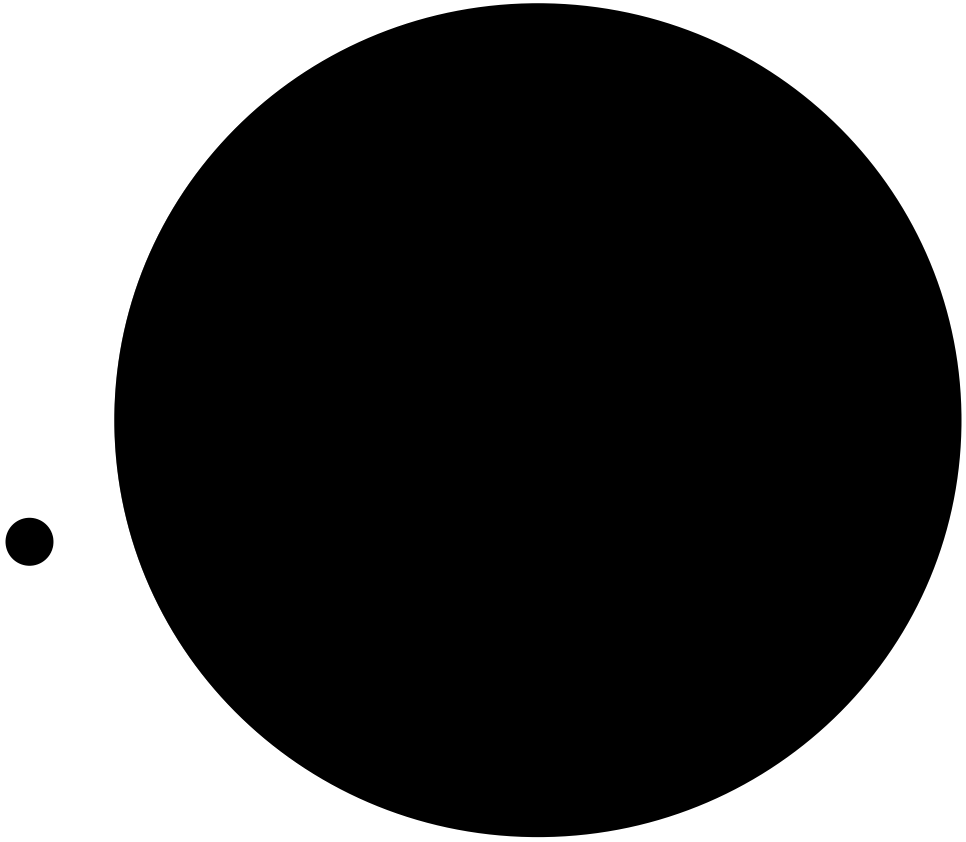
Equivalence Principle (1907)

Schwarzschild solution & Null geodesics (1916)

Kerr solution (1964)

Notion of Event Horizon (1950s/1960s)

Extreme-Mass-Ratio (EMR) merger

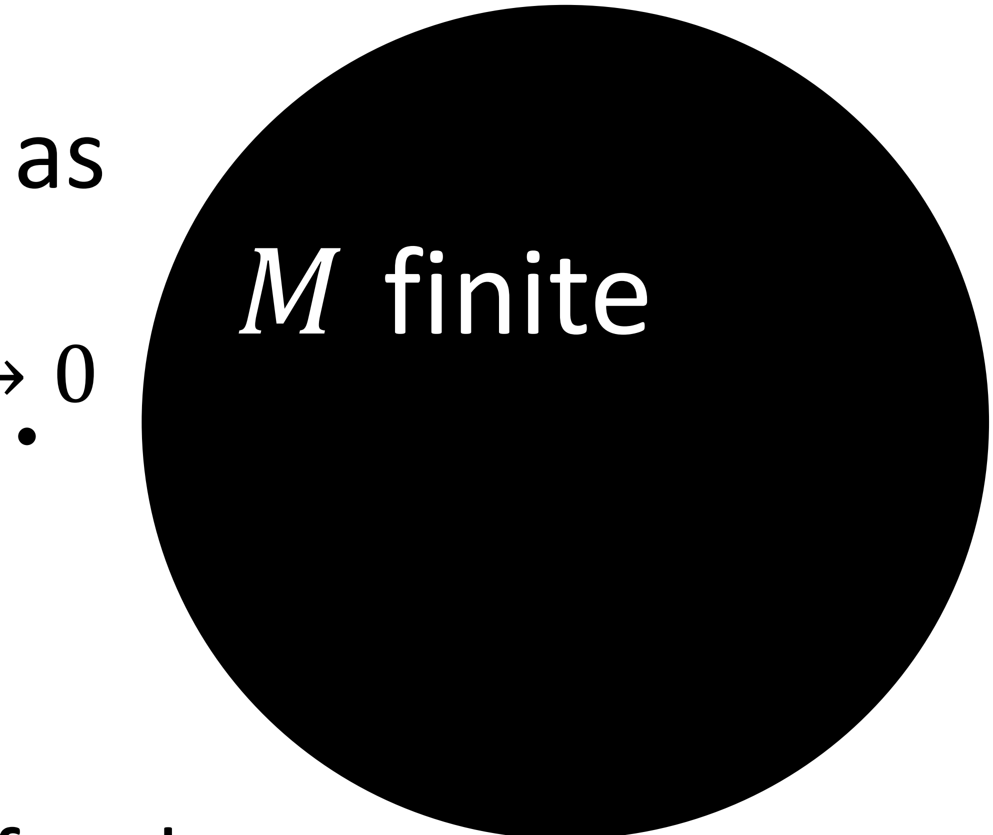


$$m \ll M$$

$m \ll M$

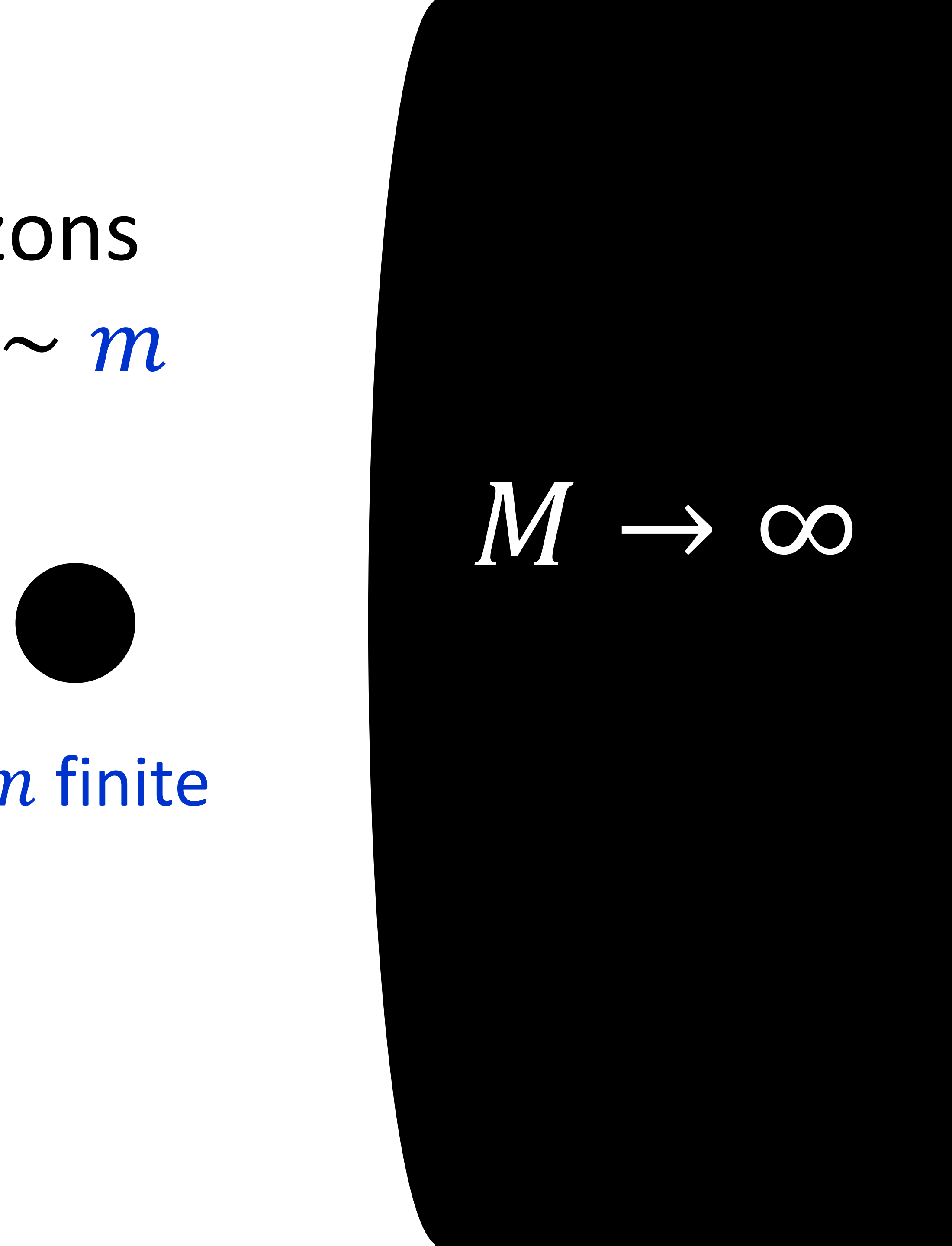
most often taken as

$m \to 0$



M sets the scale for the radiation emitted

Fusion of horizons involves scales $\sim m$



Gravitational waves?

When $M \rightarrow \infty$ the radiation zone is pushed out to infinity

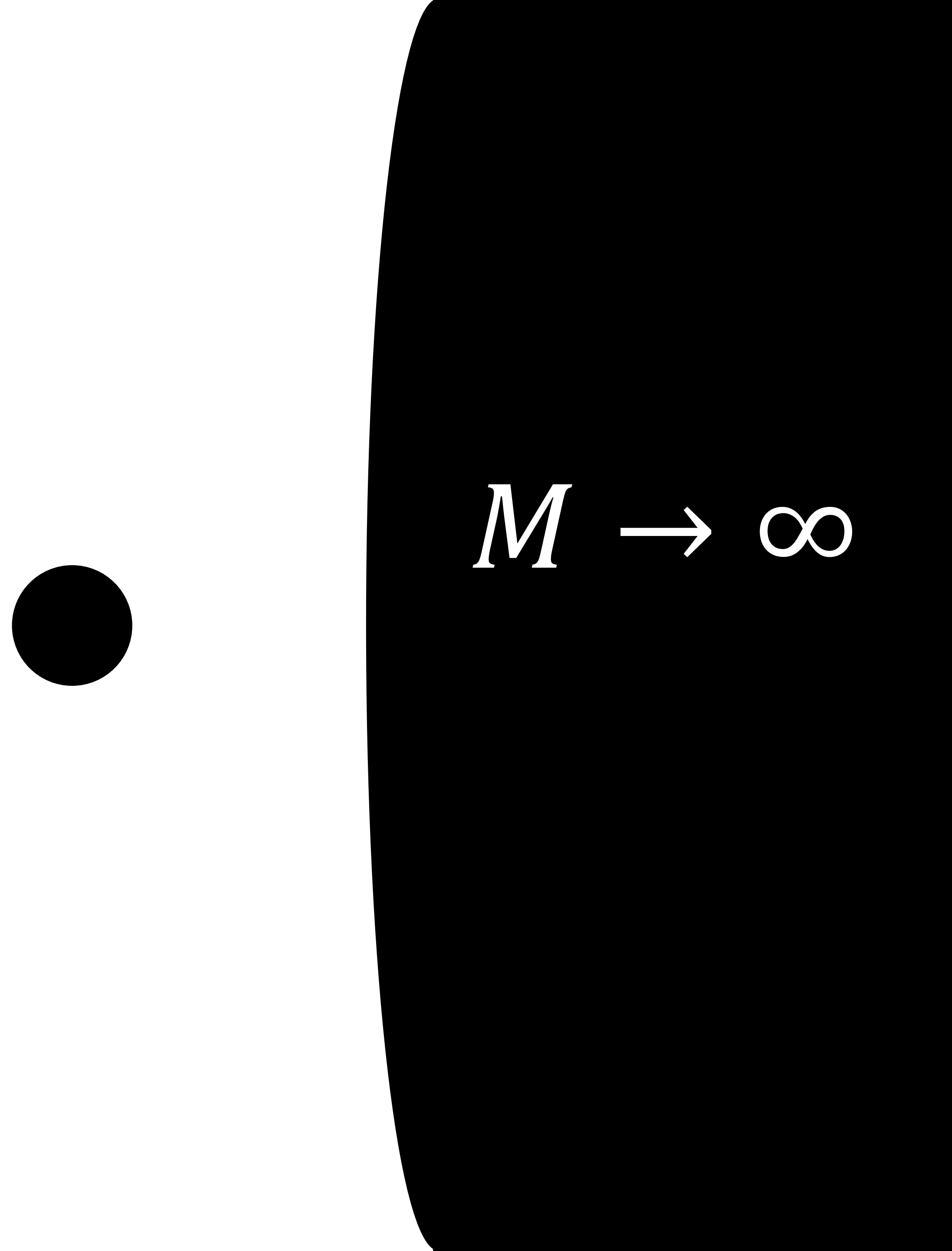
No gravitational waves in this region

Gravitational waves?

GWs will reappear if we introduce
corrections for finite small $\frac{m}{M}$

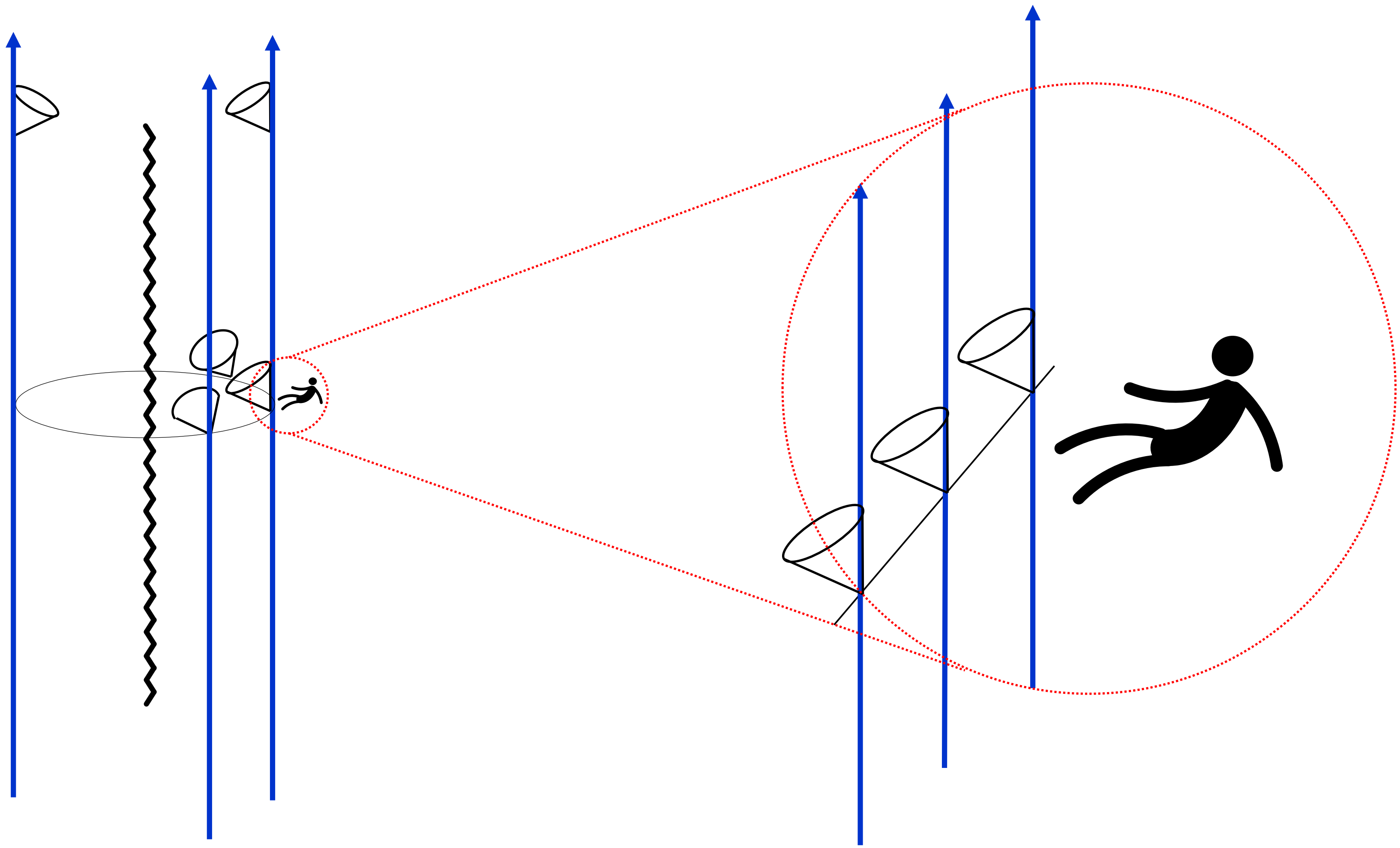
matched asymptotic expansion to Hamerly+Chen 2010
Hussain+Booth 2017

not for today


M → ∞

$$M \to \infty$$

Very large black hole / Very close to the horizon



Very close to a Black Hole

Horizon well approximated
by null plane
in Minkowski space

This follows from
the Equivalence Principle

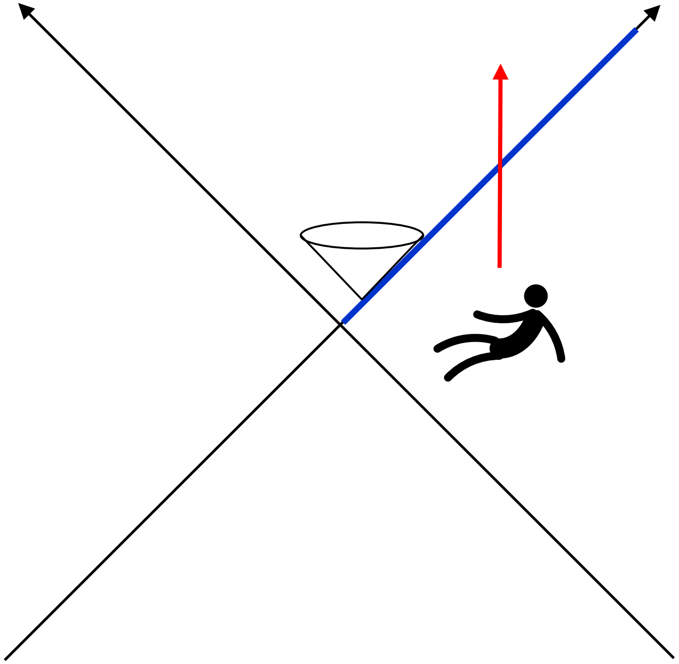
At short enough scales, geometry is
equivalent to flat Minkowski space

Curvature effects become small,
but horizon remains

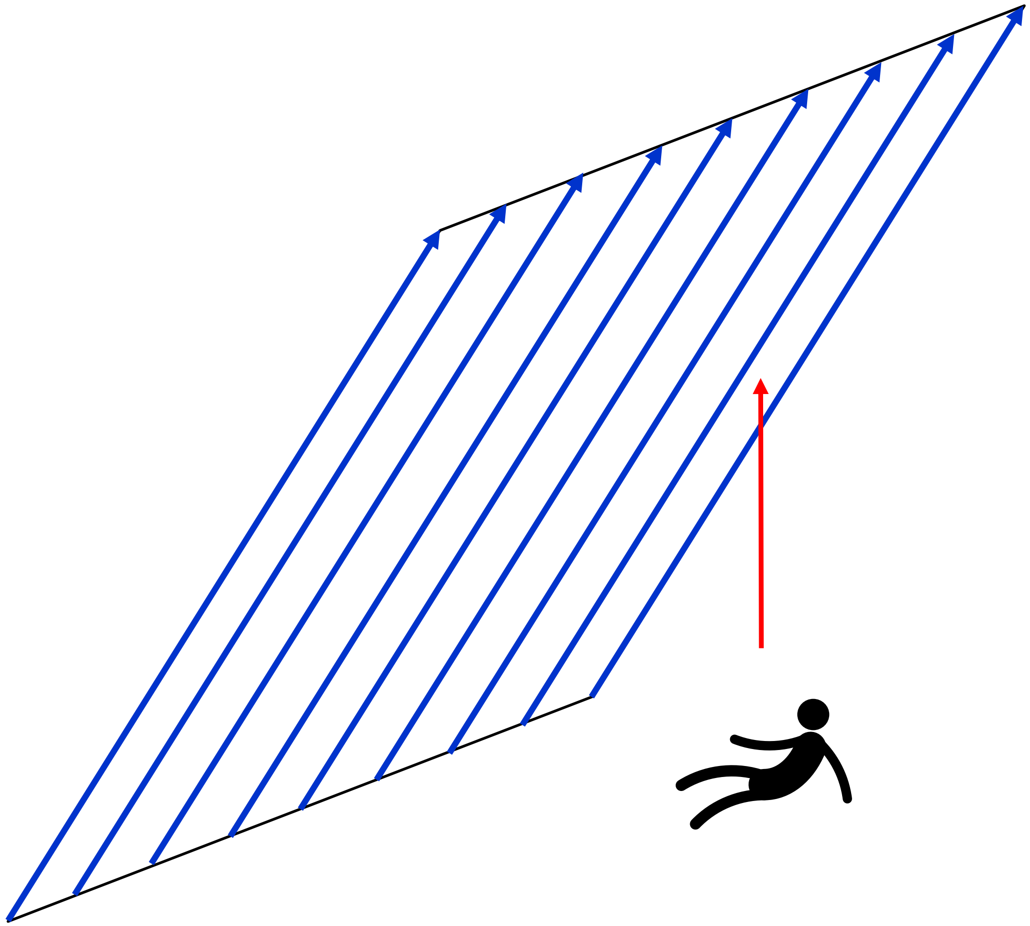
Locally gravity is equivalent to acceleration

Locally black hole horizon is equivalent to acceleration horizon

Falling into very large bh =
crossing a null plane in Minkowski space

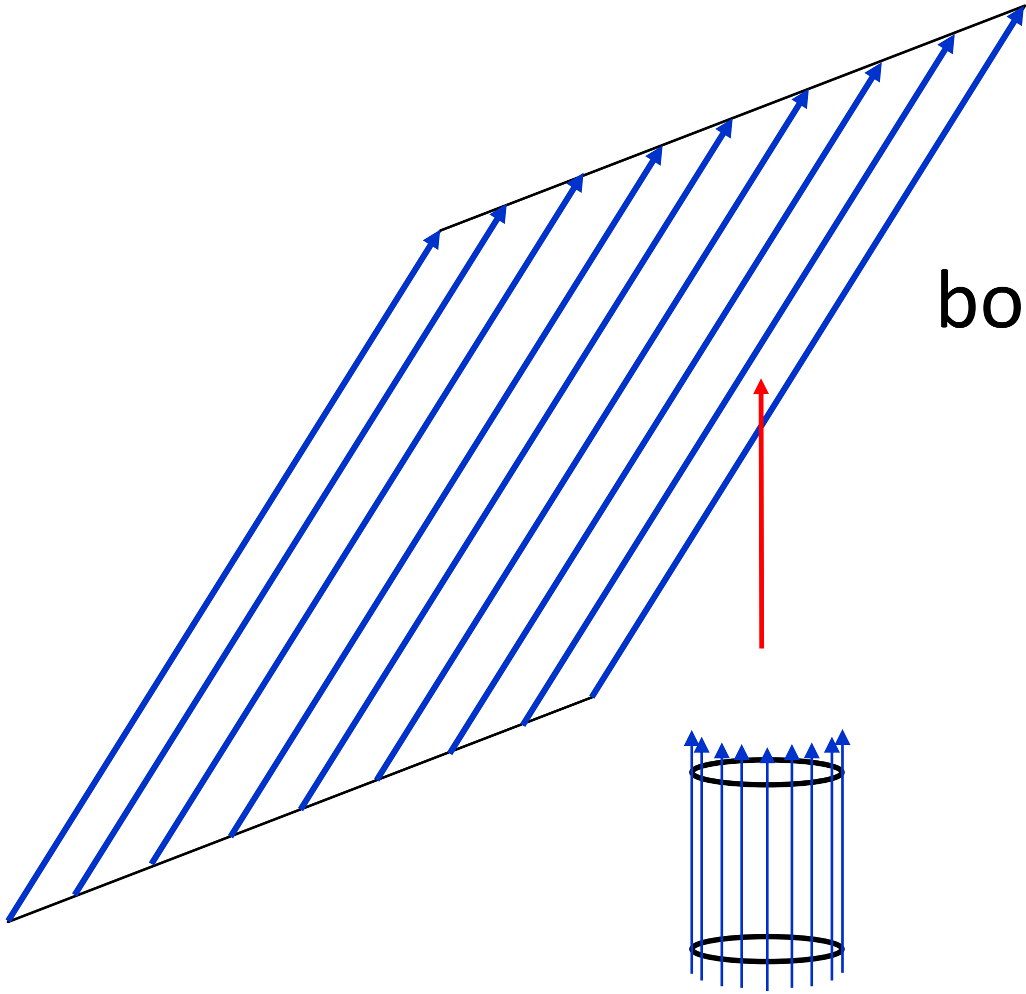


Object falling into a Large Black Hole



in rest frame of
infalling object

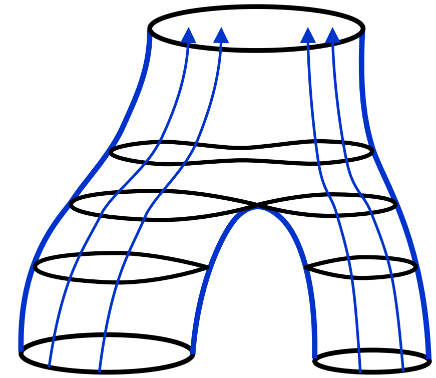
Small Black Hole falling into a Large Black Hole

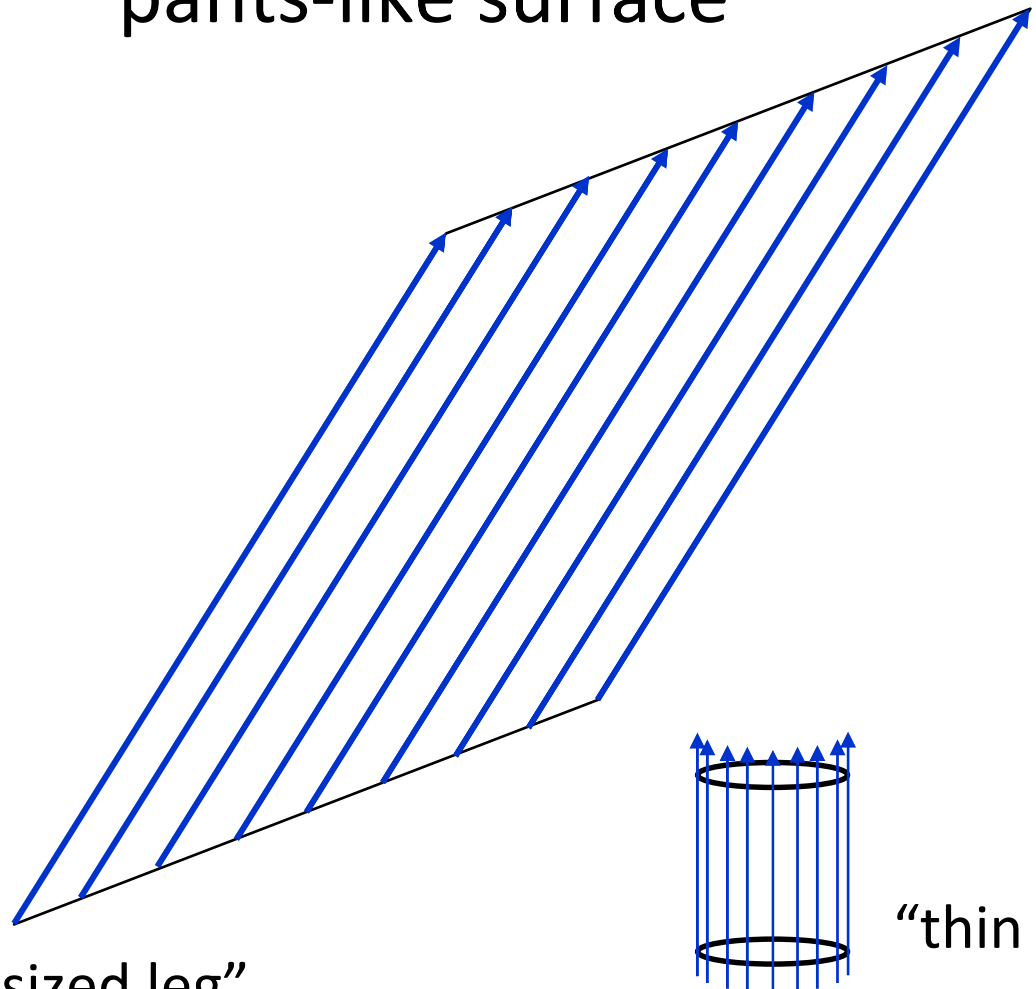


both are made of lightrays

in rest frame of small black hole

Lightrays must merge to form a pants-like surface

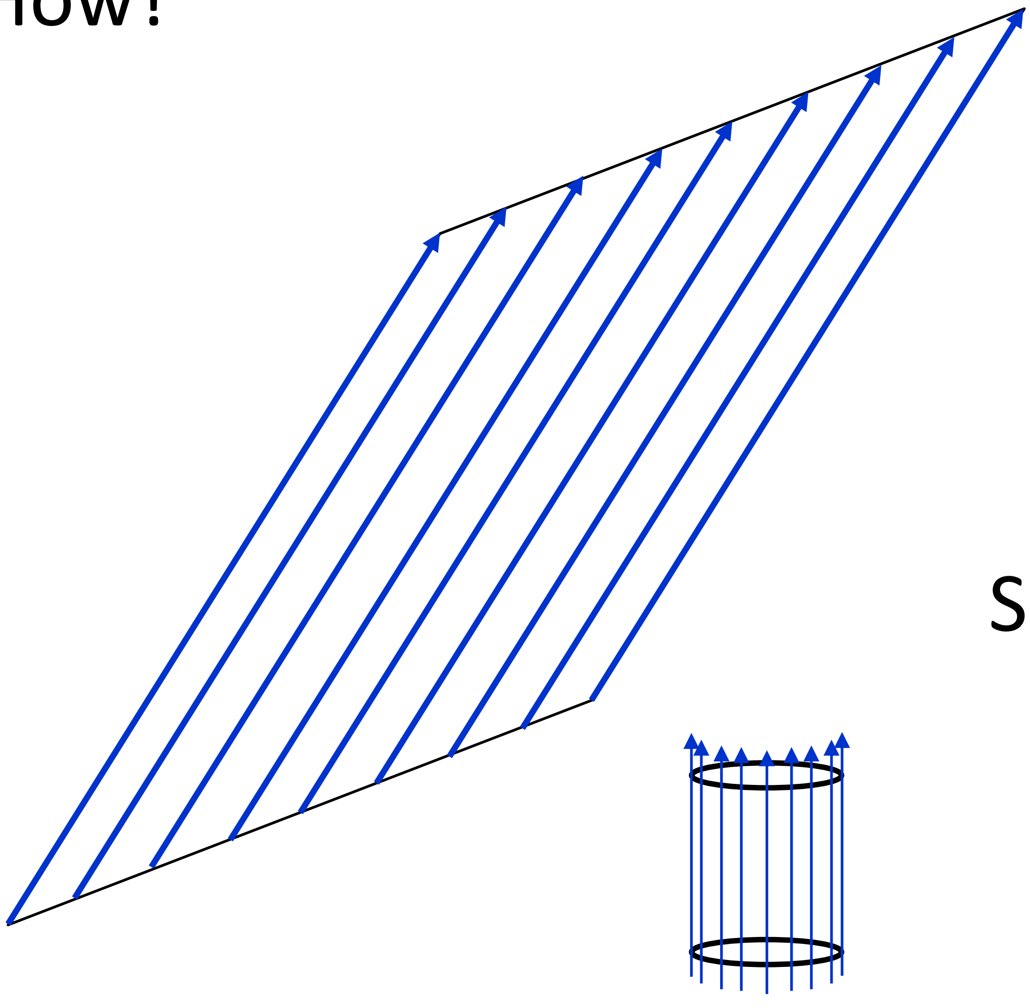




“oversized leg”

“thin leg”

How?



EH is a family of lightrays in spacetime

Small black hole: Schwarzschild solution with finite mass m

To find the pants surface:

Trace a family of null geodesics in the

Schwarzschild solution

that approach a null plane at infinity

All the equations you need to solve

$$t_q(r) = \int \frac{r^3 dr}{(r-1)\sqrt{r(r^3 - q^2(r-1))}}$$

$$\phi_q(r) = \int \frac{q dr}{\sqrt{r(r^3 - q^2(r-1))}}$$

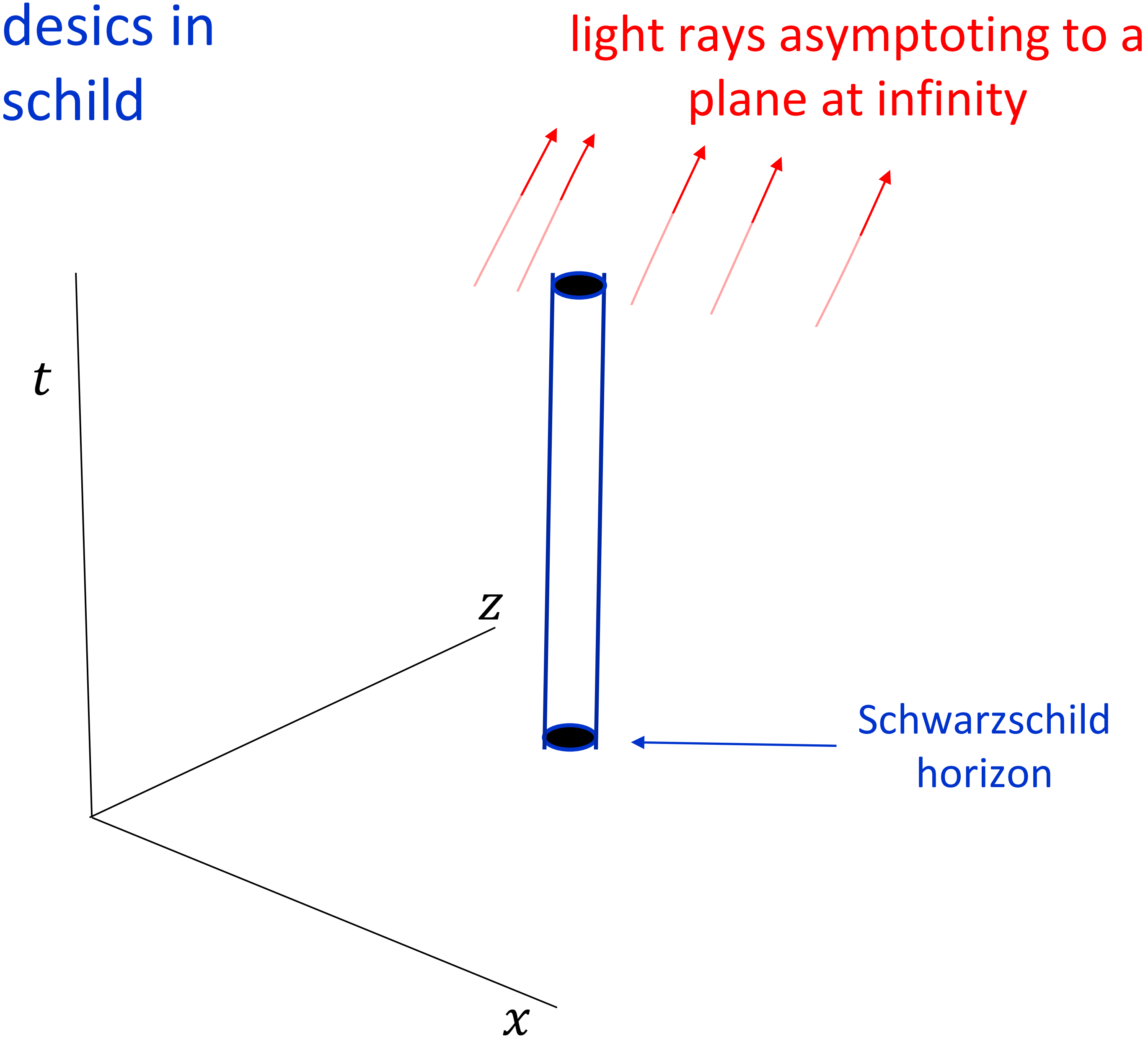
with appropriate final conditions:

$2m = 1$

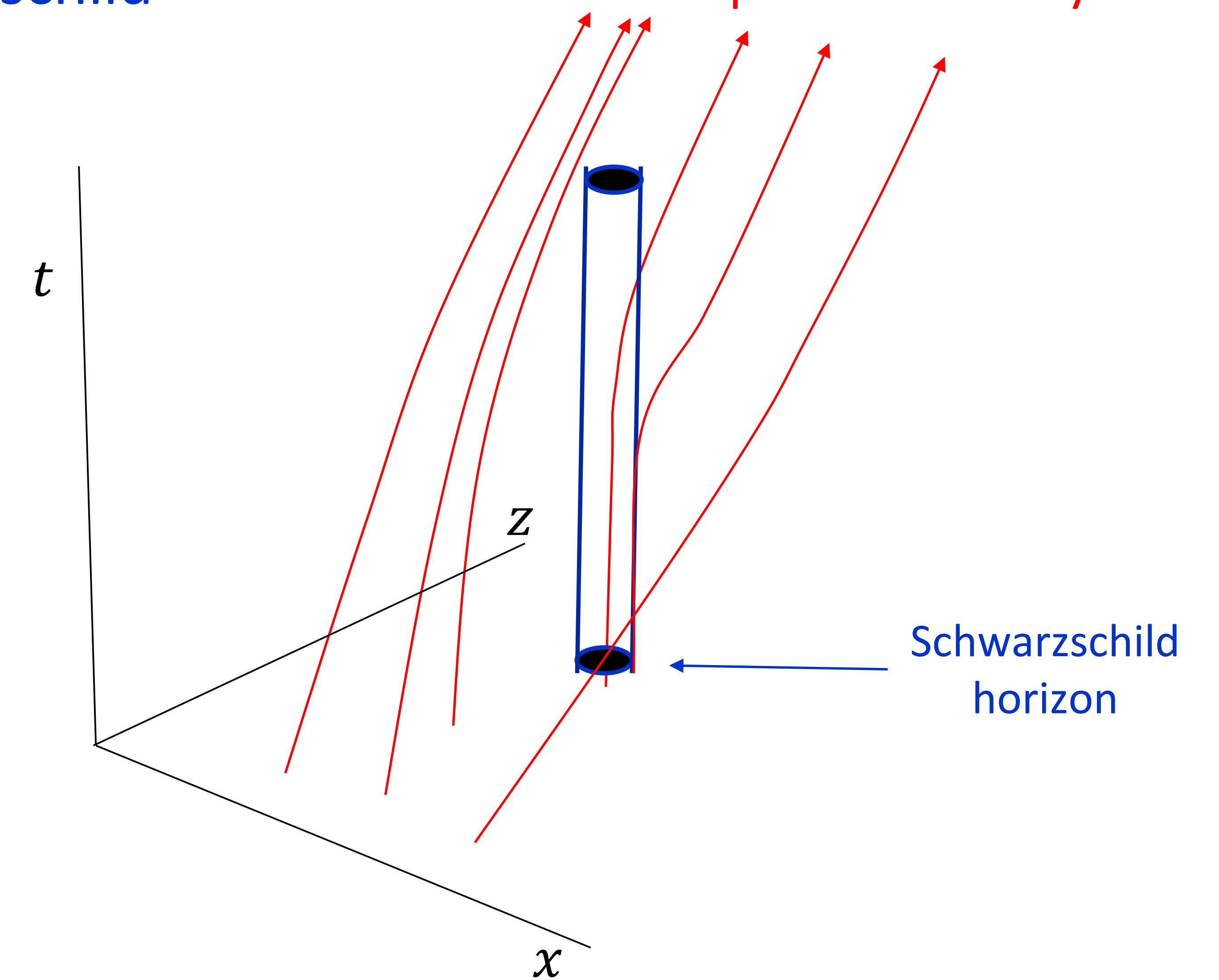
null plane at infinity

$q =$ impact parameter of lightrays at infinity

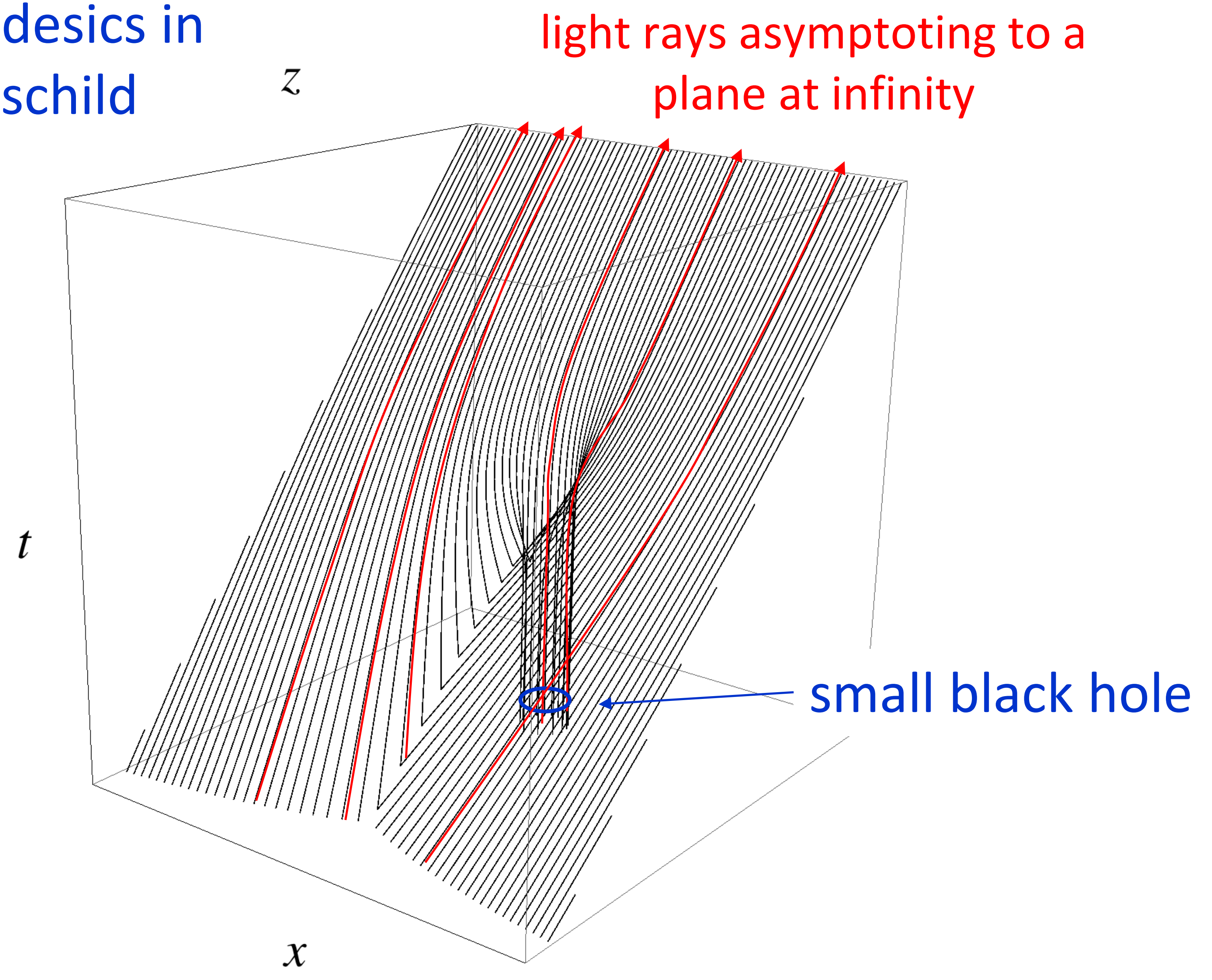
Null geodesics in Schwarzschild solution



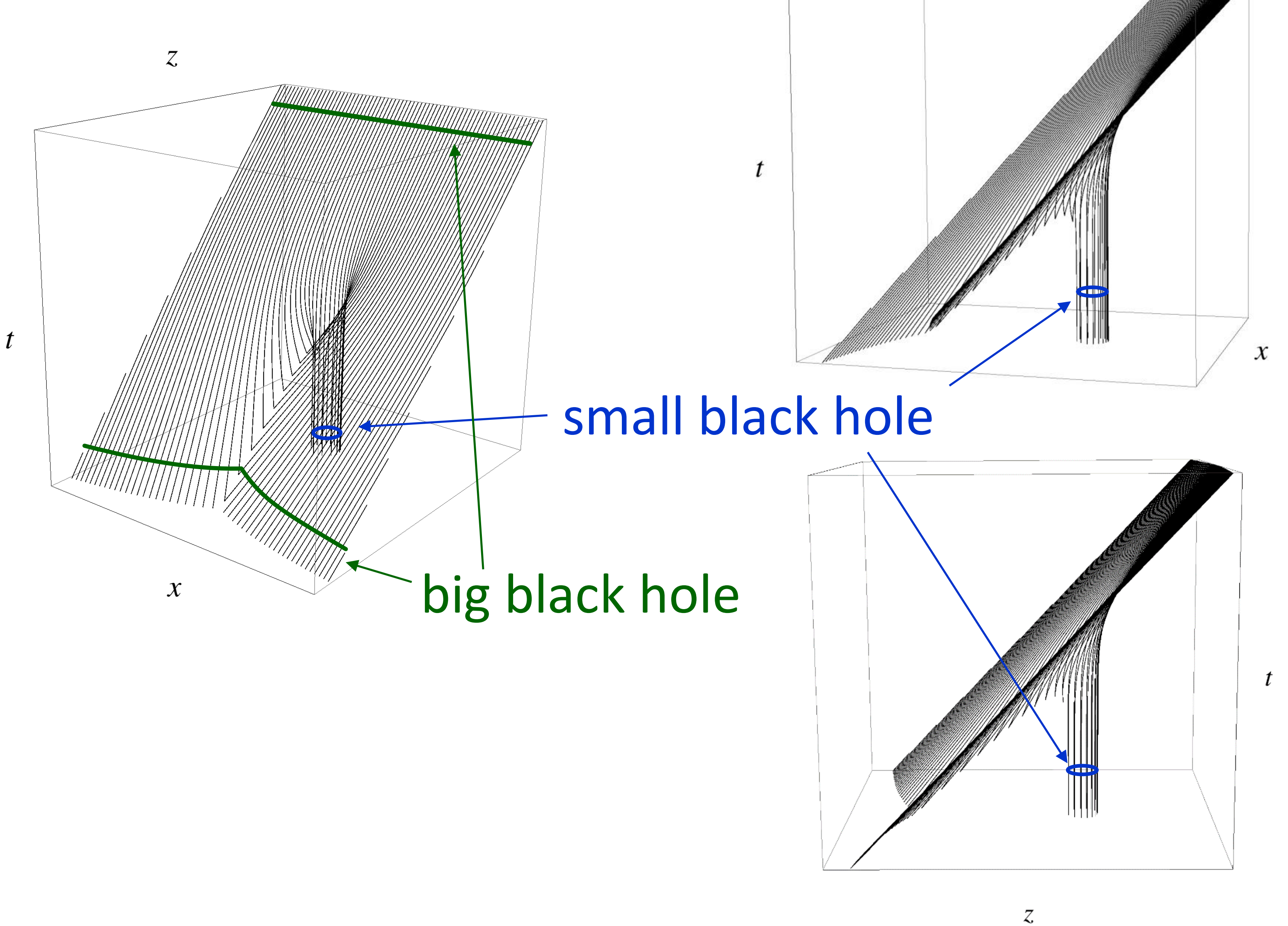
Null geodesics in Schwarzschild solution



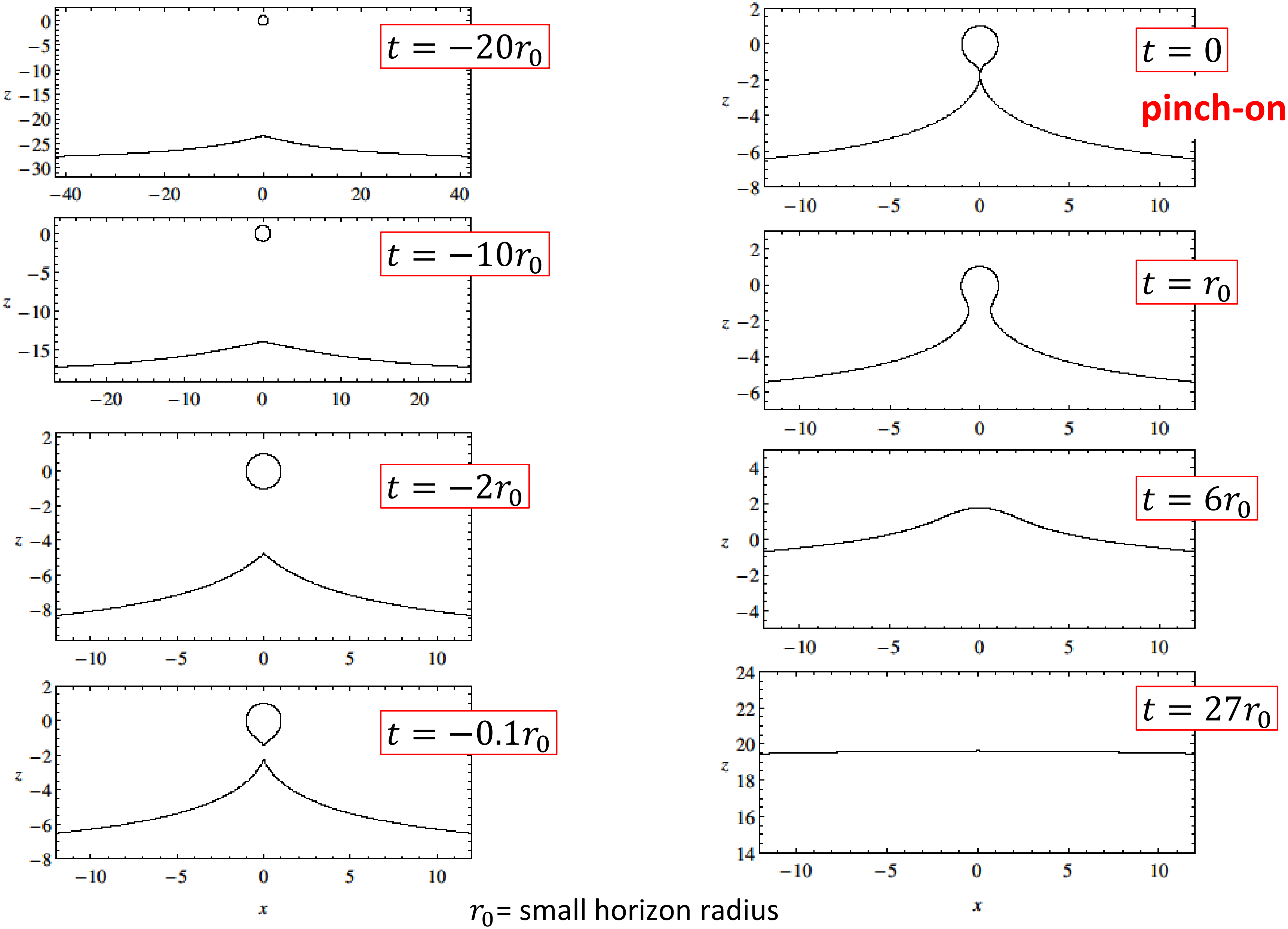
Null geodesics in Schwarzschild solution



“Pants” surface



Sequence of constant-time slices



r_0= small horizon radius

Preferred time-slicing

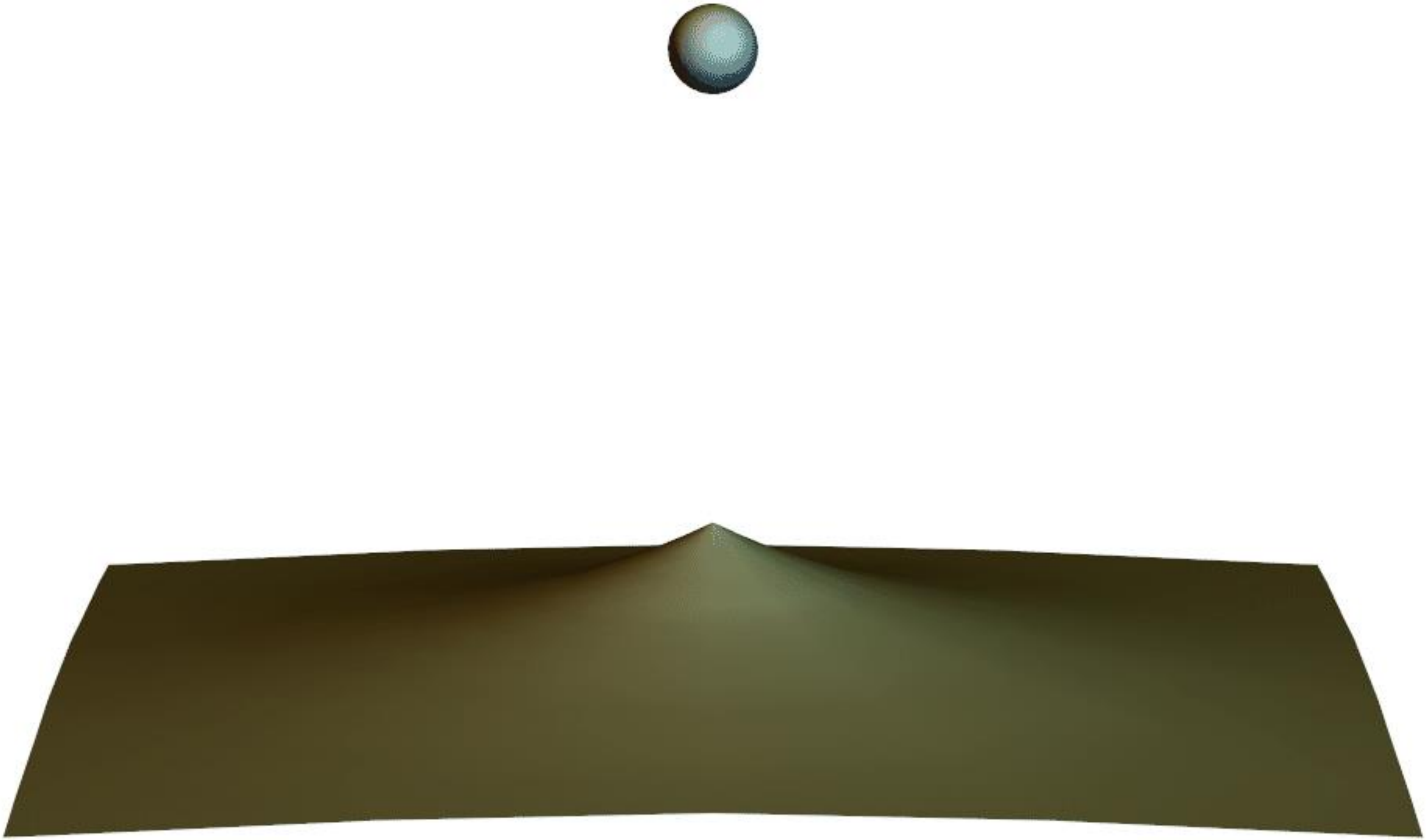
$\exists$ timelike Killing vector

Schwarzschild time

Rest-frame of small black hole

is well defined

made with *Mathematica* in a laptop computer



Complete characterization of merger

Precise quantitative results for:

Elongation of horizon at pinch

Duration of fusion

Line of caustics

Area increase

Critical behavior at pinch

Simple local model for pinch

The full monty

The ultimate description of

EMR mergers

Rotation and motion

Large black hole rotation

Relative motion in infall

Just a boost

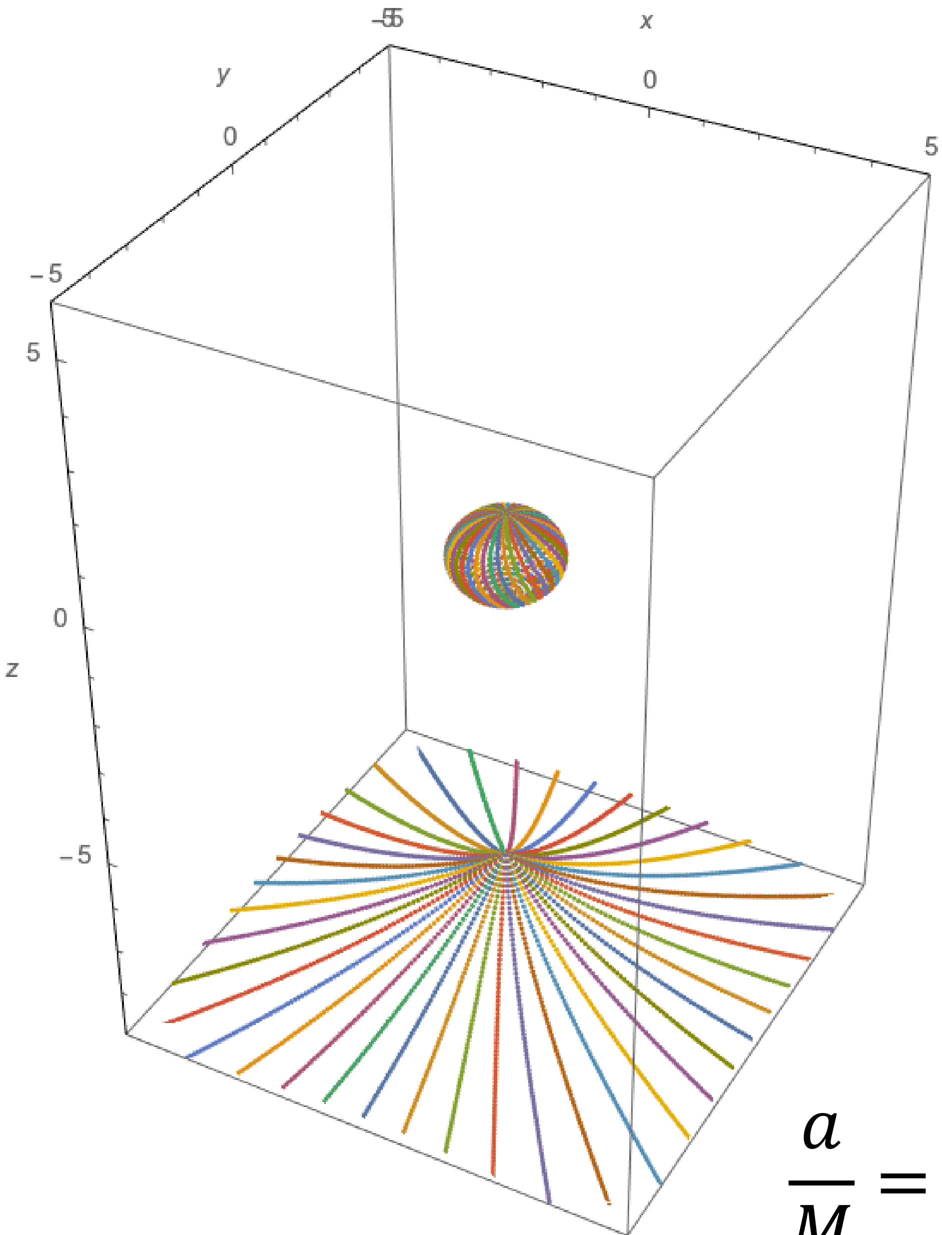
Equivalent to a rotation of the surface

Small black hole rotation

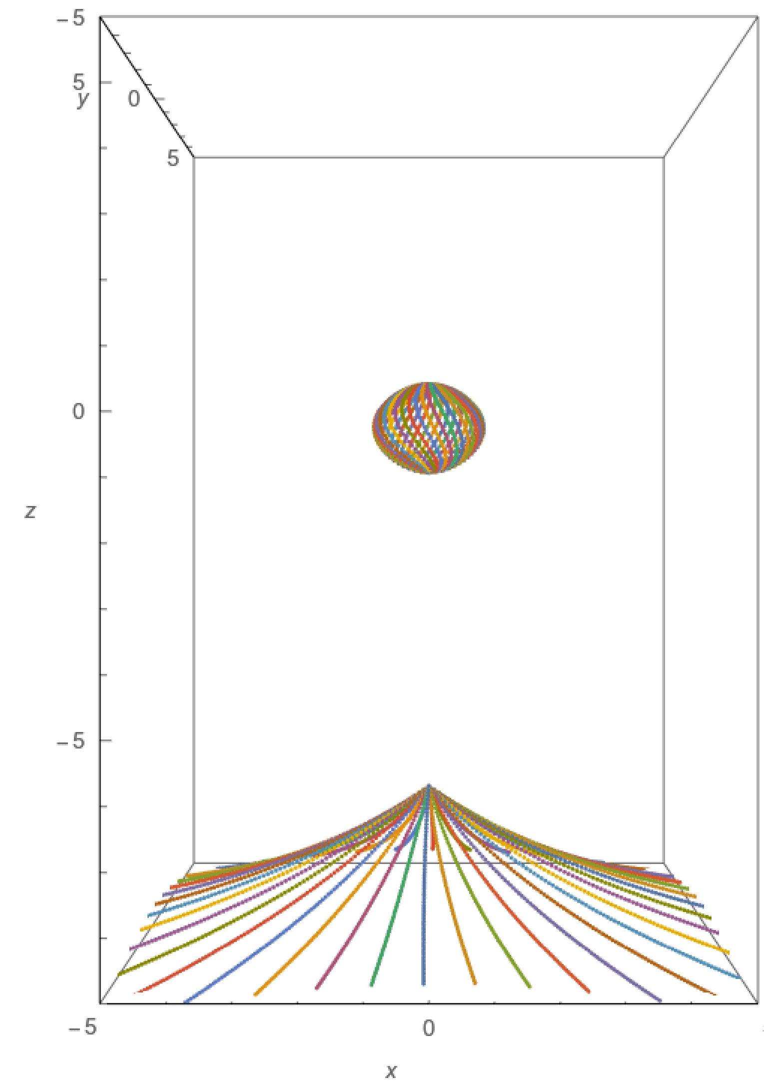
Change Schwarzschild → Kerr

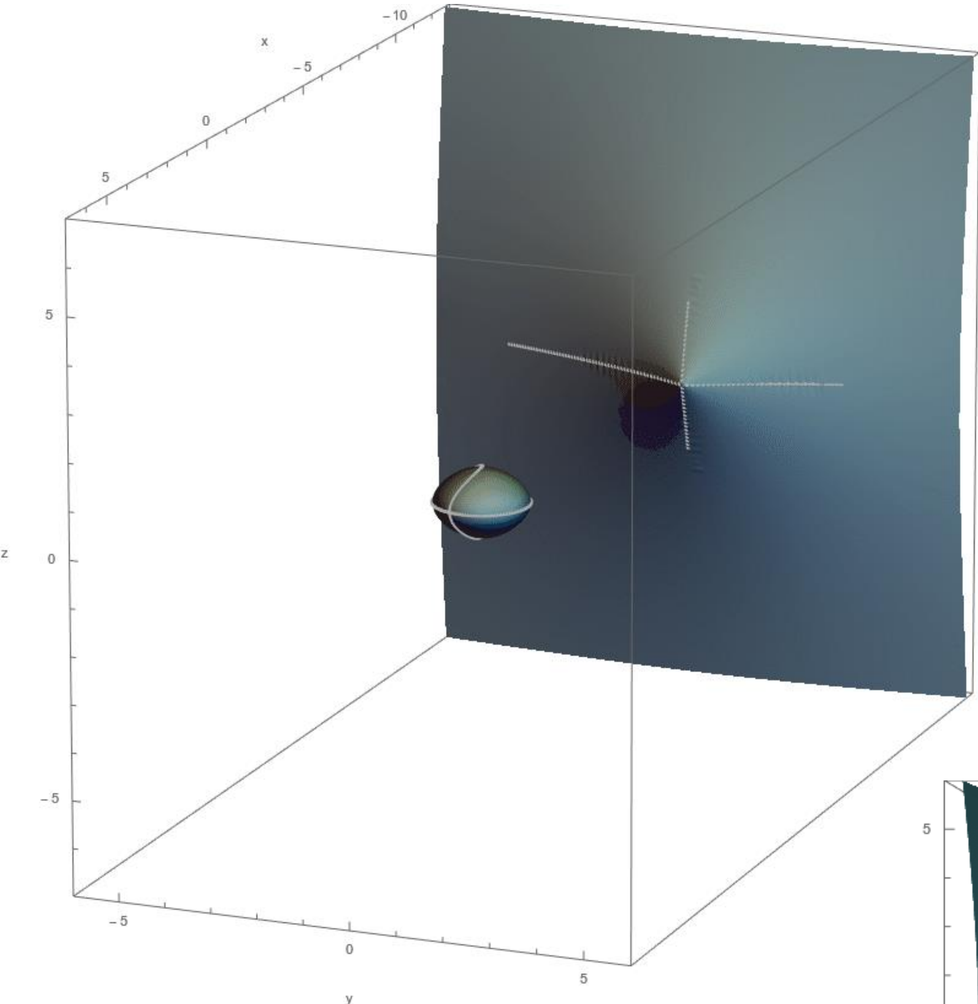
Fusion of **any EMR Black Hole binary in the Universe**

to leading order in $\frac{m}{M} \ll 1$



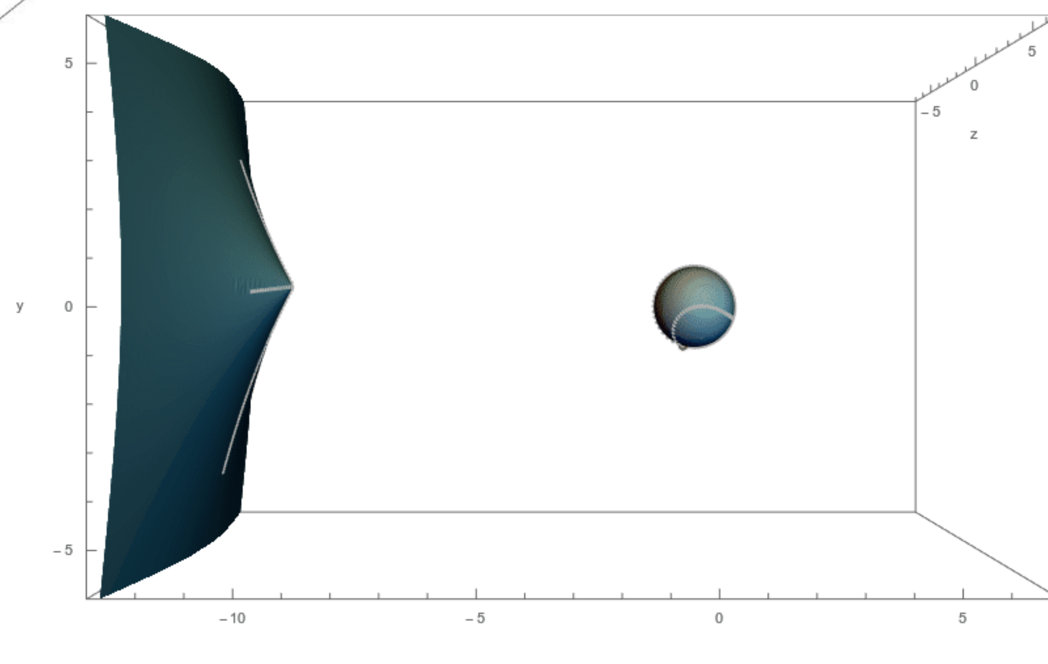
$$\frac{a}{M} = 0.8$$





$$\frac{a}{M} = 0.9$$

view from above



Final remarks

Simple, accurate, generic

description of a process that is

happening all over the Universe

Can we *observe* this?

Maybe not

Then, what is it good for?

Fusion of Black Hole Event Horizons
is a signature phenomenon of
General Relativity

Equivalence Principle allows to
capture and *understand* it easily in
a (realistic) limit

Exact construction

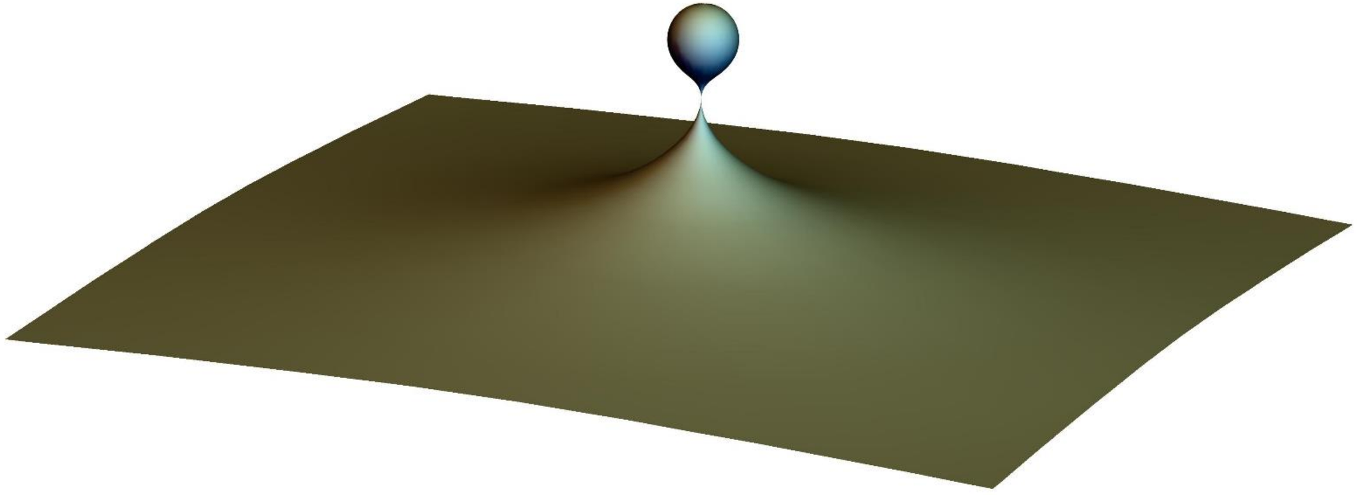
Benchmark for detailed numerical studies

First step in expansion in $\frac{m}{M} \ll 1$
to incorporate gravitational waves
(matched asymptotic expansion)

Equivalence Principle magic

Get 2 black holes

out of a geometry with only 1

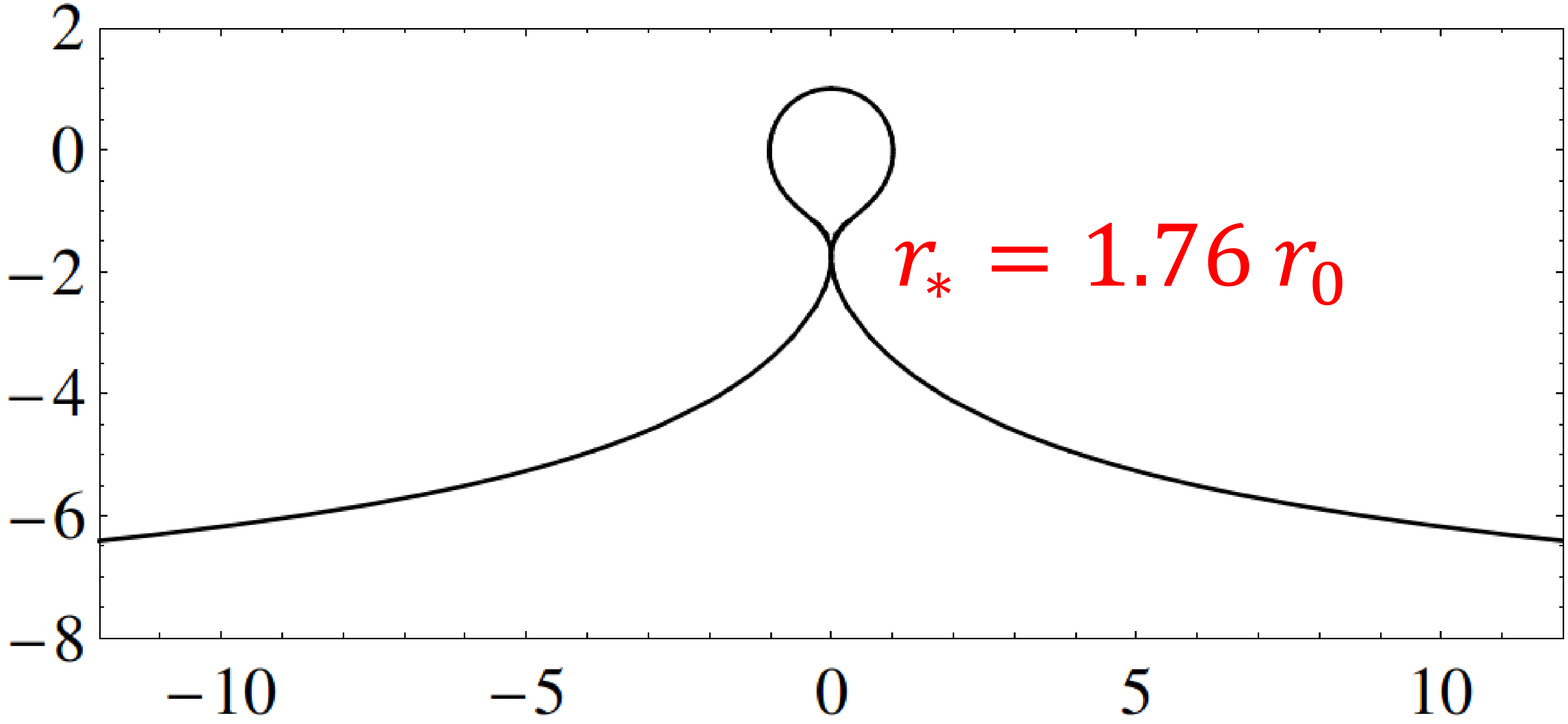


This could have been done (at least) 50 years ago!

Elongation at pinch-on

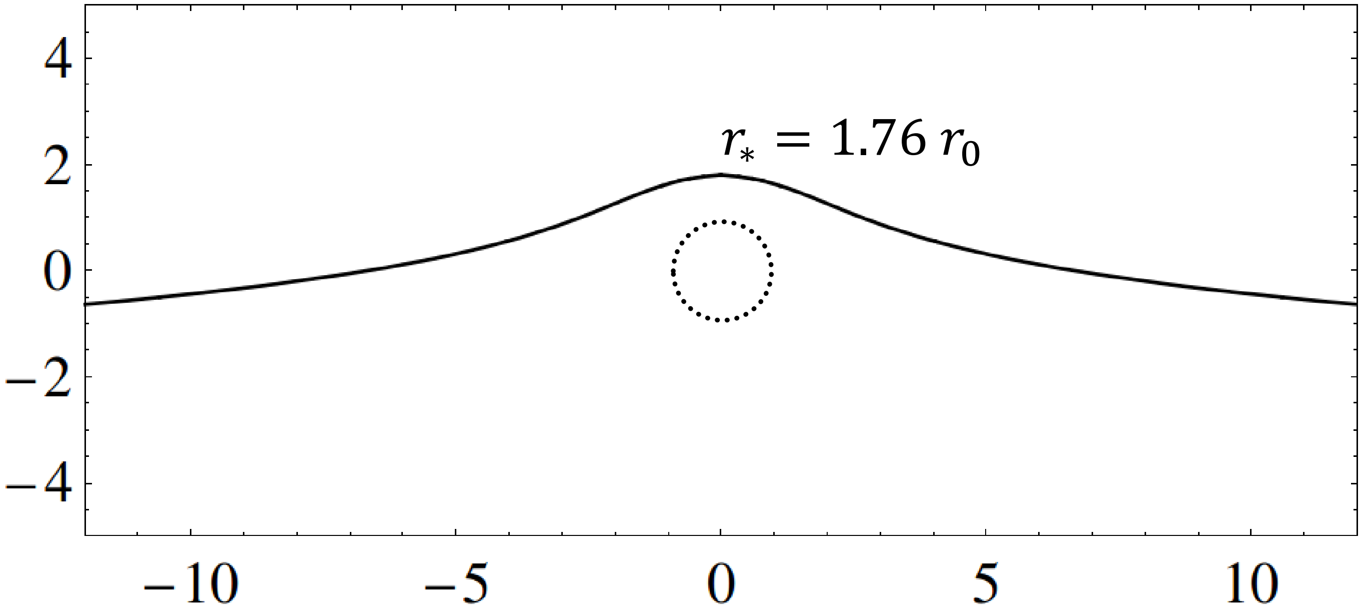
$r_0 = 2m =$ radius of small bh

$r_* =$ radius at pinch-on point



Duration of fusion

$$t = 5.95\, r_0$$



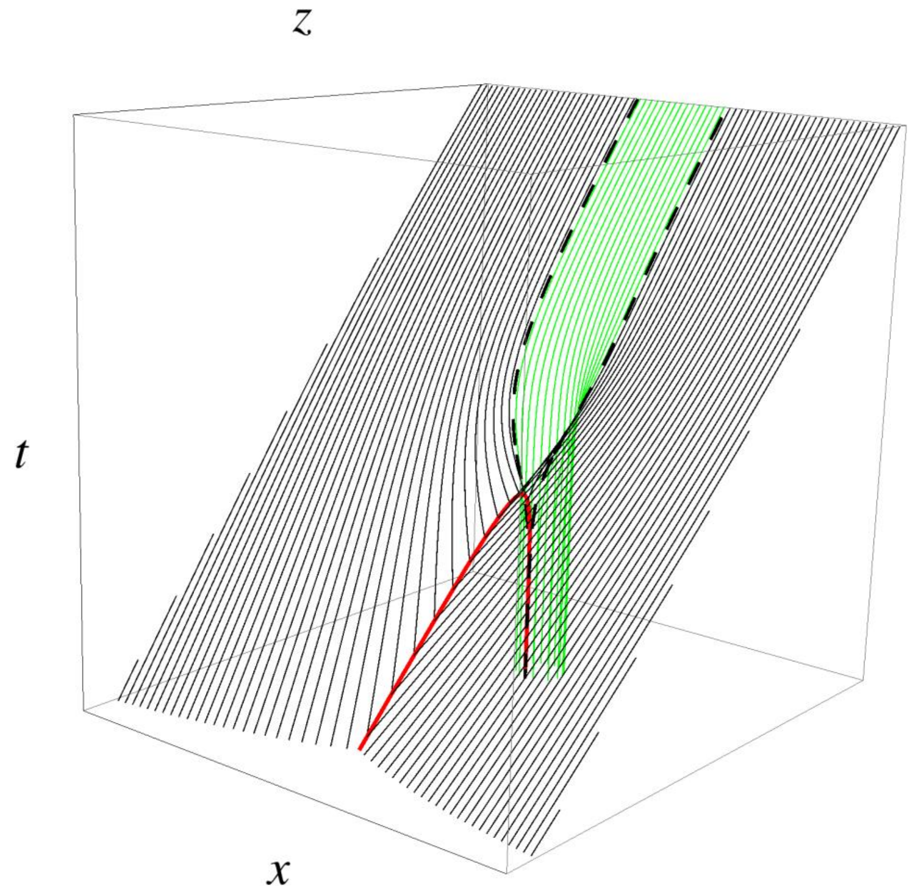
Area increase

From null-generator
expansion

$$\frac{\Delta A_{\mathrm{smallbh}}}{A_{\mathrm{smallbh}}}=0.242$$

with null-generator
addition

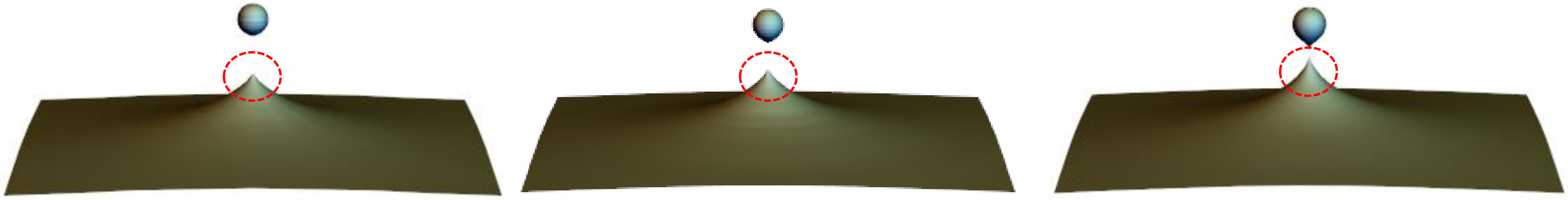
$$\frac{\Delta A_{\mathrm{smallbh}}}{A_{\mathrm{smallbh}}}=0.794$$



(total $\Delta A \simeq 32\pi Mm \to \infty$ from added generators)

Pinch-on: Criticality

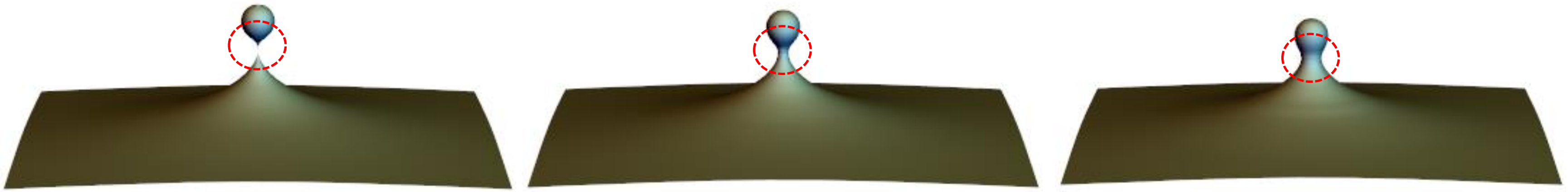
Opening angles of cones $\sim |t|^{1/2}$



∃ simple local model for pinch

Pinch-on: Criticality

Throat growth $\sim t$



$\exists$ simple local model for pinch