Gravitational Waves from Cosmic Strings

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What is a cosmic string?

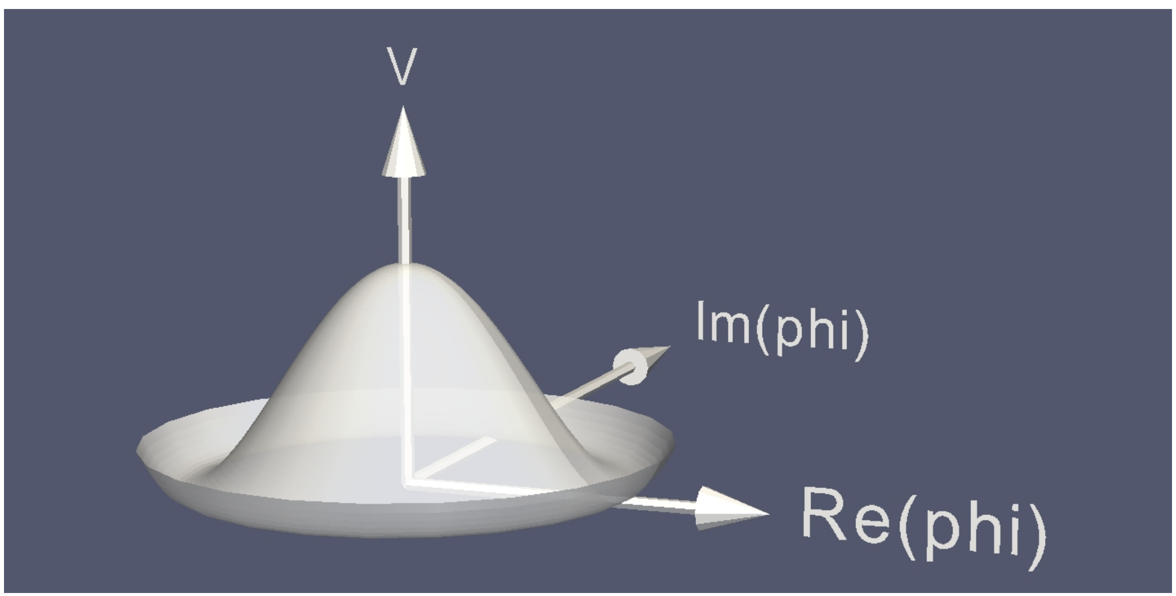
- Topological defects in field theory.
    - Relativistic version of Abrikosov vortices.
    - They exist in many extensions of the standard model.
- Cosmic Superstrings.
    - Fundamental Strings can be stretched to cosmological sizes and behave basically as classical objects.

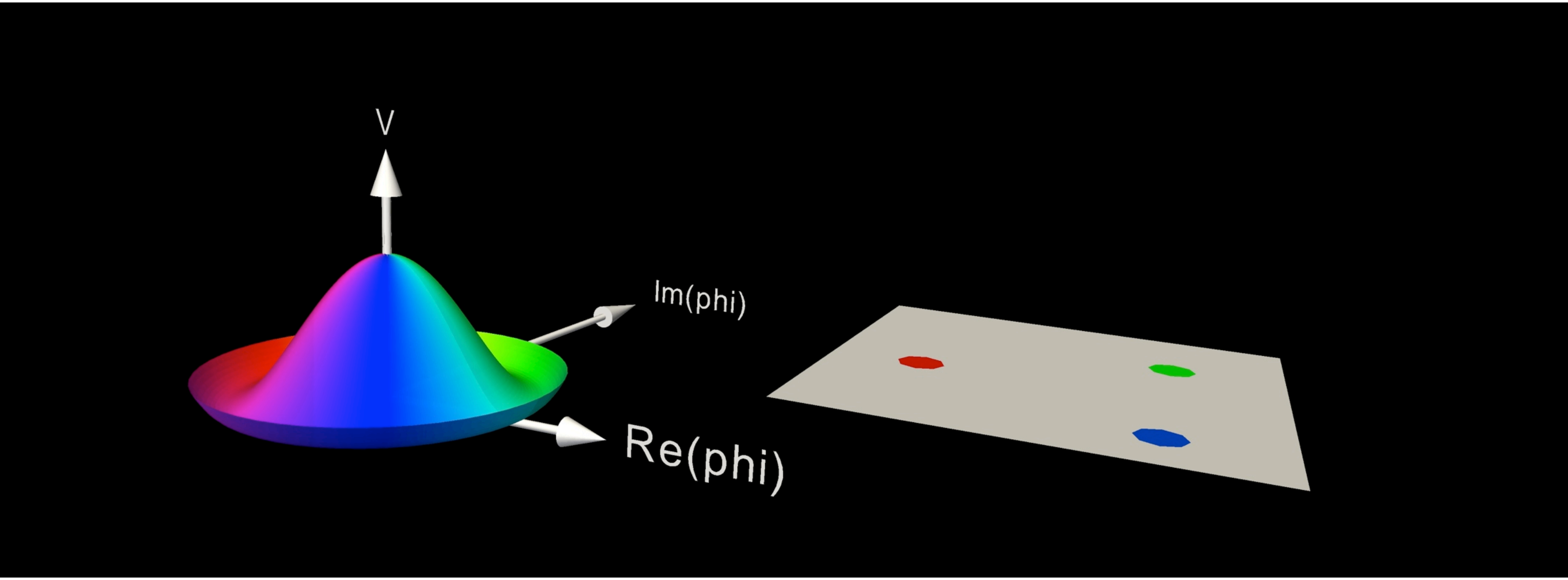
What is a cosmic string?

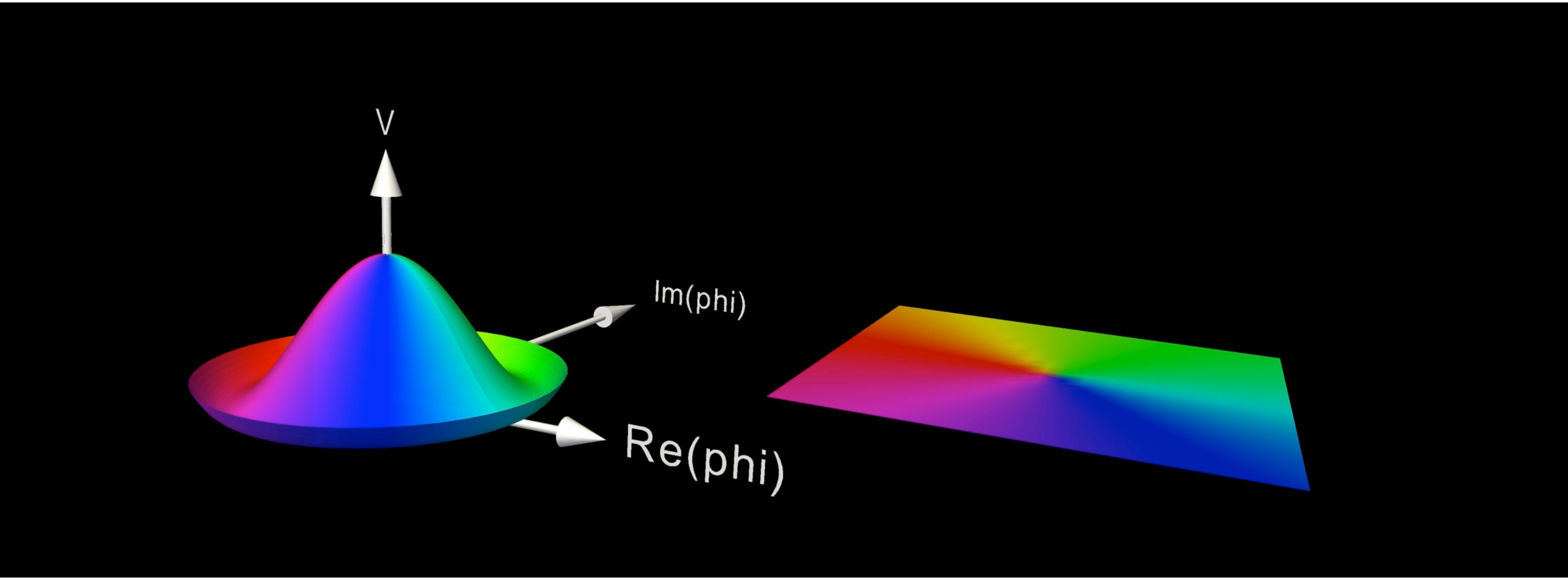
- Simplest model:

$$S_{U(1)} = \int d^4x \left[\partial_\mu \phi \partial^\mu \phi^* - V(|\phi|^2)\right]$$

$$V(\phi) = \frac{\lambda}{4}(|\phi|^2 - \eta^2)^2$$




V
Im(phi)
Re(phi)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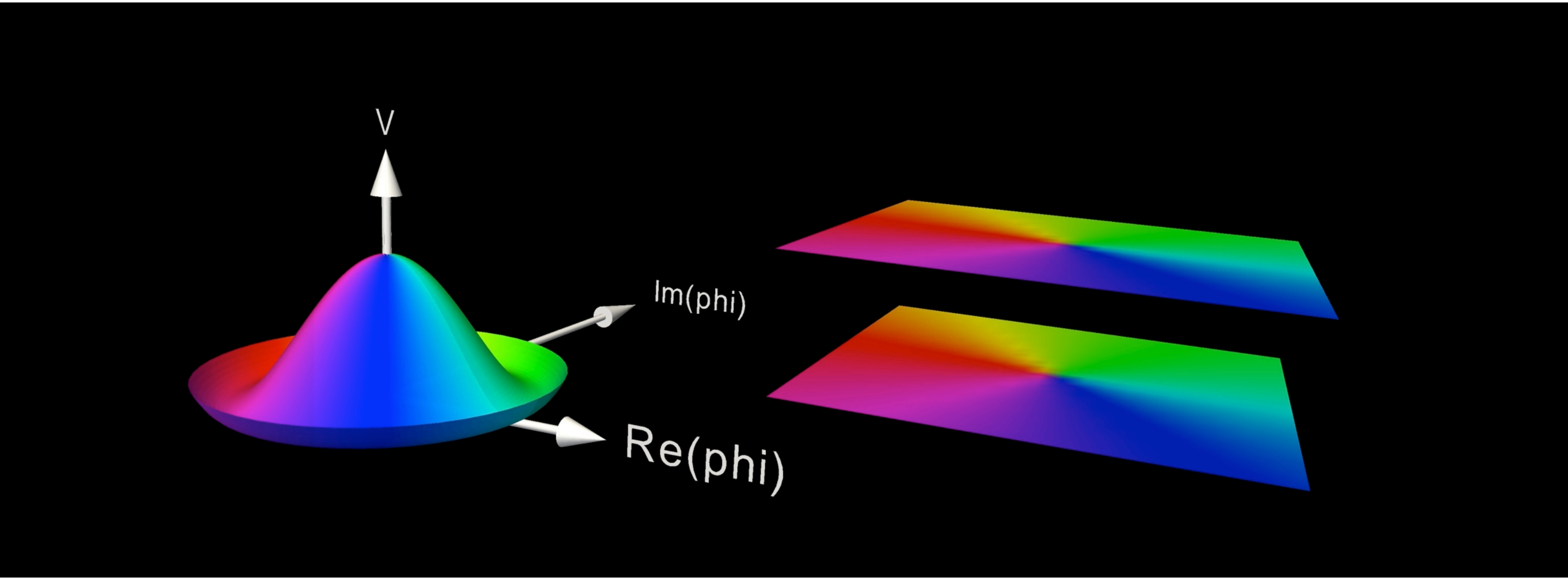
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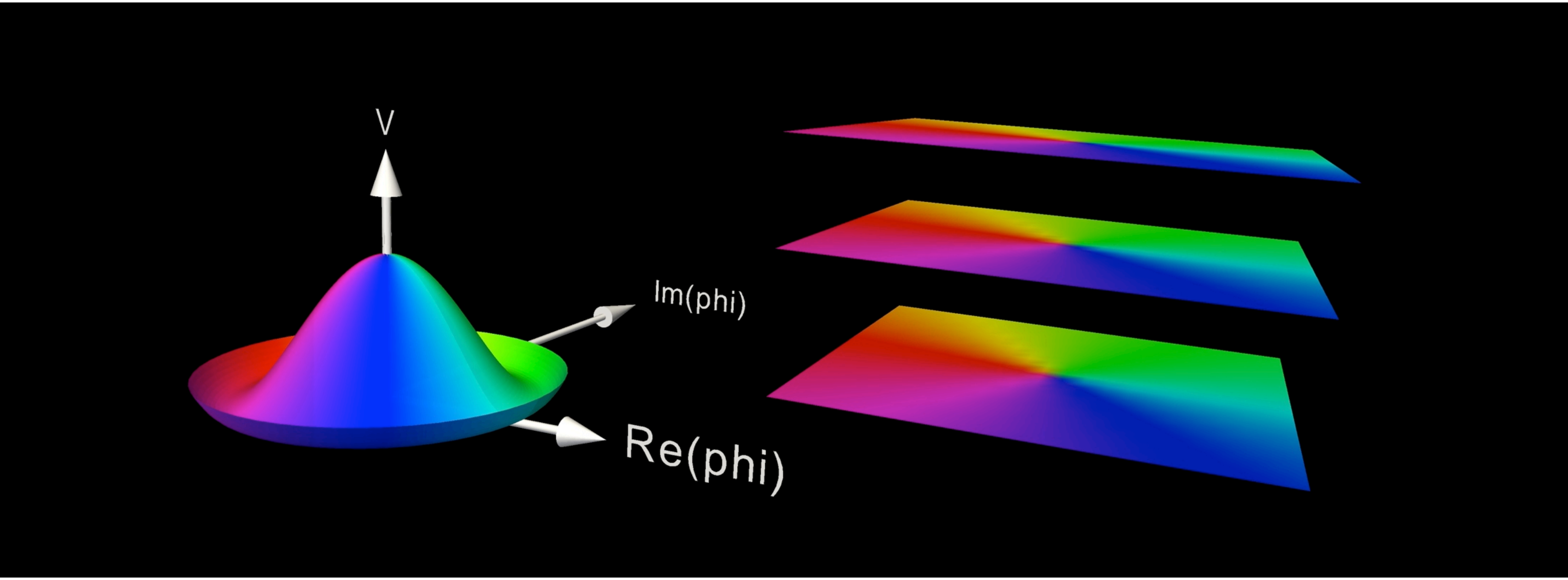
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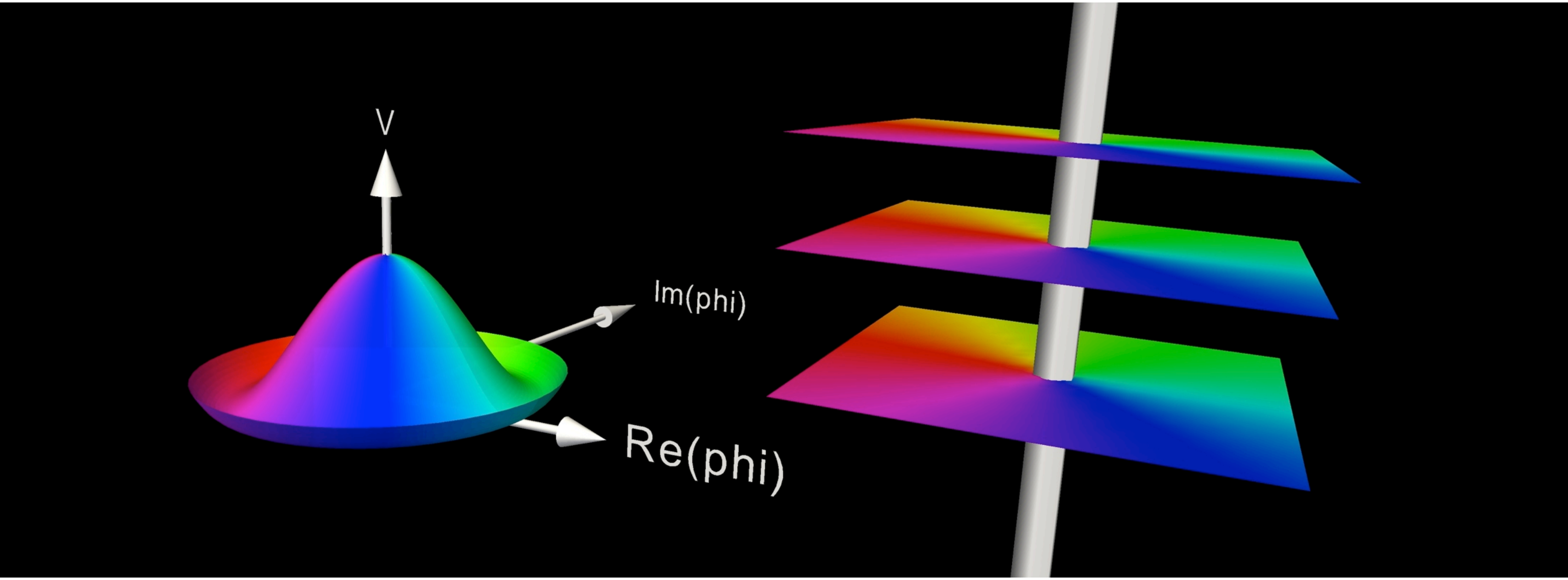

V
Im(phi)
Re(phi)

What is a cosmic string?

- Physical properties of the strings:

  - They are topological stable objects, they have no ends.

  - They are Lorentz invariant.

    Tension = Energy density per unit length

  - They are not coupled to any massless mode, except gravity.

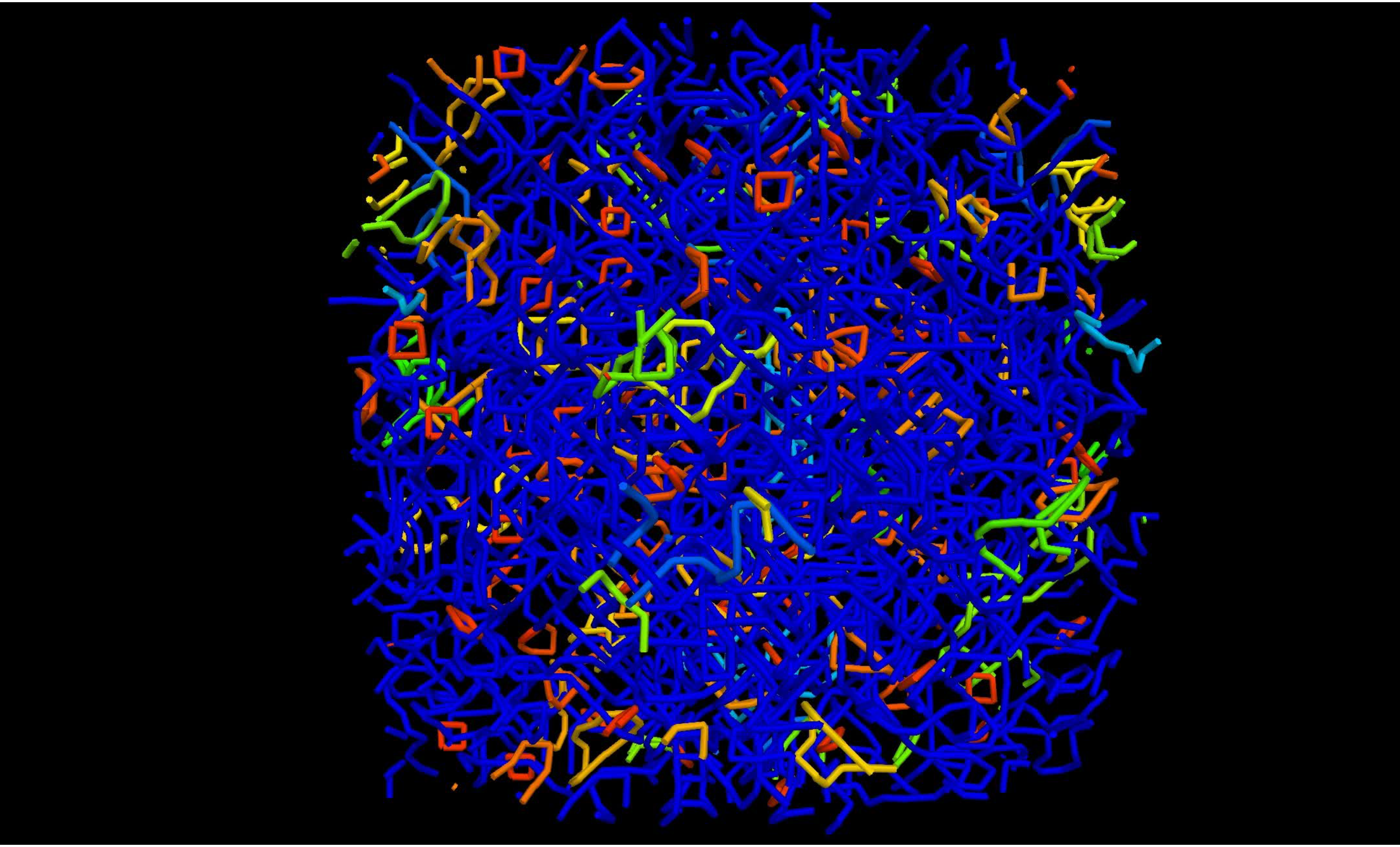
(This is the simplest version of strings that we will consider here)

The String Scale

- Thickness, energy density and tension of the string are controlled by the symmetry breaking scale. η
- For a Grand Unified Theory scale: $\eta \approx 10^{16}\mathrm{GeV}$
- Thickness: $\delta = 10^{-30}\mathrm{cm}$
- Linear mass density: $\mu = 10^{22}\mathrm{gr/cm}$
- Tension : $T = 10^{37} N$
- Gravitational effects depend on: $$G\mu = \left(\frac{\eta}{M_{Pl}}\right)^2 \sim 10^{-6}$$

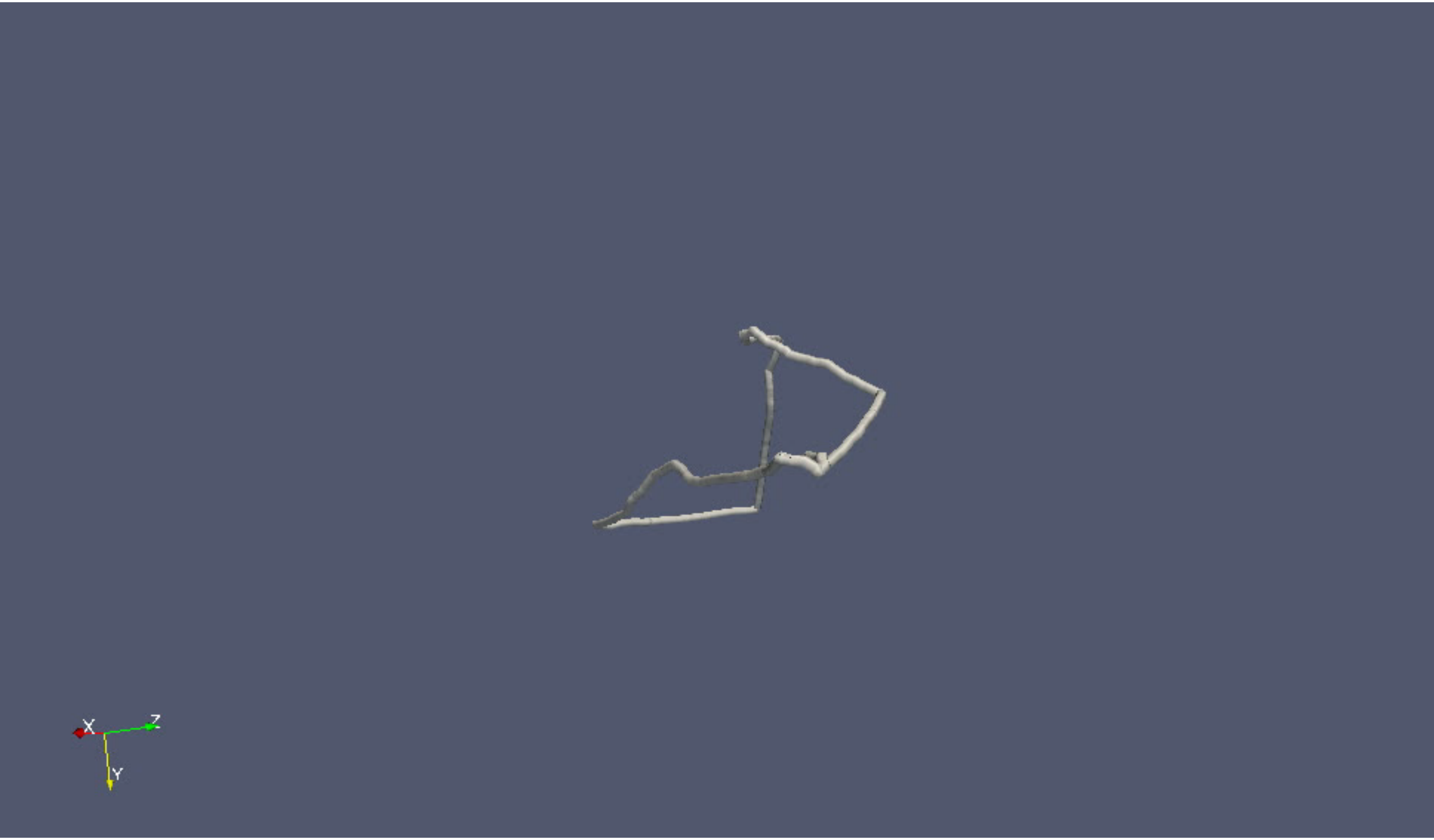
Nambu-Goto Cosmic String Networks

(B-P., Olum and Shlaer '12).



Loops from the Simulation

(B-P., Olum and Shlaer '12).



Gravitational Radiation by Loops

- The power of gravitational waves will affect the size of the loops:

$$\dot{M} \sim G(\dddot{Q})^2 \sim GM^2L^4w^6 \sim \Gamma G\mu^2$$

- The total power has been calculated with several sets of loops:

$$P \sim \Gamma G\mu^2 \qquad\qquad \Gamma \sim 50 - 100$$

- Loops will therefore shink is size or the rest mass of the loop will be:

$$m(t) \sim m(t') - \Gamma G\mu^2(t - t')$$

Gravitational Waves from the Network

- There are 2 different contributions to gravitational waves from a network of strings:

    - Stochastic background generated by all the modes in the loop.

      (Vilenkin ‘81, Hogan and Rees ’84, Caldwell et al. ‘92, Siemens et al.; Battye et al., Sanidas et al., Binetruy et al.; Blanco-Pillado et al. ‘13 and many more).

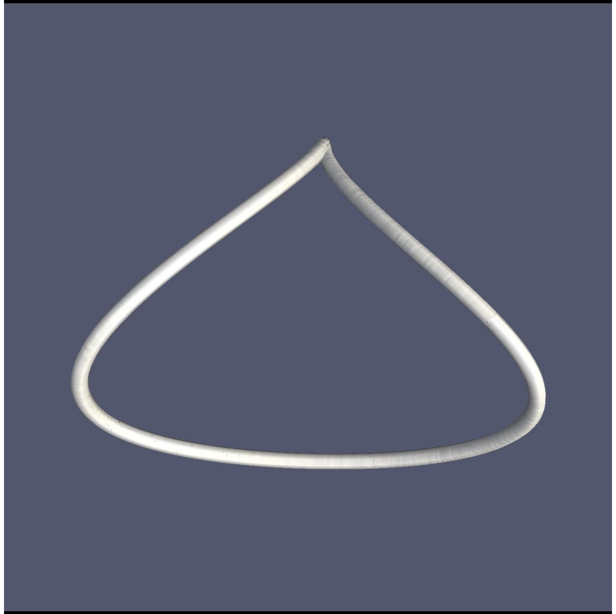
    - Burst signals from individual cusps.

      (Damour & Vilenkin ‘01).

Cosmic String Dynamics (Loops)

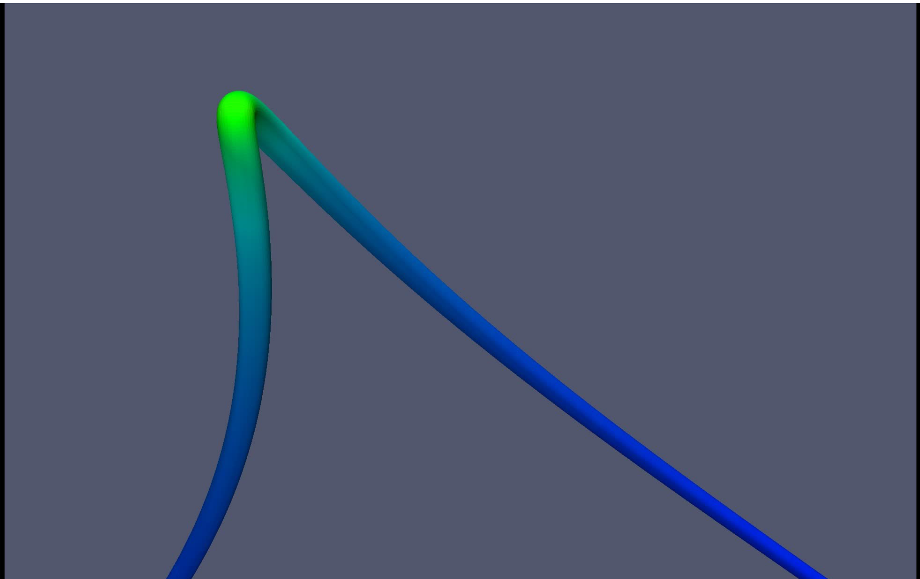
- The solutions for closed loops are periodic.
- The loops oscillate under their tension.
- The strings move typically relativistically.
- During its evolution a loop may have points where the string reaches the speed of light: A cusp

Cosmic String Cusps

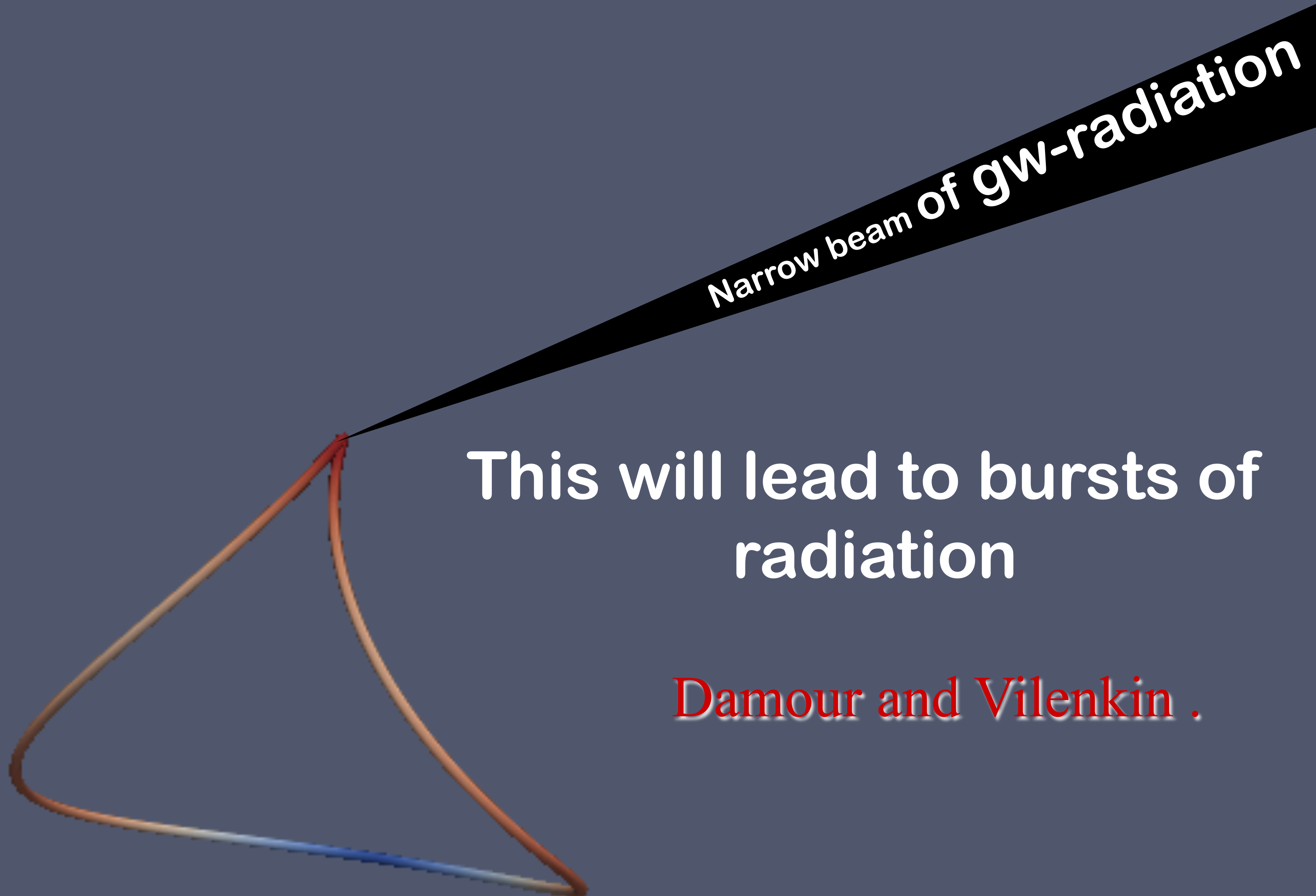


(Turok ‘84).

- Loops will typically have a cusp in each oscillation.
- The string doubles back on itself.



Gravity Waves from cusps



This will lead to bursts of radiation

Damour and Vilenkin .

Stochastic background of Gravitational Waves

- The whole network of strings contributes to the stochastic background of GW.

$$\Omega_{gw}(\ln f) = \frac{8\pi G}{3H_o^2} f \int_0^{t_0} dt \left(\frac{a(t)}{a(t_0)}\right)^3 \int_0^{m_{max}} dm\ n(t,m) \left(\frac{dP}{df}\right)$$

$n(t,m)$ ⟸ It depends directly on the number of loops.

$\left(\frac{dP}{df}\right)$ ⟸ It also depends on the spectrum of gw emission by the surviving loops.

$\left(\frac{a(t)}{a(t_0)}\right)^3$ ⟸ Cosmology

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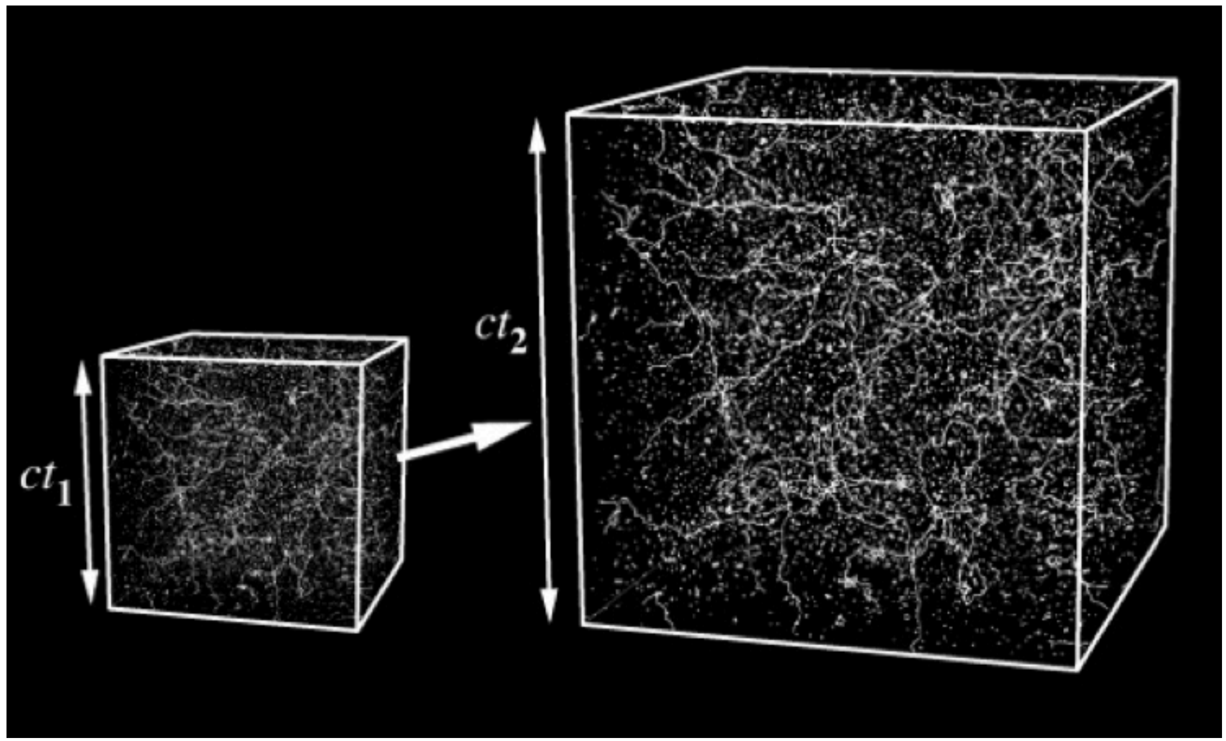
Cosmic String Networks

- As the string network evolves it reaches a scaling solution where the energy density of strings is a constant fraction of the energy density in the universe.

$$\frac{\rho_\infty}{\rho} = \text{constant}$$



- All statistical properties scale with the horizon distance.

The number of cosmic string loops

(B-P., Olum and Shlaer ‘13).

- We have been able to obtain from the simulations the scaling distribution of loops.
- At formation their are mostly formed at the size of a tenth of the horizon size.
- This allows us to calculate the loop distribution of sizes at any moment in the history of the universe:

$$\frac{n_r(t,l)}{a^3(t)} \approx \frac{0.18}{t^{3/2}(l+\Gamma\mu t)^{5/2}}$$

Gravitational Radiation by Loops

- Loops are periodic sources so they emit at specific frequencies

$$P = \sum_n P_n$$

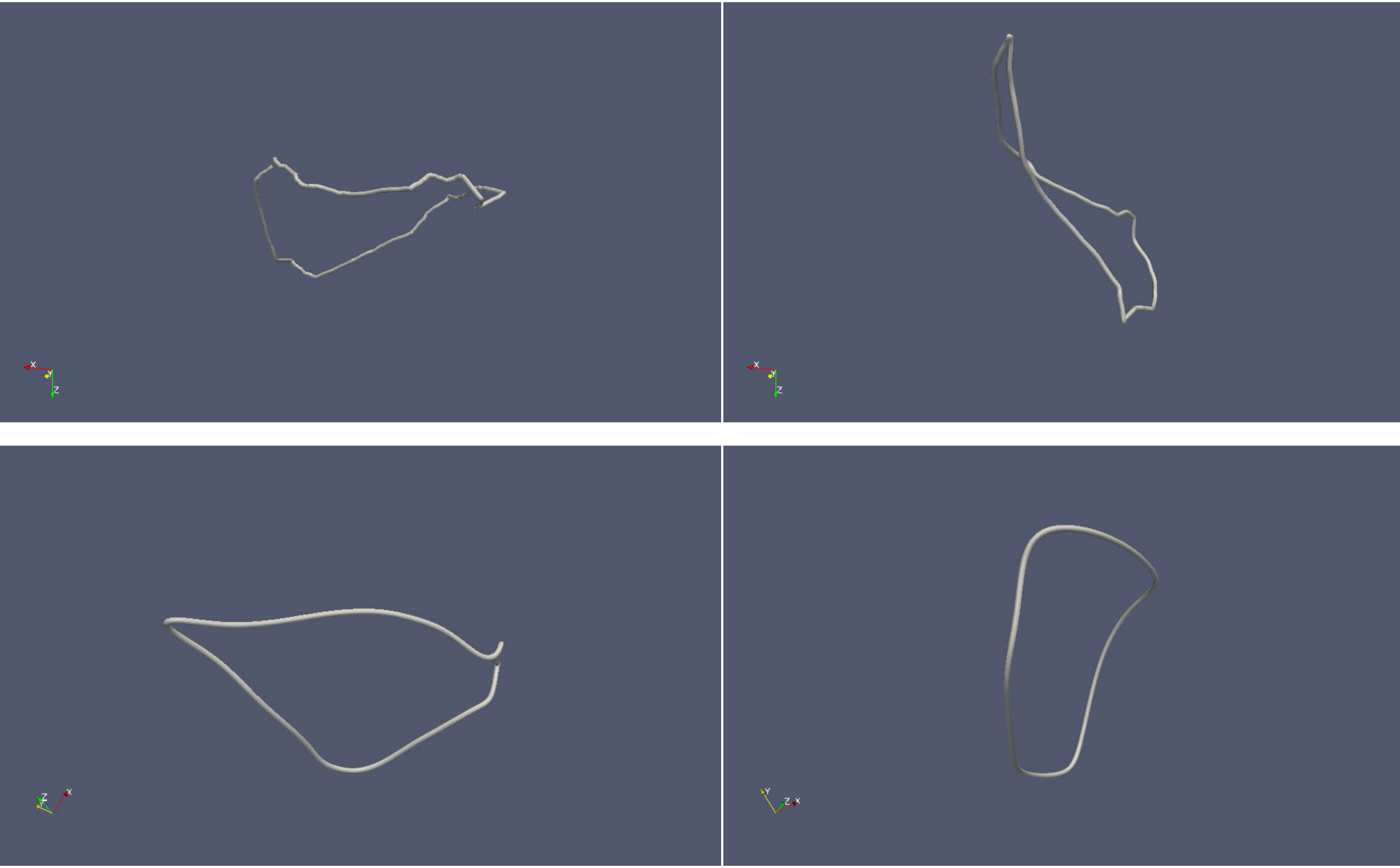
- We have to determine:

$$P_n$$

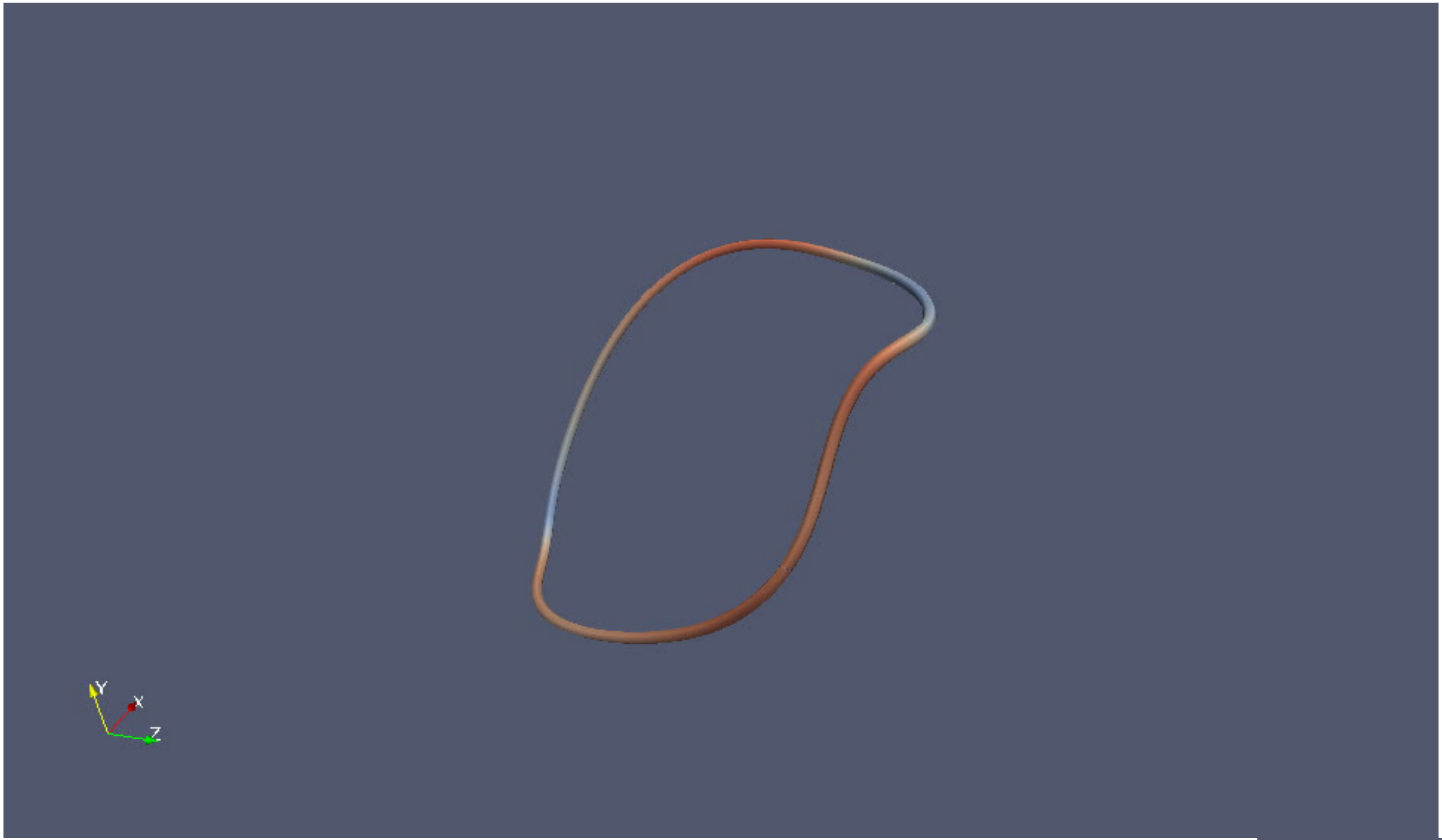
- The power of radiation does not depend on the size of the loop, only on its shape.

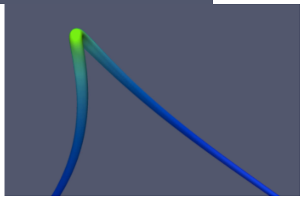
- Loops change their shape by emitting gw radiation.

Smoothing the loops



Smooth Loop

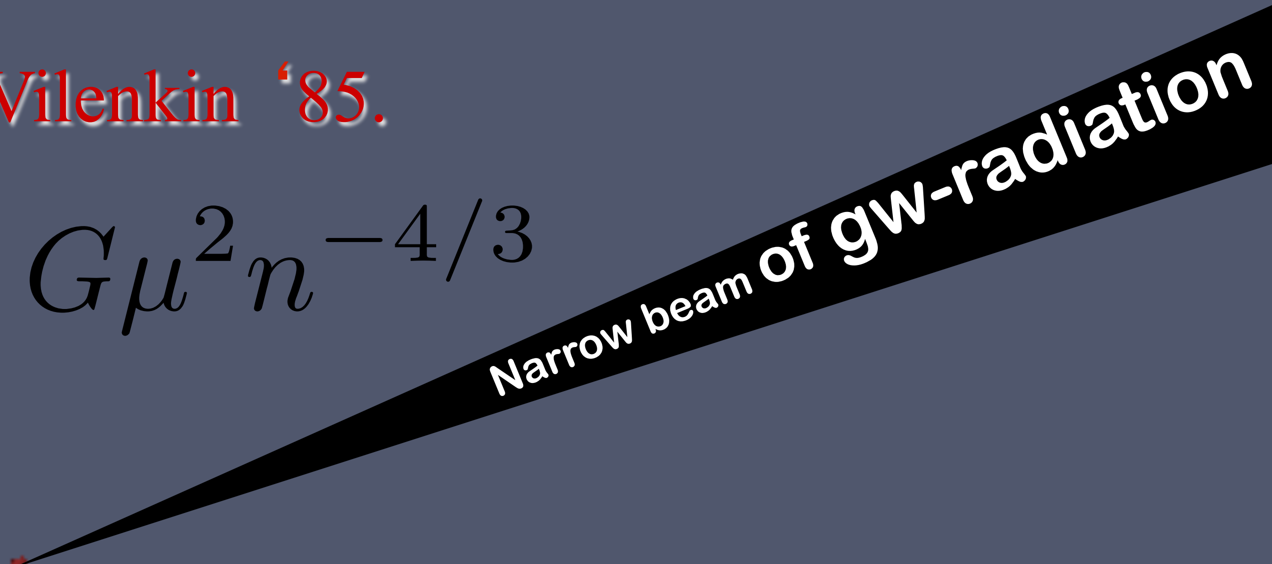


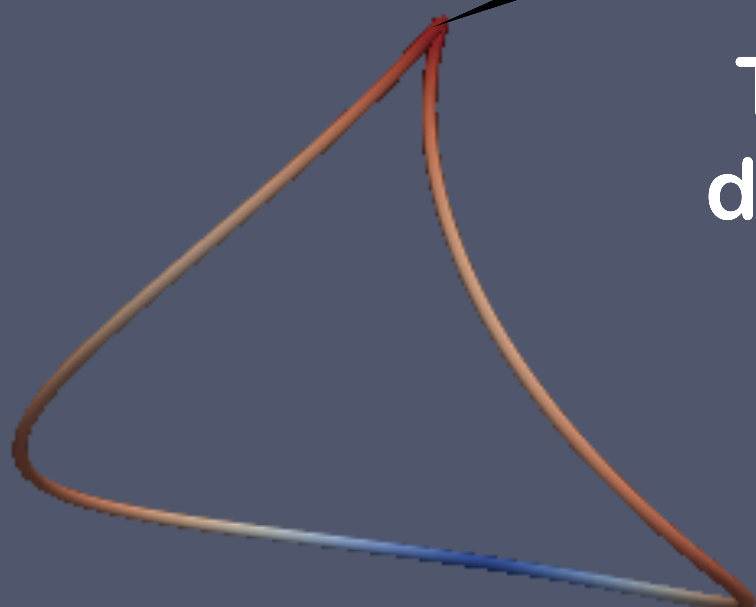


Gravity Waves from cusps

Vachaspati and Vilenkin ‘85.

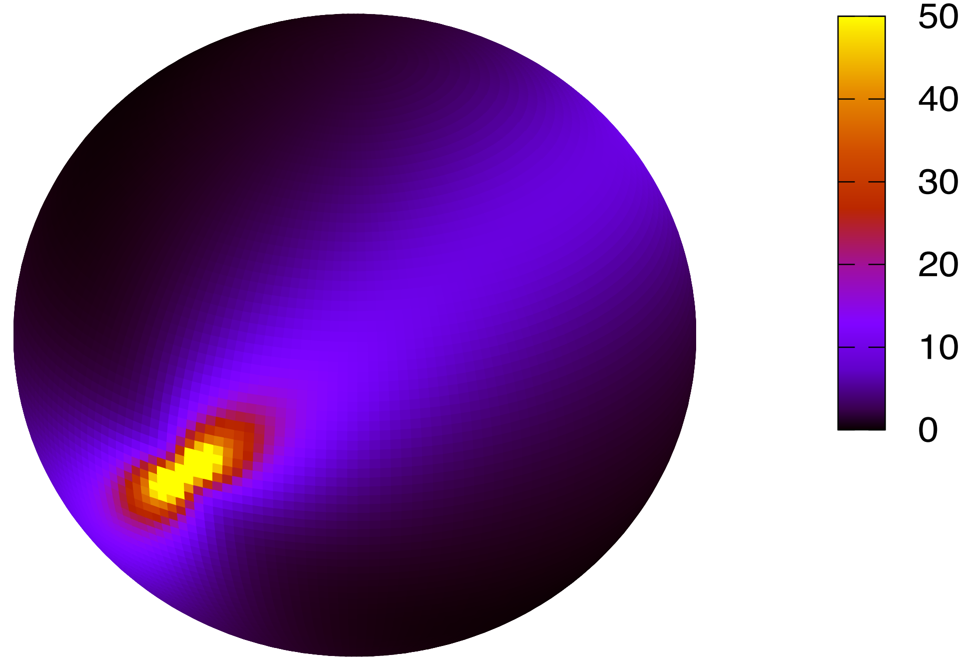
$$P_n^{cusps} \sim G\mu^2 n^{-4/3}$$





This leads to a slow decline of the power. High frequencies

Gravitational Radiation by Loops



- Angular distribution is typically highly anisotropic
- Difficult to compute.
- We obtain the average spectrum avering over more than 1000 loops.

$$P_n$$

Stochastic background of Gravitational Waves

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$$\Omega_{gw}(\ln f) = \frac{8\pi G}{3H_o^2} f \int_0^{t_0} dt \left(\frac{a(t)}{a(t_0)}\right)^3 \int_0^{m_{max}} dm \; n(t,m) \left(\frac{dP}{df}\right)$$

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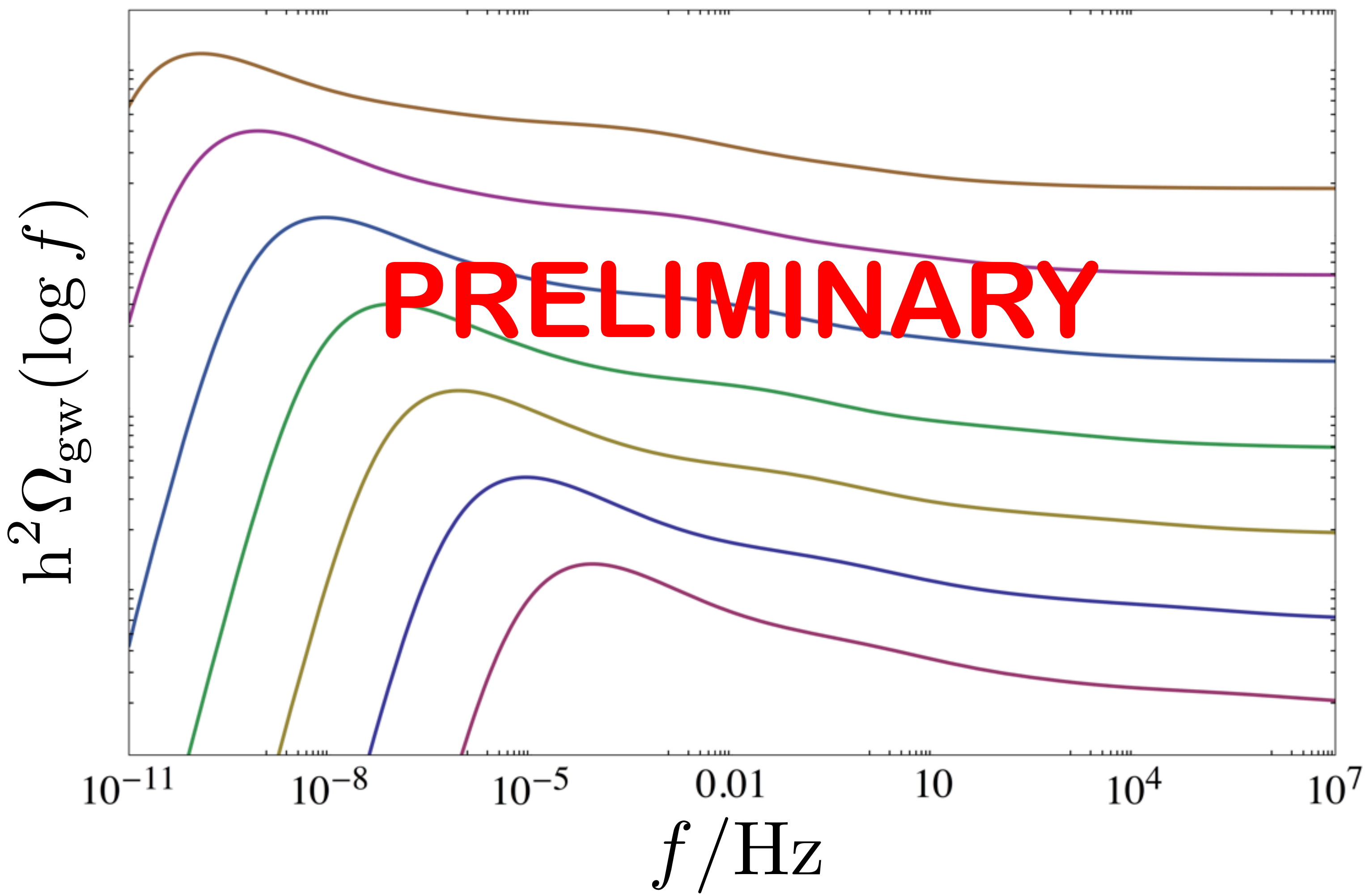

PPTA bound: $G\mu < 10^{-11}$

Conclusions

- We are entering an era of precision cosmology in cosmic string simulations.
- We have reached a consensus on the number and size of the important loops consistent with other NG simulations.
- We have computed the average gw power spectrum of loops from the simulation after they have been smooth by a (toy-model) backreaction.
- We can impose important constraints on the scale of the string from PTA.
- More data will constrain the models even more or detect cosmic strings!