IKERBASQUE & University of the Basque Country (Bilbao)

Smoking guns of a bounce in MTG through the spectrum of GWs

The Iberian Gravitational Wave Meeting 2017

Mariam Bouhmadi-López
in collaboration with J. Morais (EHU) and A. B. Henriques (IST-Portugal)
Phys.Rev. D87 (2013) 10, 103528 and work in progress

Bilbao 16th of May 2017

Index

Motivation

- In a homogeneous and isotropic universe filled with matter that fullfils the SEC, even the WEC (except for the case $p = -\rho = const.$), a Big Bang singularity is reached by tracing back the history of such a universe.
- A simple (or even simplistic) way of tackling the issue is by remplacing the Big Bang with a Bounce. Novello and Perez Bergliaffa 2008
- The disadvantage of a bounce in a (spatially flat, open or *closed*) FLRW universe is that a matter content violating NEC has to be invoked. Molina-París and Visser 1999
- However, we expect that gravity at those energy scale will be modified and/or quantum effect must arrise.
- Therefore, the bounce could be a consequence of a modification of gravity at those scales.
- We implement the previous idea within an $f(R)$ metric model.
- $f(R)$ theory is one of the simplest ways of modifiying gravity. In addition, (i) it includes Starobinsky inflationary model, which is seating at the sweet spot of the current observations aiming to constrain the inflationary cosmological zoo, and (ii) some $f(R)$ model can as well explain the current speed up of the universe.
- We will look for potential imprints of a bounce on the spectrum of the stochastic cosmological background of GWs

$f(R)$ Gravity

- The action:

$$S = \frac{1}{2\kappa^2}\int f(R)\sqrt{-g}\mathrm{d}^4x + S^{(m)}$$

- The equation of motion:

$$f_R G_{\mu\nu} = \kappa^2 T_{\mu\nu} + \text{geometrical} \quad \text{corrections}$$

- Friedmann equation:

$$H^2 = \frac{\kappa^2}{3f_R}\left(\rho - \frac{f(R)-f_R R}{2\kappa^2} - 3H\frac{f_{RR}\dot{R}}{\kappa^2}\right)$$

- Raychaudhuri equation:

$$2\dot{H} + 3H^2 = \frac{\kappa^2}{f_R}\left(p + \frac{f(R)-f_R R}{2\kappa^2} + \frac{f_{RRR}\dot{R}^2 + f_{RR}\ddot{R} + 2Hf_{RR}\dot{R}}{\kappa^2}\right)$$

- The reddish term could play the role of DE, inflation or (and why not?) induce **Bounce**

Capozziello and Faraoni 2009 and many mores ...

A kinematical reconstruction of a bouncing solution within $f(R)$-1-

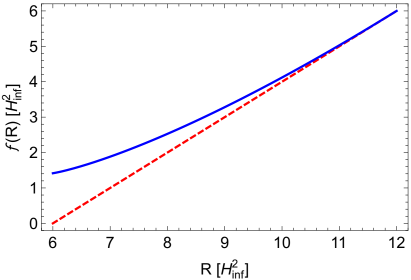
- We assume a bounce ($\dot{a}(t_b) = 0$ and $\ddot{a}(t_b) > 0$) and set $t_b = 0$:

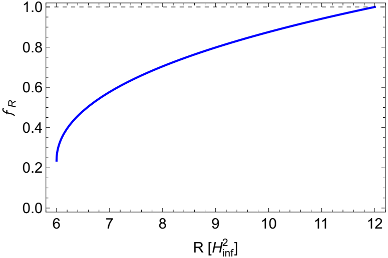
$$a(t) = a_b \cosh\left[H_{\rm inf} t\right]$$

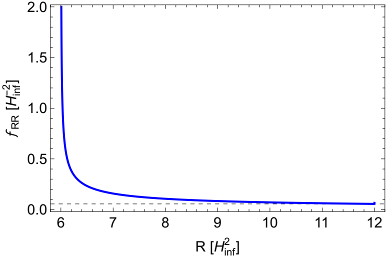
- For a spatially flat FLRW universe, this solution is asymptotically de Sitter, therefore, standard slow-roll inflation is guaranteed
- For the choseen scale factor, the modified Friedmann equation implies two solutions for $f(R)$
- Some physical criteria have to be imposed to choose an appropriate $f(R)$ function
  - We imposed $G^{(\rm eff)}$ is always positive
  - We imposed that asymptotically we recover the Hilbert-Einstein action, i.e. $f(R)$ is a linear function of R
- Under this condition we got a unique solution

$$f(R) = C\sqrt{R/H_{\rm inf}^2 - 3}\cos\left\{\frac{\sqrt{3}}{2}\left[\pi - \arccos\left(\frac{9 - R/H_{\rm inf}^2}{3}\right)\right]\arccos\left(\sqrt{\frac{3}{2}\frac{R/H_{\rm inf}^2 - 6}{R/H_{\rm inf}^2 - 3}}\right)\right\}$$

A kinematical reconstruction of a bouncing solution within $f(R)$-2-







GWs spectrum & Bogoliubov coefficients-1-

- The stochastic GWs spectrum in an expanding universe can be calculated using the method of the continuous Bogoliubov coefficients Parker 69, Starobinsky 79, Allen 88
- Tensor perturbations:

$$h_{ij}(\eta,\mathbf{x}) = \sqrt{\kappa}\sum_{p=1}^{2}\int \frac{d^3k}{(2\pi)^{3/2}a(\eta)\sqrt{2k}}[a_p(\mathbf{k},\eta)\varepsilon_{ij}(\mathbf{k},p)e^{i\mathbf{k}.\mathbf{x}}\xi(\mathbf{k},\eta) + herm.conj.],$$

where $k = \mid \mathbf{k} \mid = 2\pi a/\lambda = \omega a$; $p \rightarrow$ polarizations of the gravitational-waves, ε_{ij} is the polarization tensor, a_p the annihilation operator and ξ the mode function for the GWs

- The evolution of ξ depends on the gravitational theory
- Bogoliubov coefficient α and β determine the evolution of $a(\mathbf{k},\eta)$ from an initial creation and annihilation operators.

$$a(\mathbf{k},\eta) = \alpha(k,\eta)A(\mathbf{k}) + \beta^*(k,\eta)A^\dagger(\mathbf{k}),$$

where $|\alpha|^2 - |\beta|^2 = 1.$

- The graviton density is $|\beta|^2$

GWs spectrum & Bogoliubov coefficients-2-

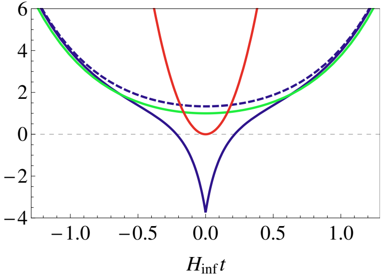
- The relative logarithmic energy spectrum of the GW's reads

$$\Omega_{\mathrm{GW}}(\omega,\eta_0)=\frac{\hbar\kappa^2}{3\pi^2c^5H^2(\eta_0)}\omega^4|\beta(\eta_0)|^2.$$

- The evolution of β depends on the gravitational theory
- It can be shown that $|\beta|^2=|X-Y|^2/4$ where $(z=a\sqrt{f_R})$

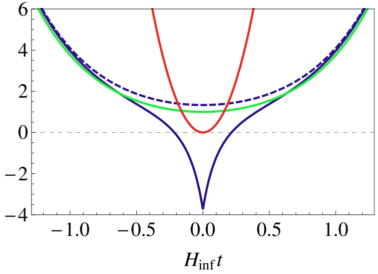
$$X'=-ikY,\quad Y'=-\frac{i}{k}\left(k^2-\frac{z''}{z}\right)X$$

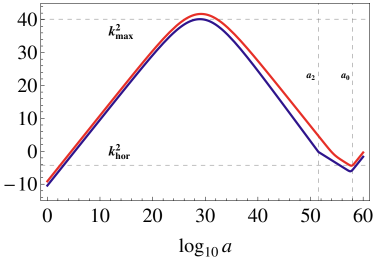
(i) the potential z''/z (continuous blue curve) and its asymptotic behaviour (discontinuous blue curve); (ii) the potential a''/a (green curve); (iii) the comoving wave-number k_H^2 (red curve); as functions of the cosmic time and near the bounce. Vertical axis in units of $a_b^2H_{\mathrm{inf}}^2$



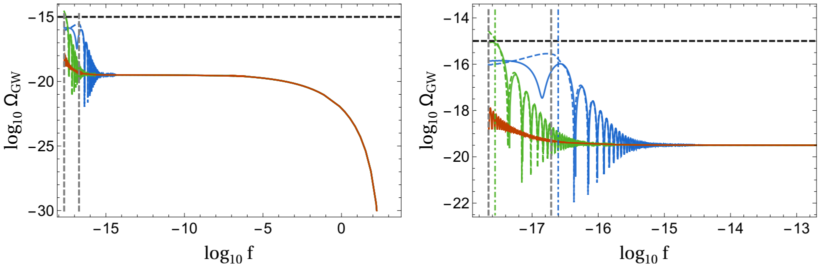
Initial conditions

- Within the asymptotic regime (a de Sitter-like expansion) exact solutions for X and Y can be found
- Those solution can be determined univocaly by taking into account that for large k they converge to a Bunch-Davies-like solution





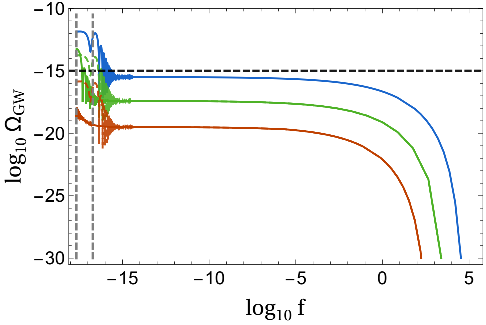
Effect of a_b



$E_{\rm inf} = 1.5 \times 10^{15}$ GeV, $a_{\rm ini} = 10 a_b$
$a_b = 100 \times 10^4$ (blue), 18×10^4 (green), 2×10^4 (brown)
Continuous (discontinuous) curve corresponds to $f(R)$ (GR)

$E_{\rm inf} \equiv \left(\frac{3}{\kappa^2} H_{\rm inf}^2\right)^{1/4}$

Effect of $H_{\rm inf}$ -1-

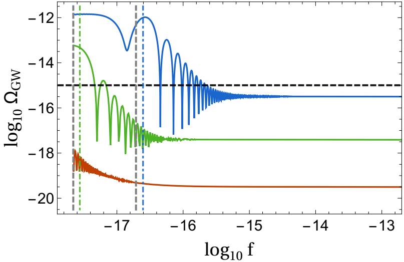


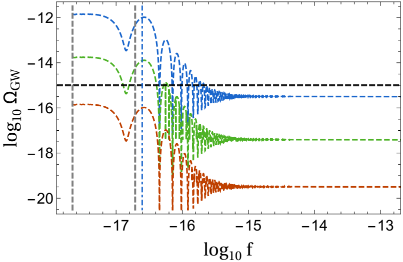
Blue: $E^{(1)}_{\rm inf} = 1.5 \times 10^{16}$ GeV, $a^{(1)}_b = 2 \times 10^4$, $a_{\rm ini} = 10a^{(1)}_b$

Green: $E^{(2)}_{\rm inf} = 0.5 \times 10^{16}$ GeV: (i) $a^{(1)}_b = 2 \times 10^4$, $a_{\rm ini} = 10a^{(1)}_b$ (continuous curve) (ii) $a^{(2)}_b = a^{(1)}_b H^{(1)}_{\rm inf}/H^{(2)}_{\rm inf}$, $a_{\rm ini} = 10a^{(2)}_b$ (discontinuous curve)

Brown: $E^{(3)}_{\rm inf} = 1.5 \times 10^{15}$ GeV: (i) $a^{(1)}_b = 2 \times 10^4$, $a_{\rm ini} = 10a^{(1)}_b$ (continuous curve) (ii) $a^{(3)}_b = a^{(1)}_b H^{(1)}_{\rm inf}/H^{(3)}_{\rm inf}$, $a_{\rm ini} = 10a^{(3)}_b$ (discontinuous curve)

Effect of $H_{\rm inf}$ -2-



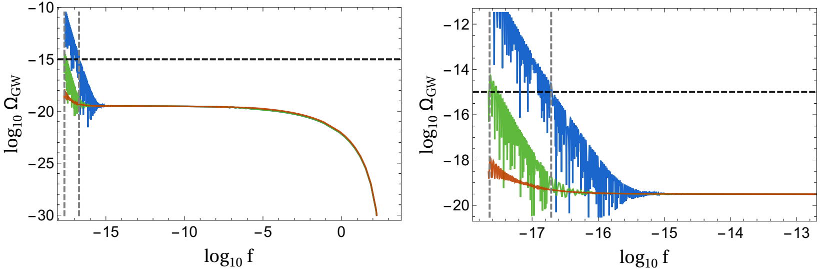


blue: $E^{(1)}_{\rm inf} = 1.5 \times 10^{16}$ GeV, $a^{(1)}_b = 2 \times 10^4$, $a_{\rm ini} = 10a^{(1)}_b$

green: $E^{(2)}_{\rm inf} = 0.5 \times 10^{16}$ GeV: (i) $a^{(1)}_b = 2 \times 10^4$, $a_{\rm ini} = 10a^{(1)}_b$ (continuous curve) (ii) $a^{(2)}_b = a^{(1)}_b H^{(1)}_{\rm inf}/H^{(2)}_{\rm inf}$, $a_{\rm ini} = 10a^{(2)}_b$ (discontinuous curve)

Brown: $E^{(3)}_{\rm inf} = 1.5 \times 10^{15}$ GeV: (i) $a^{(1)}_b = 2 \times 10^4$, $a_{\rm ini} = 10a^{(1)}_b$ (continuous curve) (ii) $a^{(3)}_b = a^{(1)}_b H^{(1)}_{\rm inf}/H^{(3)}_{\rm inf}$, $a_{\rm ini} = 10a^{(3)}_b$ (discontinuous curve)

Effect of $t_{\rm ini}$ or $a_{\rm ini}$



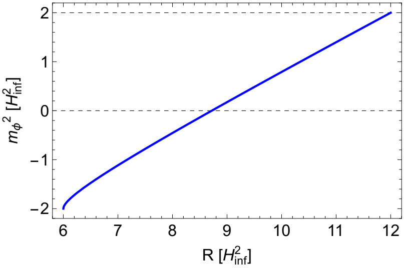
$E_{\rm inf} = 1.5 \times 10^{15}$ GeV, $a_b = 2 \times 10^4$
$a_{\rm ini} = 50a_b$ (blue), $20a_b$ (green), $10a_b$ (brown)

Conclusions-1-

- The presence of a bounce originates an oscillatory regime with various peaks appearing on the low frequency range of the energy spectrum of the GWs
- On the very low frequency ranges, the spectrum has characteristic imprints of $f(R)$ corrections at the perturbative level
- The position and shape of the peaks depend on the parameters of the model: a_b, $H_{\rm inf}$, $a_{\rm ini}$

Conclusions-2-

- Although promising, the model might present some scalar instabilities due to the scaleron mass



- A possibility to overcome this issue is to consider non-local theories of $f(R)$ Biswas, Conroy, Koshelev, Mazumdar 2014