Simple procedure for eccentricity reduction in Binary Black Hole Simulations

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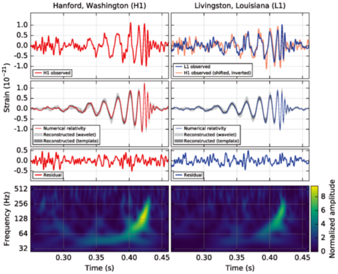
Contents

1. Introduction
2. A method to compute the eccentricity
3. On a simple procedure to reduce the eccentricity
4. Conclusions

Introduction. Gravitational Wave motivation



- In the nice talk of Alicia: both observations consistent with GW signal produced by the coalescence of two BHs.
- Both are compatible with low eccentricity systems.
- Most of the binary systems are expected to have circularized by the time their gravitational wave signals enter the frequency band of the LIGO and Virgo detectors.

Introduction. Framework

- Eccentricity Reduction $\rightarrow$ within the phenomenological waveform modelling efforts presented by Sascha, Geraint and Cecilio in their fine talks.
- BBH simulations are run with BAM code in order to get waveforms.
- Imperfections of NR initial data $\rightarrow$ waveforms with residual eccentricity.
- Main Objective: **Establish a systematic procedure to reduce eccentricity.**

Introduction. Framework

- We restrict this talk to non-precessing binaries.
- Bowen-York initial data are used in BAM code.
- Initial data given by: masses, positions and momenta of the two BHs.
- Given a case, defined by **fixed masses and positions**, we want to **develop a systematic procedure to change the momenta** to reduce the eccentricity.
- **Main Strategy**:
  1) Set up a procedure that computes the actual eccentricity.
  2) Compute analytically the correction factors of the momenta from PN formalism.
  3) Test the magnitude of the eccentricity reduction in NR simulations.

Introduction. Eccentricity estimators.

- Non-precessing binary with zero eccentricity $\rightarrow$ orbital variables vary monotonically.
- In numerical simulations, small eccentricity $\rightarrow$ small residual oscillations with amplitude proportional to the eccentricity are added monotonically changing orbital variables.
- The determination of these residual oscillations is needed to measure the eccentricity .
- A generic eccentricity estimator for the orbital frequency, $\varepsilon_\Omega(t)$ is:

$$\varepsilon_\Omega(t) = D(t) + e_\Omega \cos(\Omega_r t + \Phi) \quad (1)$$

- We define an eccentricity estimator of the form:

$$e_\Omega(t) = \frac{\Omega(t) - \Omega_{\text{fit}}}{2\Omega_{\text{fit}}} \quad (2)$$

- Election based on the nearly gauge-independence of the orbital frequency, Ω.

Results. Fitting model

- A fit to the data (NR or PN) is necessary to compute $e_\Omega(t)$. (*Mathematica NonlinearModelFit function*).
- We use an Ansatz based on the TaylorT3 approximant [Buonanno et al. Phys.Rev.D80,084043(2009)]

$$\begin{aligned} \theta &= [\eta | T_{merg} t_0 - t|/5]^{-1/8} \\ A &= \frac{a_1 \theta^3}{16\pi}\left(1 + a_2\theta^2 + a_3\theta^3 + a_5\theta^5\right) \\ ansatz &= A + a_6 \cos\left(\Omega_0 \omega_1 t + t_1\right) \end{aligned} \tag{3}$$

- T_{merg} is an scale of the merger time (for NR typically $\approx 3000M$).
- t_0, a_1, a_2, a_3, a_5,a_6, ω_1 and t_1 are unknown coefficients to fit.
- Ω_0 is the 3.5 PN orbital frequency for quasi-circular orbits.

Results. Fitting model

- The ansatz can be identified with the residual between the data and our model based on $3.5PN$ approximant.

$$\mathcal{R}(t) = \Omega(t) - \Omega_0 \tag{4}$$

- Once the model is fitted to data, e_Ω can be calculated from the amplitude of the oscillations,

$$e_\Omega = \frac{1}{2}\frac{a_6}{\Omega_0}, \tag{5}$$

- and the corresponding error,

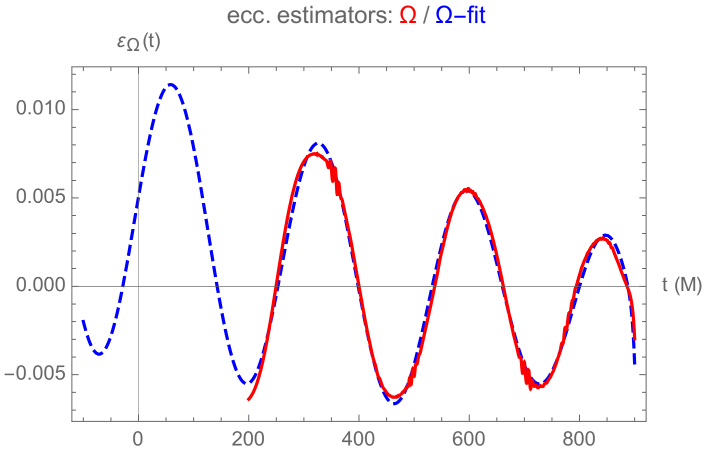
$$\delta e_\Omega = \frac{1}{2}\frac{\delta a_6}{\Omega_0}. \tag{6}$$

Results. Eccentricity from NR simulations

- Simulations were run in MareNostrum, CESGA and UIB clusters.

Case	$\lvert e_\Omega \rvert \times 10^{-3}$	$\delta e_\Omega \times 10^{-5}$
q4._-0.2_0.5_D40D11_T_96_408	7.62	1.09
q4._-0.5_0.5_D40D11_T_96_408	0.96	1.013
q4._-0.8_0.8_D40D11_T_96_384	8.00	1.06
q1.2_-0.8_0._D40D11.8633_T_96_432	5.5	7.5
q1.75_-0.55_0.5_D40D11.2383_T_96_432	1.35	1.409
q1.2_-0.5_-0.8_D40D12.2525_T_96_432	7.014	5.676
q1.2_0.8_0._D40D11.3399_T_96_432	4.097	3.888
q1.75_0.55_-0.5_D40D11.568_T_96_432	1.680	2.349
q1.5_0.85_-0.85_-0.17_T_64_240	2.796	2.211
q1.5_0.85_-0.85_-0.17_T_80_400	2.979	2.136
q2._0.75_0.75_D11.11_96	5.227	3.602
q2._-0.75_-0.75_D12.6_96	1.364	62.385
q1.2_-0.85_0.85_0.07_T_96_480	8.021	3.790
q1.5_-0.50_0.50_0.1_T_64_400	2.361	1.414
q2_0.85_0_0.283333_T_80_400_it1	2.437	43.065
q2_-0.85_0.85_0.283_T_80_440_it1	2.133	6.418

- **Case:** q1.2_-0.85_0.85_0.07_T_96_480



Results. On reducing the eccentricity

- Following [Pürrer et al. Phys.Rev.D85,124051(2012)] an iterative scheme to reduce the eccentricity is worked out.
- An adjustement of (p_r^0, p_t^0) required to reduce eccentricity of simulations.
- **Basic Idea**:
- **First iteration:**
  a) Modify factors (λ_t, λ_r) the initial values (p_r^0, p_t^0) such that:

$$e_M(\lambda_r p_r^0, \lambda_t p_t^0) = e_{NR}^0 \tag{7}$$

  b) In practice, eccentricity estimators are very noisy.
  Better to take the residuals. $\rightarrow \mathcal{R}_M^{\lambda}(t) \approx \mathcal{R}(t)$.

- **Second iteration:**
  a) Update the parameters of the numerical simulation and perform a second simulation using:

$$
\begin{aligned}
p_r^1 &= p_r^0/\lambda_r^0 \\
p_t^1 &= p_t^0/\lambda_t^0
\end{aligned}
\tag{8}
$$

  b) The result of the procedure is $e_{NR}^1 < e_{NR}^0$.

- **Next iterations:**
  a) Repeat the process iteratively updating the momenta:

$$
\begin{aligned}
p_r^{i+1} &= p_r^i/\lambda_r^i \\
p_t^{i+1} &= p_t^i/\lambda_t^i
\end{aligned}
\tag{9}
$$

Results. Limitations of the iterative procedure

- For eccentricity reduction purposes only **inspiral runs** are needed.
- The value of the eccentricity cannot be lowered beyond the error of the eccentricity estimator $\mathcal{O}\left(10^{-5}\right)$.
- The GW signal is usable only **after junk radiation** has passed at around $200M$.
- Computational cost of NR runs is high.
- PN approximation is accurate enough during inspiral and computationally cheaper.
- Using PN, undesired gauge and numerical effects are avoided.

- Solve numerically 3.5PN Hamilton equations.
- Estimate e_Ω and try to reduce it.
- Use the 1PN energy and angular momentum to derive a formula to compute λ_t.

$$\mathcal{E} = -\frac{M}{r} + \frac{P_r^2}{2\eta^2 M^2} + \frac{P_t^2}{2\eta^2 M^2} + \gamma \left[\frac{(3-9\eta)P_r^4}{8\eta^4 M^4} + \frac{(3-9\eta)P_r^2 P_t^2}{4\eta^4 M^4} \right.$$
$$\left. + \frac{3(1-3\eta)P_t^4}{8\eta^4 M^4} + \frac{M^2}{2r^2} + \frac{\frac{(2\eta+3)P_r^2}{2\eta^2 M} + \frac{(\eta+3)P_t^2}{2\eta^2 M}}{r} \right], \tag{10}$$

$$\mathcal{J} = \frac{rP_t}{\eta M} + \gamma \left[\frac{(1-3\eta)rP_t^3}{2\eta^3 M^3} + \frac{(1-3\eta)rP_r^2 P_t}{2\eta^3 M^3} + \frac{(\eta+3)P_t}{\eta} \right]$$

- $P_t = P_\phi / r$ is the tangential momentum and $\gamma = 1/c^2$.

- At 1PN and linear order in eccentricity, e_Ω can be written as:

$$e_\Omega(t) = \frac{\Omega(t) - \Omega(0)}{2\Omega(0)} = e_t \frac{1 + e_\phi / e_t}{2} \tag{11}$$

- The eccentricities can be written in terms of $\mathcal{E}$ and $\mathcal{J}$ as:

$$\begin{aligned} e_\phi^2 &= 1 + \frac{2\mathcal{E}\left(\gamma\mathcal{E}\left(\frac{\eta}{2} - \frac{15}{2}\right) + 1\right)\left(\mathcal{J}^2 - 6\gamma M^2\right)}{M^2} \\ e_t^2 &= 1 + \frac{2\mathcal{E}\left(\gamma\mathcal{E}\left(\frac{17}{2} - \frac{7\eta}{2}\right) + 1\right)\left(\gamma(2 - 2\eta)M^2 + \mathcal{J}^2\right)}{M^2} \end{aligned} \tag{12}$$

Results. Testing iterative procedure using PN

- Combining the previous formulae e_Ω can be written in terms of P_r and P_t:

$$e_\Omega(t) = \frac{1}{4\eta^4 r\sqrt{\eta^4 - r^2 P_t^2\left(P_r^2+P_t^2\right)+2\eta^2 r P_t^2}}\left[-12\gamma(\eta-2)\eta^6 \right.$$
$$+ r^3 P_t^2\left(P_r^2+P_t^2\right)\left[-4\eta^2+5\gamma\eta\left(P_r^2+P_t^2\right)+13\gamma\left(P_r^2+P_t^2\right)\right] \quad (13)$$
$$+ 2\eta^2 r^2 P_t^2\left[4\eta^2+\gamma\eta\left(P_r^2+2P_t^2\right)-40\gamma\left(P_r^2+P_t^2\right)\right]$$
$$\left. + 2\eta^4 r\left(2\eta^2-6\gamma P_r^2+\gamma\eta\left(3P_r^2-4P_t^2\right)+38\gamma P_t^2\right)\right]$$

- Then we can write $P_r = \lambda_r P_0^r$ and $P_t = \lambda_t P_0^t$, where P_0^t and P_0^r denote initial values for the momenta.

Results. Testing iterative procedure using PN

- We can deduce a formulae for $\lambda_t(e_\Omega, P_r, P_t)$ setting $\lambda_r = 1$, and using QC initial values,

$$P_r^0 = 0, \qquad P_t^0 = \eta M \sqrt{M/r_0}, \tag{14}$$

- Newtonian linear eccentricity order:

$$\lambda_t^{\text{Newt}} = 1 + \text{sign}\frac{|e_\Omega|}{2}, \tag{15}$$

- 1PN linear eccentricity order:

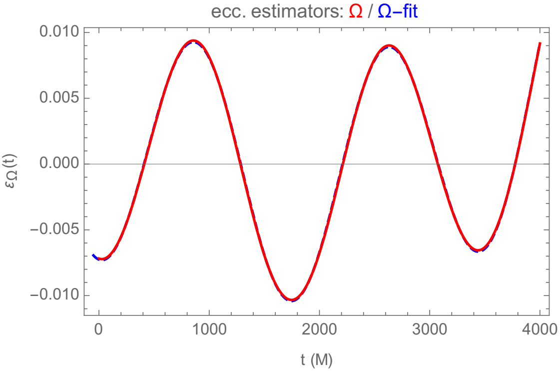
$$\lambda_t^{\text{1PNLin}} = 1 + \text{sign}\frac{|e_\Omega|}{2\left[1 + \frac{\eta+13}{r_0}\right]}, \tag{16}$$

- 1PN quadratic eccentricity (e_t and e_ϕ) order:

$$\lambda_t^{\text{1PNQuad}} = \sqrt{1 + \text{sign}\frac{4r_0\,|e_\Omega|}{21\eta + 4r_0 + 4}}, \tag{17}$$

Results. Testing iterative procedure using PN

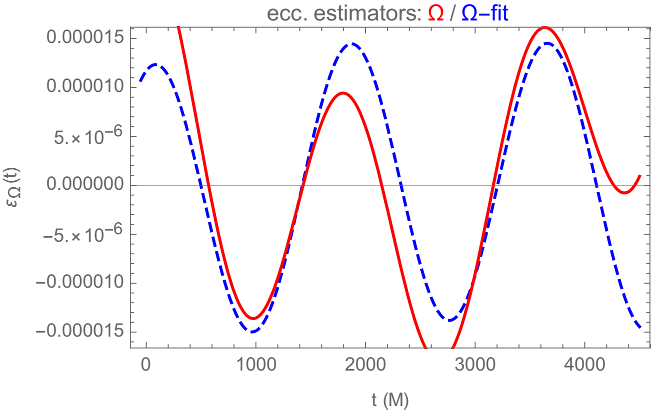
- *sign* is $+1$ or -1 depending if the eccentricity reaches a minimum or maximum at the origin.
- Qualitative behavior at origin $\rightarrow$ mixture of P_t and P_r.



- In that case $sign = -1 \rightarrow P_t$ has to increase.

Results. Testing iterative procedure using PN

- Test to check the validity of the previous formulae.
  1) Generate a 1PN simulation with very low eccentricity.
- Example case: q4._S1_-0.8_S2_0.8. $e_\Omega = (1.43 \pm 0.01) \times 10^{-5}$

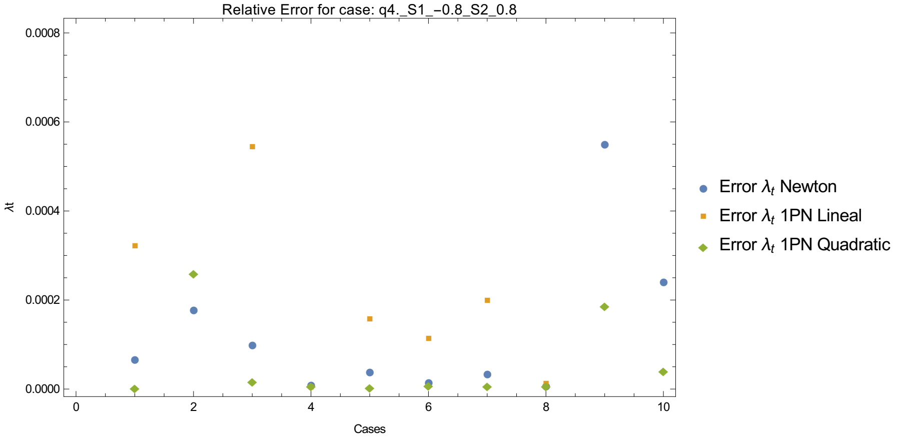


Results. Testing iterative procedure using PN

2) Set $\lambda_r = 1$ and change manually λ_t.
3) Check which formula predicts the best answer.

λ_t^{theo}	λ_t^{Newt}	$\lambda_t^{\text{1PN Lin}}$	$\lambda_t^{\text{1PN Quad}}$	$E_{rel}\ \lambda_t^{\text{Newt}}$	$E_{rel}\ \lambda_t^{\text{1PN Lin}}$	$E_{rel}\ \lambda_t^{\text{1PN Quad}}$
1.0015	1.00157	1.00118	1.0015	6.66×10^{-5}	3.2×10^{-4}	3.26×10^{-6}
1.009	1.00918	1.00691	1.00874	1.77×10^{-4}	2.07×10^{-3}	2.60×10^{-4}
1.0025	1.0026	1.00196	1.00248	9.92×10^{-5}	5.43×10^{-4}	1.78×10^{-5}
1.00002	1.00002	1.00002	1.00002	8.62×10^{-6}	2.77×10^{-6}	7.59×10^{-6}
1.00075	1.00079	1.00059	1.00075	3.86×10^{-5}	1.56×10^{-4}	3.68×10^{-6}
0.9995	0.999486	0.999613	0.999508	1.44×10^{-5}	1.13×10^{-4}	8.06×10^{-6}
0.9991	0.999067	0.999298	0.999107	3.33×10^{-5}	1.97×10^{-4}	7.34×10^{-6}
0.999975	0.999982	0.999986	0.999982	6.68×10^{-6}	1.12×10^{-5}	7.48×10^{-6}
0.9915	0.990954	0.993194	0.991314	5.50×10^{-4}	1.70×10^{-3}	1.87×10^{-4}
0.9955	0.995261	0.996434	0.995459	2.40×10^{-4}	9.38×10^{-4}	4.13×10^{-5}

Results. Testing iterative procedure using PN



Conclusions

- A simple method to compute the eccentricity from simulations has been set.
- New Ansatz based on TaylorT3 approximant $\rightarrow$ reliable measure of eccentricity and value at the origin.
- Qualitative behavior of e_Ω at origin $\rightarrow$ estimation of the mixture of P_t and P_r.
- Use a general iterative procedure consisting in adjusting $(\lambda_r P_r^0, \lambda_t P_t^0)$.
- Prohibitive cost of NR simulations $\rightarrow$ first tests within PN approximation.
- Formulas for λ_t are computed from 1PN Lagrangian.
- A reduction of the eccentricity is observed.
- **Future Work**: Compute analytical correction for P_r^0, analyse NR simulation results, extend to precessing cases,...