Toward calibrating phenomenological waveform models with subdominant harmonics to Numerical Relativity

Cecilio García, Sascha Husa, Geraint Pratten, Marta Colleoni, Xisco Jiménez Forteza

Universitat de les Illes Balears

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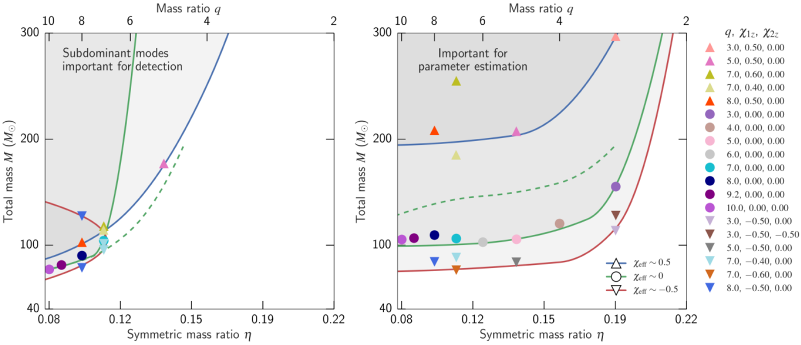
Are Higher Modes important?

- Standard GWs searches and parameter estimation for CBC use waveforms models that neglect Higher Modes
- Loss of detection rates and biases in parameter estimation [1,2]



(a) For detection (b) For parameter estimation

[1]Vijay Varma et al (2014)
[2]J.C. Bustillo et al (2015)

Multimode Decomposition

$$h(t,\theta,\varphi;\Xi) = \sum_{l=2}^{\infty}\sum_{m=-l}^{l} Y_{l,m}^{-2}(\theta,\varphi) h_{l,m}(t,\Xi)$$

- $Y_{l,m}^{-2}(\theta,\varphi)$: spin -2 weighted spherical harmonics
- Non-precessing case $\Rightarrow$ equatorial symmetry

$$h_{l,m}(t,\Xi) = (-1)^l h_{l,-m}^{*}(t,\Xi)$$

- Intrinsic parameters: $\Xi = \{M, q, \chi_1, \chi_2\}$
- Decomposition of each mode in amplitude and phase

$$h_{l,m}(t,\Xi) = A_{l,m}(t,\Xi) e^{-i\psi_{l,m}(t,\Xi)}$$

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How to compute $h_{l,m}$?

- Post-Newtonian (PN) approximation for the inspiral
- Numerical Relativity (NR) for the whole regime
- PN + NR $\rightarrow$ **Hybrids waveforms**
- NR is very time consuming
- Model calibrated to some hybrid waveforms to cover a wider parameter space:

$\downarrow$

Phenomenological Models

$\downarrow$

Fit an Ansatz to the Hybrids

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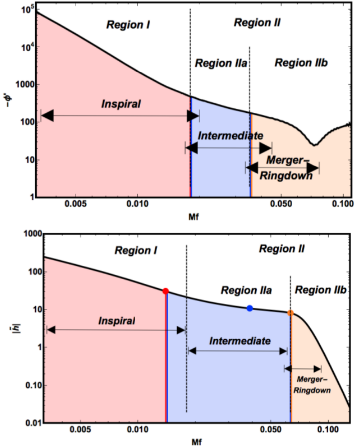
PhenomD model



- S. Khan, S. Husa et al (2015)
- Inspiral $\rightarrow$ PN + fit to hybrids
- MRD $\rightarrow$ Fit to hybrids
- Important parameters:
  - MECO frequency
  - RingDown frequency
  - Damping frequency

MECO frequency

- It defines the transition from inspiral to the plunge
- Minimum Energy Circular Orbit
- Energy [3] of the orbit given by Post-Newtonian and Kerr solution

$$E(v) = \frac{E^{n-PN}}{\eta} - \left(\sum_{i=0}^{i=2n} E^{Kerr}(v^i) \right) + E^{Kerr} \tag{1}$$

- $f_{MECO} = \frac{v_{min}^3}{\pi M}$

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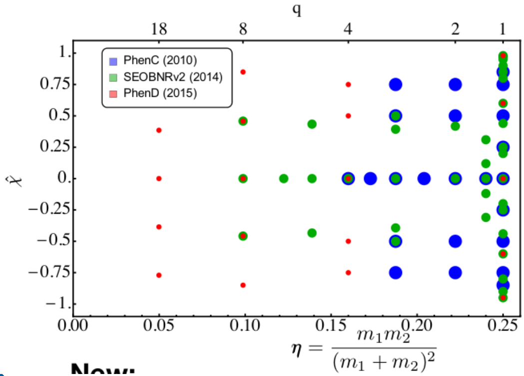
[3] M. Cabero, A.B. Nielsen, A.P. Lundgren and C.D. Capano (2016)

How PhenomD works

- Parameter space of the Hybrids Waveforms to calibrate the model



- Calibration:
  10 BAM, 9 SXS
- Verification:
  23 SXS, 6 BAM

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PN Amplitude in FD

- Under Stationary Phase Approximation:

$$\tilde{h}^{lm}(f) = \int_{-\infty}^{\infty} h^{lm}(t) e^{2\pi i f t} dt \approx A^{lm}(x) \sqrt{\frac{2\pi}{m\ddot{\phi}(x)}} e^{i\psi^{lm}(f)} \quad (2)$$

- Complex time-domain PN amplitude[4]
- $x = \omega^{2/3} = \left(\frac{2\pi f}{m}\right)^{2/3}$, $\omega = \dot{\phi}$
- $\ddot{\phi} = \frac{3}{2}\sqrt{x}\dot{x}$, $\dot{x} = TaylorT4(x)$
- Real frequency-domain PN amplitude → **TaylorF2**:

$$\tilde{A}^{lm}(f) = |A^{lm}(x)| \sqrt{\frac{2\pi}{m\ddot{\phi}(x)}} \quad (3)$$

[4] Blanchet et al (2008), Arun et al (2011), Buonanno et al (2013)

How PhenomD works

- Modular model: calibrate each region to Hybrid data
  - **Inspiral**: FD post-Newtonian expresions + higher-order terms fit to hybrids

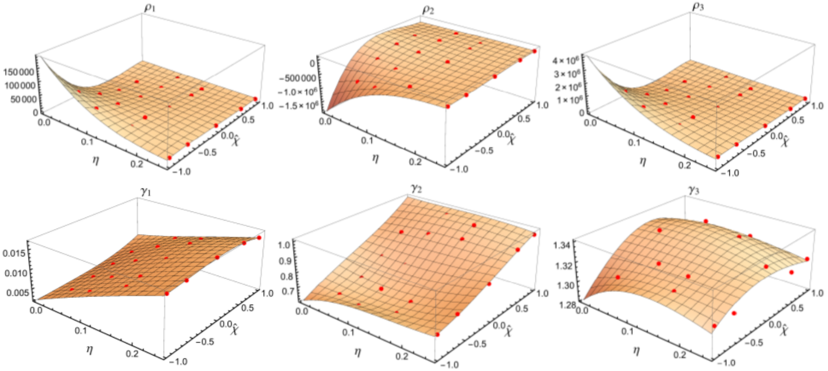
$$A_{ins}^{22}(f) = A_{PN}^{22}(f) + A_0^{22}(f) \sum_{i=1}^{3} \rho_i f^{(6+i)/3} \tag{4}$$

  - **MRD**: phenomenological ansatz

$$\frac{A_{MR}^{22}(f)}{A_0^{22}(f)} = \gamma_1 \frac{\gamma_3 f_{DAMP22}}{(f - f_{RD22})^2 + (\gamma_3 f_{DAMP22})^2} e^{-\frac{\gamma_2 (f - f_{RD22})}{\gamma_3 f_{DAMP22}}} \tag{5}$$

  - **Intermediate region**: 4 order polynomial interpolation, match the model and its derivative at the ends of the interval

Amp coefficients across parameter space



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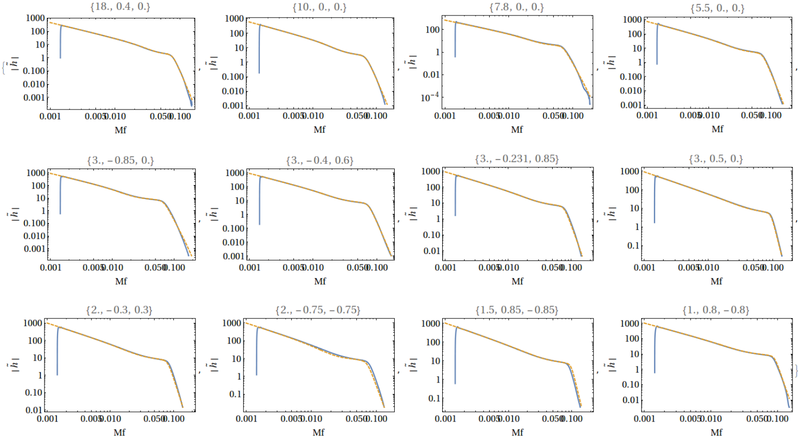
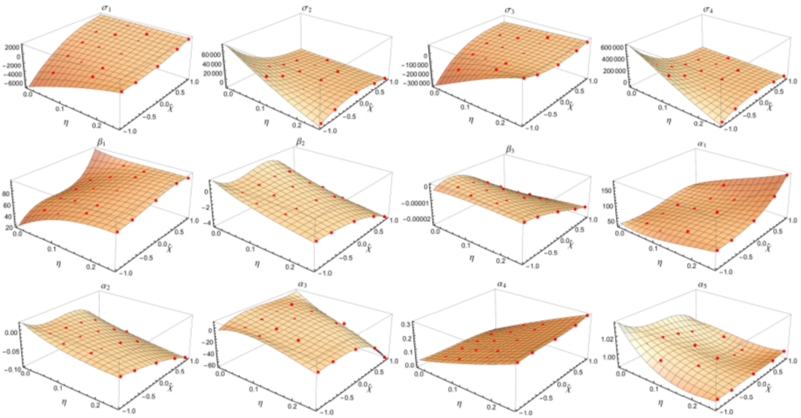
Comparison NR and PhenomD

- The {2,2} mode is well described by PhenomD



Phase coefficients

Same philosophy with slightly different ansatzs.



Phase for HM

- **Inspiral**
  - Multipole phases are related by a simple scaling

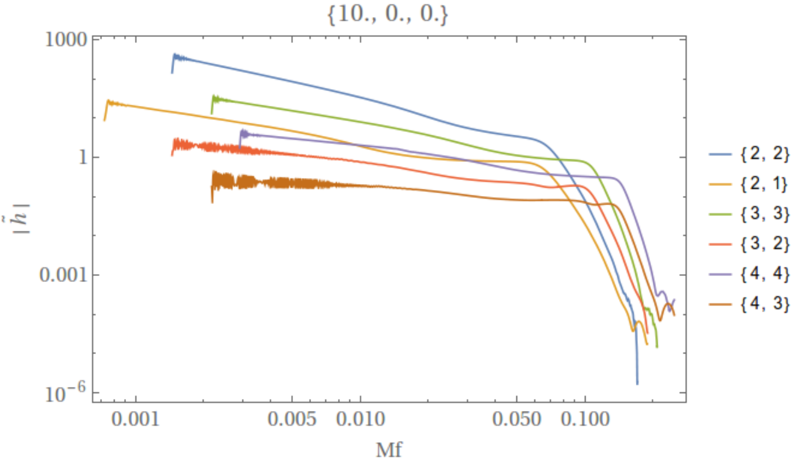
$$h_{lm}(t) = A_{lm}(t)e^{i\psi_{lm}(t)},\ \psi_{lm} = m\phi_{Orb} \tag{6}$$

  - SPA yields in the FD to:

$$f_{lm} \overset{SPA}{\approx} \frac{m}{2} f_{22} \tag{7}$$

  - $\phi_{lm}(f) = \frac{2}{m}\phi_{22}(\frac{m}{2}f)$

Inspiral for HM: $\{q, \chi_1, \chi_2\}$=$\{10, 0, 0\}$ (SXS)



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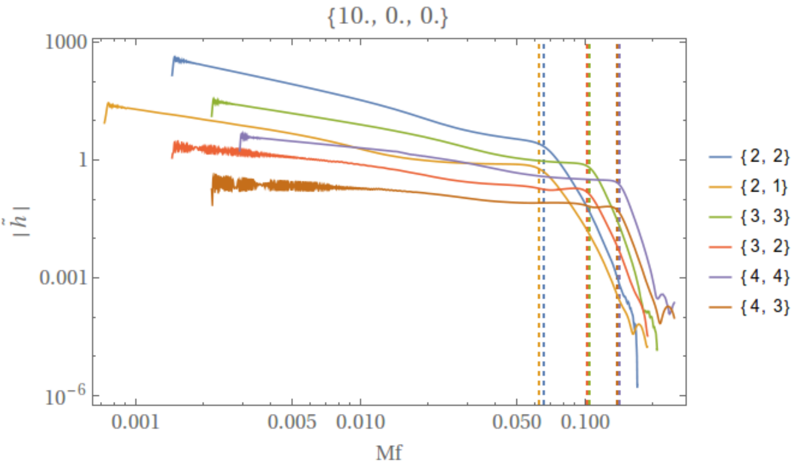

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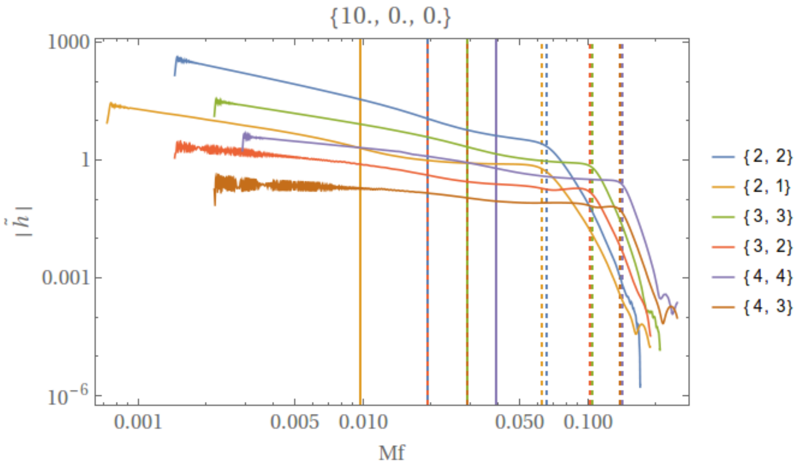
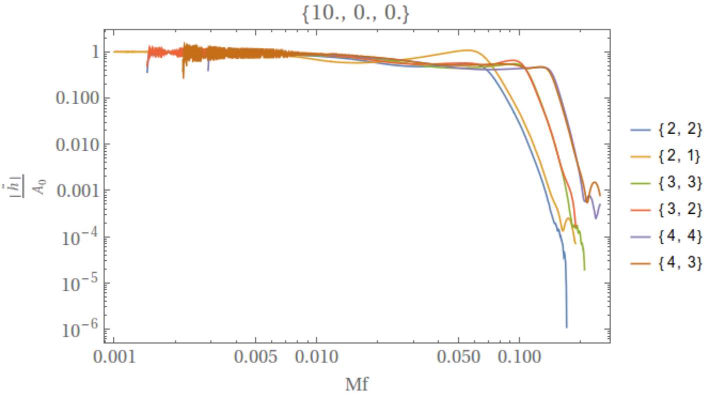
Inspiral for HM: $\{q, \chi_1, \chi_2\}$=$\{10, 0, 0\}$ (SXS)

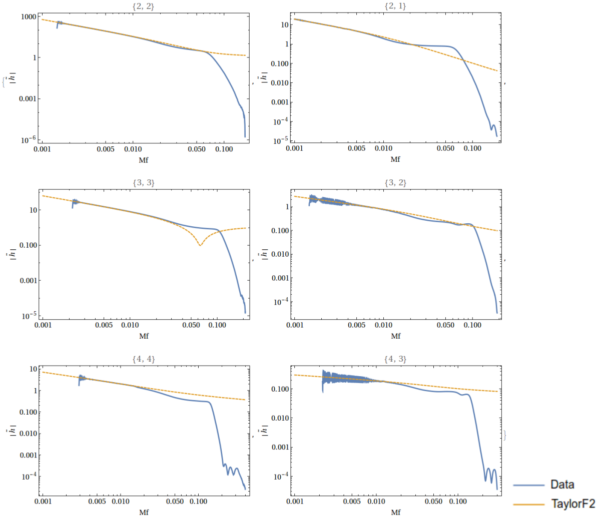


Inspiral for HM: $\{q, \chi_1, \chi_2\}$=$\{10, 0, 0\}$ (SXS)

- Do we recover Post-Newtonian limit for low frequencies?



Inspiral for HM: $\{q, \chi_1, \chi_2\}$={10, 0, 0} (SXS)



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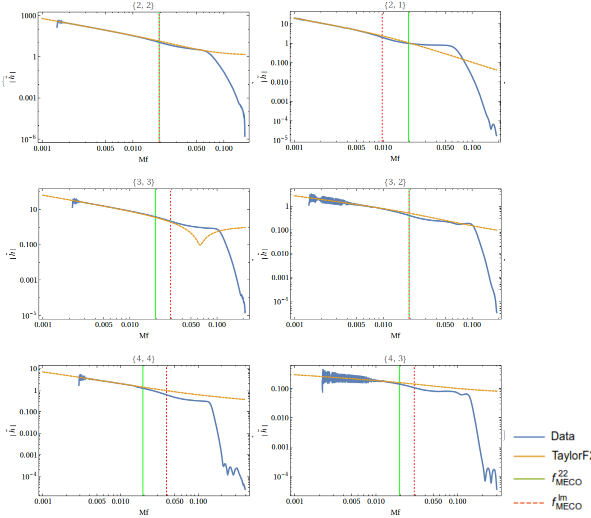
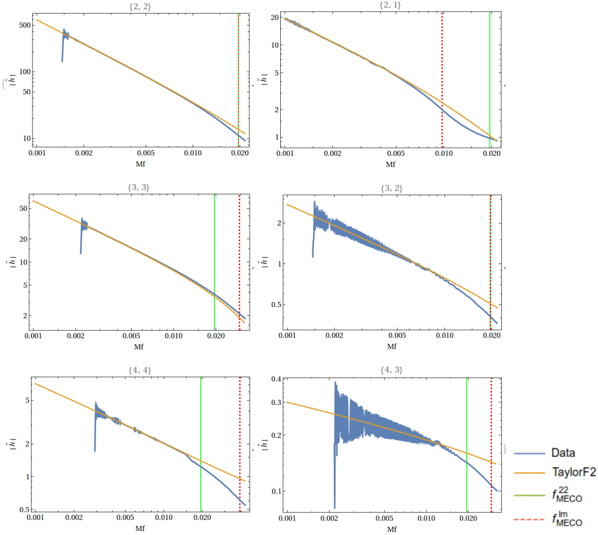
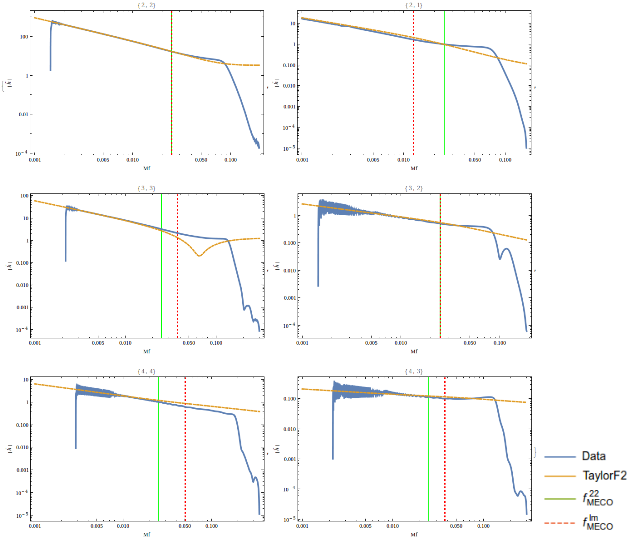


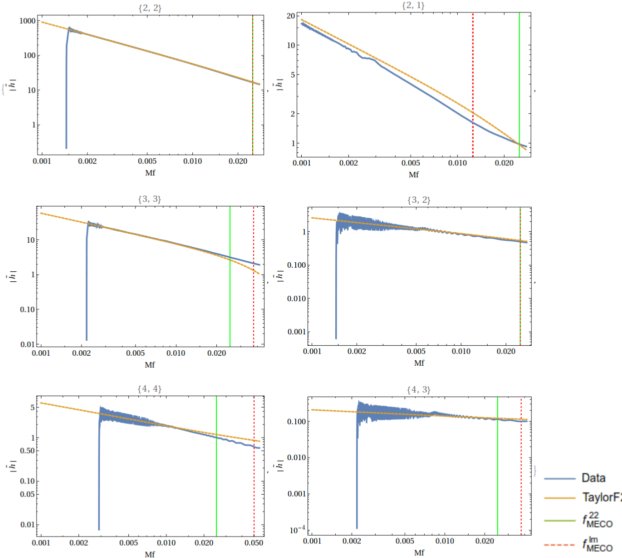
Inspiral for HM: $\{q, \chi_1, \chi_2\}$={10, 0, 0} (Zoom)



Inspiral for HM: $\{q, \chi_1, \chi_2\}$=$\{3, 0.3, 0.3\}$ (SXS)



Inspiral for HM: $\{q, \chi_1, \chi_2\}$=$\{3, 0.3, 0.3\}$ (Zoom)



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Inspiral for Higher Modes

- We recover the PN limit for low frequencies
- Near the f_{MECO}, disagreement between TaylorF2 and data grows
- Add higher orders of PN, which are not know yet
- We calibrate them to hybrid data
- Ansatz for the inspiral:

$$A^{lm}_{ins}(f) = A^{lm}_{PN}(f) + A^{lm}_{0}(f) \sum_{i=1}^{3} \rho^{lm}_{i} f^{(6+i)/3} \tag{8}$$

- Ongoing work: the fit will be done soon

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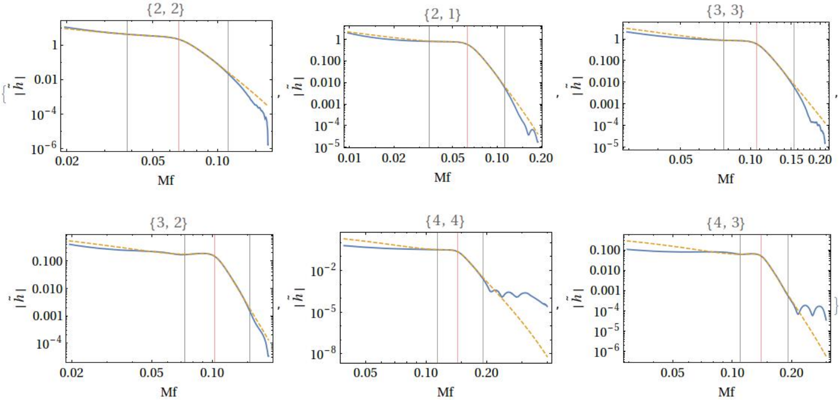
Merger-Ringdown for Higher Modes

- Same ansatz for the rescaled amplitude than for {2,2} but changing parameters.

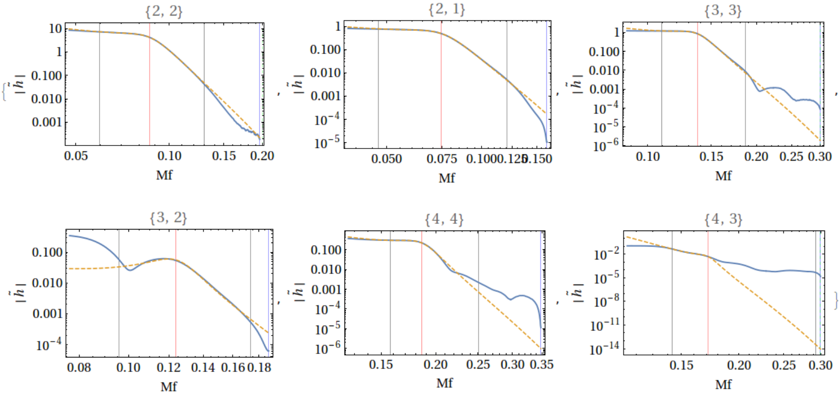
$$\frac{A^{lm}_{MR}(f)}{A^{lm}_{0}(f)} = \gamma_1^{lm} \frac{\gamma_3^{lm} f_{DAMPlm}}{(f - f_{RDlm})^2 + (\gamma_3^{lm} f_{DAMPlm})^2} \exp^{-\frac{\gamma_2^{lm}(f - f_{RDlm})}{\gamma_3^{lm} f_{DAMPlm}}} \tag{9}$$

- Fit range
  $finf = f_{RDlm} - 2f_{DAMPlm}$
  $fsup \rightarrow$ where amplitude decays to $1/100$

MRD fit for HM: $\{q, \chi_1, \chi_2\}=\{10,0,0\}$ (SXS)



MRD fit for HM: $\{q, \chi_1, \chi_2\}$={3,0.3,0.3} (SXS)



Next steps

- Connect Inspiral fit with MRD fit through as for the {2,2} mode
- 1 set of coefficients for each mode: $\{\rho^{lm}_{1,2,3}, \gamma^{lm}_{1,2,3}\}$
- Repeat the process for all the cases we have ($\sim$ 260 BAM+SXS)
- With the set of coefficients $\{\rho^{lm}_{1,2,3}, \gamma^{lm}_{1,2,3}\}\times$Ncases, do a fit over parameter space to get $\rho^{lm}_i, \gamma^{lm}_i$ as functions of $\{\eta, \chi_1, \chi_2\}$

Conclusions

- Higher Modes are important for detection and parameter estimation and have to be modelled
- The HM amplitude needs to be calibrated to NR data
- The inspiral region is well described by PN theory but the fit may improve results
- The ansatz for MRD based on PhenomD works well
- Forthcoming work, repeat procedure over the whole set of NR to have a good model over the parameter space