Gravitational Waves in theories with non-minimal coupling between matter and curvature

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SFRH/BD/102820/2014   UID/FIS/04650/2013

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FCT Fundação para a Ciência e a Tecnologia
MINISTÉRIO DA CIÊNCIA, TECNOLOGIA E ENSINO SUPERIOR
UNIÃO EUROPEIA
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Iberian Gravitational Wave Meeting, Bilbao, 2017

Outline

- Why not GR?
- The non-minimal coupling between matter and curvature (NMC)
- Inflation in NMC theories
- GW in $f(R)$ theories
- GW in NMC theories

Why not GR?

Successes:

- Solar System constraints;
- GPS;

But there were still some conundrums:

- Large scale data requires DM and DE;
- It lacks a consistent high energy version;

Alternative theories of gravity:

- f(R)
- Horndeski gravity;
- Jordan-Brans-Dicke;
- NMC [Bertolami, Böhmer, Harko, Lobo 2007]...

The non-minimal coupling between matter and curvature (NMC)

$$S = \int \left[\kappa f_1\left(R\right) + \left(1 + f_2\left(R\right)\right)\mathcal{L}\right]\sqrt{-g}d^4x \quad , \tag{1}$$

where $\kappa = M_P^2/2$.
Varying the action relatively to the metric $g_{\mu\nu}$:

$$\begin{aligned} 2\left(\kappa F_1 - F_2\rho\right)\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right) =& \left(1 + f_2\right)T_{\mu\nu} + \kappa\left(f_1 - F_1R\right)g_{\mu\nu} + \\ & + F_2\rho R g_{\mu\nu} + 2\Delta_{\mu\nu}\left(\kappa F_1 - F_2\rho\right) \end{aligned} \tag{2}$$

where $F_i \equiv df_i/dR$, and $\Delta_{\mu\nu} \equiv \nabla_\mu\nabla_\nu - g_{\mu\nu}\Box$.
One recovers GR by setting $f_1(R) = R$ and $f_2(R) = 0$.

Using the Bianchi identities, one finds the covariant non-conservation of the energy-momentum tensor:

$$\nabla_\mu T^{\mu\nu} = \frac{F_2}{1+f_2}\left(g^{\mu\nu}\mathcal{L} - T^{\mu\nu}\right)\nabla_\mu R \qquad (3)$$

For a perfect fluid, the extra force due to the NMC can be expressed as:

$$f^\mu = \frac{1}{\rho + p}\left[\frac{F_2}{1+f_2}(\mathcal{L} - p)\nabla_\nu \mathrm{R} + \nabla_\nu p\right] h^{\mu\nu}, \qquad (4)$$

with $h^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$ being the projection operator.

Degeneracy-lifting of the Lagrangian choice [O. Bertolami, F. S. N. Lobo, J. Páramos, 2008]

Mimicking Dark Matter (galaxies, clusters) [O. Bertolami, J. Páramos, 2010; O. Bertolami, P. Frazão, J. Páramos, 2013]

Cosmological Perturbations [O. Bertolami, P. Frazão, J. Páramos, 2013]

Preheating scenario after inflation [O. Bertolami, P. Frazão, J. Páramos, 2011]

Modified Friedmann equation [O. Bertolami, J. Páramos, 2013]

Modified Layzer-Irvine equation and virial theorem [O. Bertolami, C. Gomes, 2014]

Inflationary dynamics [C. Gomes, O. Bertolami, J.G. Rosa, arXiv:1611.02124 [gr-qc]] ...

Inflation in NMC

[Gomes, Rosa and Bertolami arXiv:1611.02124 [gr-qc]]

We are interested in the following formulation for the NMC action (Einstein + pure non-minimal coupling):

$$S = \int \left[R + \left(1 + f_2\left(R\right)\right)\mathcal{L}\right]\sqrt{-g}d^4x \ . \tag{5}$$

For a homogeneous scalar field the time component of the covariant non-conservation of the energy gives at lowest order:

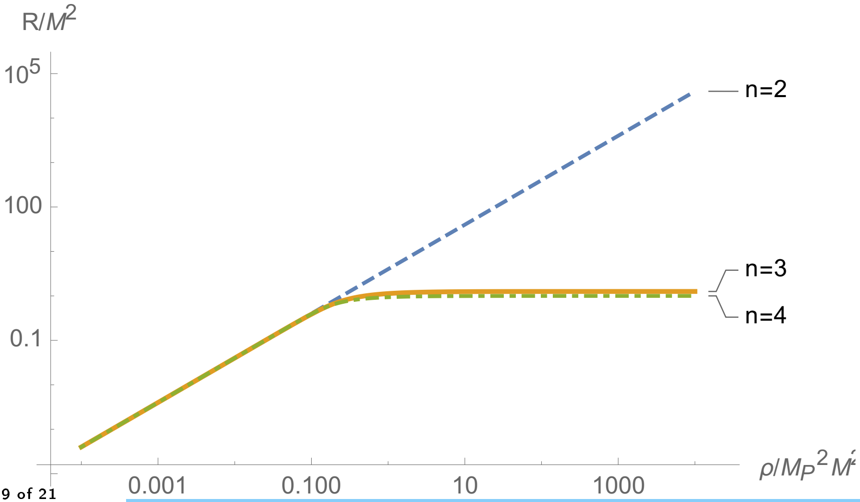
$$\ddot{\phi} + 3H\dot{\phi} + V'\left(\phi\right) \approx 0 \ . \tag{6}$$

Taking the slow-roll limit, for $\mathcal{L} = p \simeq -\rho$, we obtain a modified Friedmann equation:

$$\boxed{H^2 \simeq \left(\frac{1 + f_2}{1 + \frac{2F_2\rho}{M_P^2}} \right) \frac{\rho}{3M_P^2}} \,. \tag{7}$$

Once specified the form of the non-minimal coupling function, the Friedmann equation exhibits different behaviours for densities lower or greater than a critical value which depends on $f_2(R)$.

$$f_2(R) = \left(\frac{R}{M^2}\right)^n$$



Cubic non-minimal coupling function

For $f_2(R) = (R/M^2)^3$, defining $H_0^2 = 2^{1/3} M^2/12$ and $\tilde{\rho} = \rho / M_P^2 H_0^2$, the modified Friedmann equation yields:

$$H^2 = H_0^2 \frac{-2^{2/3}\tilde{\rho} + \left(\tilde{\rho}^3 + \sqrt{\tilde{\rho}^3(4+\tilde{\rho}^3)}\right)^{2/3}}{2^{1/3}\tilde{\rho}\left(\tilde{\rho}^3 + \sqrt{\tilde{\rho}^3(4+\tilde{\rho}^3)}\right)^{1/3}} \ . \tag{8}$$

which for $\rho \gtrsim M_P^2 M^2$ (high density regime):

$$H^2 \simeq H_0^2 \left(1 - \frac{1}{6 \times 2^{2/3}} \frac{1}{\tilde{\rho}}\right) \ . \tag{9}$$

Hilltop models $V = V_0 \left[1 - \frac{\gamma}{n} \left(\frac{\phi}{M_P} \right)^n \right]$

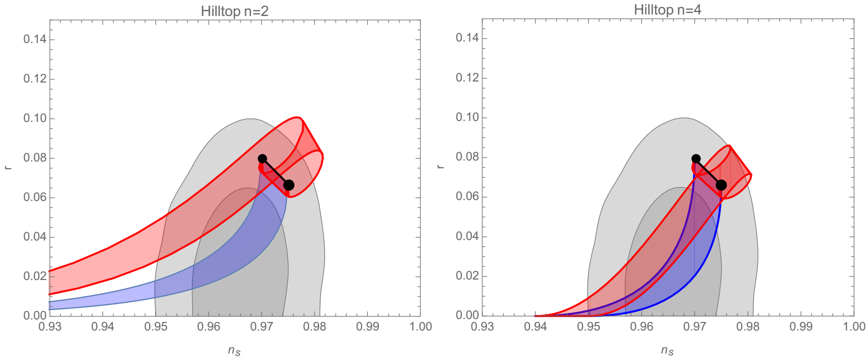
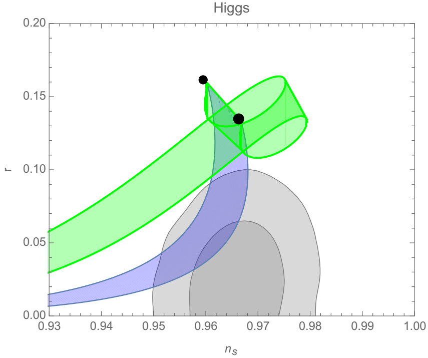


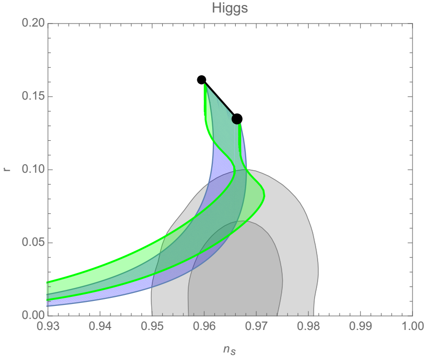
Figure: The GR results are shown in Blue, whilst NMC's ones are in Red. $x \equiv V_0/M_P^2 H_0^2 = 5$.

Higgs model

$$V(\phi) = \lambda(\phi^2 - v^2)^2 = V_0 \left[1 - \frac{\gamma}{2}\left(\frac{\phi}{M_P}\right)^2 + \frac{\gamma^2}{16}\left(\frac{\phi}{M_P}\right)^4\right]$$



(a) $x = V_0/M_P^2 H_0^2 = 5$



(b) $x = V_0/M_P^2 H_0^2 = 2.$

Chaotic models $V = V_0 \left(\frac{\phi}{M_P}\right)^n$

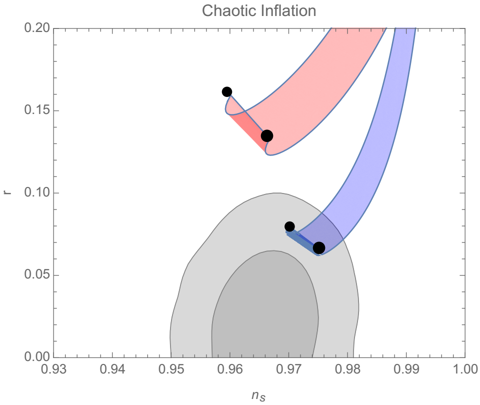


Figure: Chaotic $n = 1$ models in Blue, and $n = 2$ in Red. Both in NMC theories.

Consistency relation

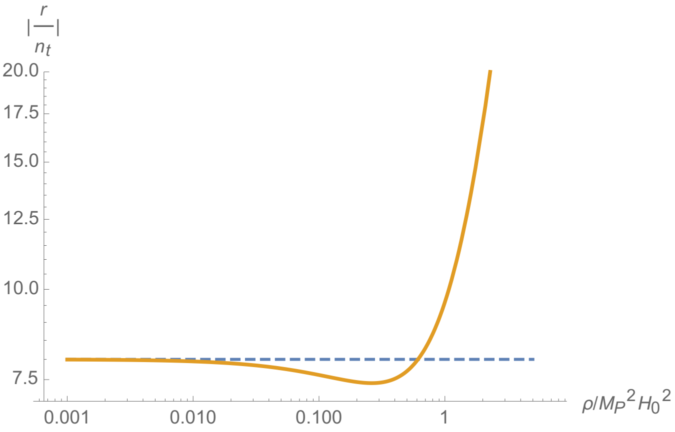


Figure: In dashed blue it is shown GR prediction, whilst is solid orange the NMC behaviour is plotted.

Gravitational waves in $f(R)$ theories [Alves, Miranda, de Araújo, 2009]

- Null tetrad:

$$\mathbf{k} = \frac{1}{\sqrt{2}}(\mathbf{e}_t + \mathbf{e}_z) \quad , \quad \mathbf{l} = \frac{1}{\sqrt{2}}(\mathbf{e}_t - \mathbf{e}_z)$$

$$\mathbf{m} = \frac{1}{\sqrt{2}}(\mathbf{e}_x + i\mathbf{e}_y) \quad , \quad \bar{\mathbf{m}} = \frac{1}{\sqrt{2}}(\mathbf{e}_x - i\mathbf{e}_y)$$

- Newman-Penrose quantities:

$$\Psi_2 = -\frac{1}{6}R_{lklk} \quad , \quad \Psi_3 = -\frac{1}{2}R_{lkl\bar{m}}$$

$$\Psi_4 = -R_{l\bar{m}l\bar{m}} \quad , \quad \Phi_{22} = -R_{lml\bar{m}}$$

- Pretrov classification $E(2)$: II_6, III_5, N_3, N_2, O_1, O_0.

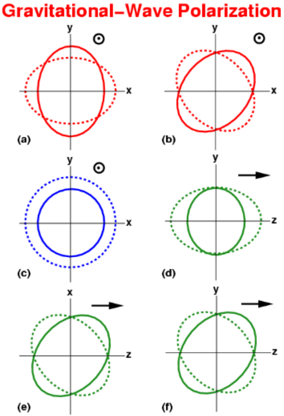


Figure: $Re\Psi_4$ $a)$, $Im\Psi_4$ $b)$, Φ_{22} $c)$, Ψ_2 $d)$, $Re\Psi_3$ $e)$, $Im\Psi_3$ $f)$. [Alves, Miranda, de Araújo 2009]

Example: $f(R) = R - \alpha R^{-\beta}$

- $\alpha = 0$: GR and N_2.
- $\alpha \neq 0$ and $\beta \in]-\infty, -2] \cup [1, +\infty[$: most general case, II_6, a longitudinal mode Ψ_2 and the others are observer-dependent.

In any $f(R) \neq R$ model there is a longitudinal model [Capozziello et al. 2008] and a perpendicular one, Φ_{22}, or breathing mode. So that one can always find a Lorentz observer that measures the six polarisation modes.

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But in the Palatini formalism $f(R)$ theories have only two tensor modes for polarisation as GR!

Gravitational waves in NMC theories

[Bertolami, Gomes, Lobo, in preparation...]

Ongoing work:

- In the far-field (no matter): NMC become $f_1(R)$;
- Other regions: matter $\rightarrow$ so NMC plays a role;
- It is of class II_2, with differences on the longitudinal mode (highly non-trivial!)

$$\omega = \omega\left(\delta R, \delta \mathcal{L}\right) \tag{10}$$

which can decouple into two independent modes, under certain conditions.

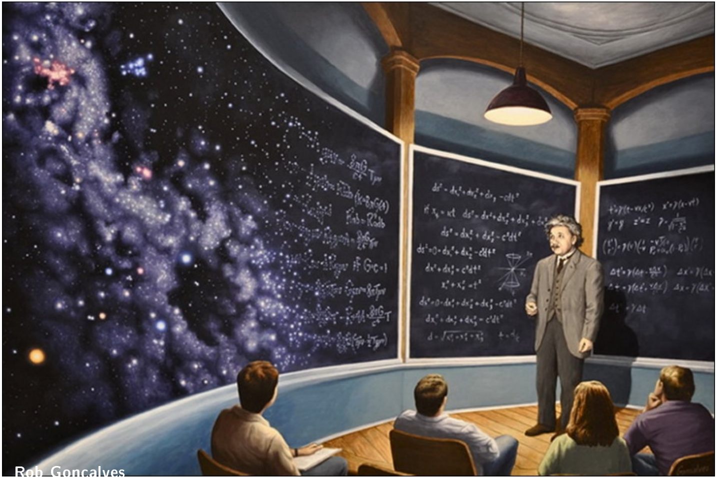
For black holes (Schwarzschild and Reissner–Nordstrom), the NMC dresses some quantities as [Bertolami, Cadoni, Porru, 2015]:

- 'bare' GR m , Q $\rightarrow$ dressed $(1 + f_2(R \rightarrow \infty))m$, $\frac{Q}{\sqrt{1+f_2(0)}}$

Back to GWs: tensor perturbations:

$$\Box(h_{\mu\nu} - \frac{1}{2}h\eta_{\mu\nu} - \omega\eta_{\mu\nu}) = \textit{dressed source term + fluctuations from matter} \tag{11}$$

Thank you for your attention!



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