Improving Aligned Spin Phenomenological Models: Extreme Mass Ratios and Double Spins

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Overview Of Current Status

Phenomenological Models:

- Focus here is on aligned spin waveform models

$$\chi_{iL} = \frac{c\,\mathbf{S}\cdot\hat{\mathbf{L}}}{Gm_i^2}$$

- Spins *aligned* with orbital angular momentum
- Orbital plane fixed $\hat{L} = \mathrm{const}$

- $\mathrm{IMRPhenomD}$ calibrated to 19 hybrids (*uncalibrated* SEOBNRv2)
- Calibrated to mass ratios 1:18
- Calibrated to spins $|a/M| \sim 0.85\ (0.98 \text{ for equal mass})$
- Target model mismatch $\lesssim 10^{-2}$
- Accurate aligned spin models *necessary*:
  - Avoid confusion with subdominant effects (e.g. higher modes)
  - Basis for precessing phenomenological models $\mathrm{IMRPhenomP}$
  - Parameterised *tests of GR* need accurate/low bias waveform models

Model Construction: Frequency Regimes

Signal Amplitude & Phase:

- Waveform is function of binary parameters:

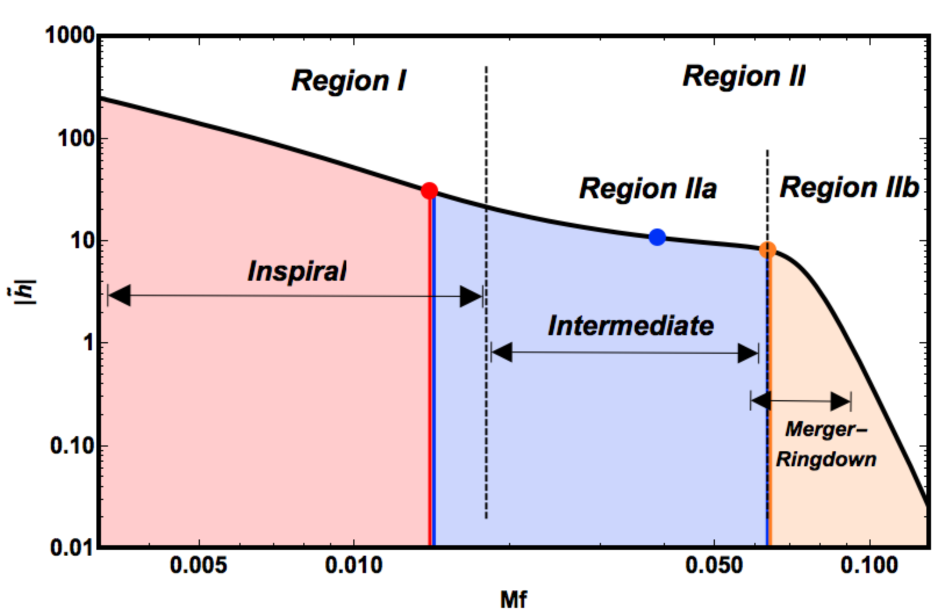
$$\Xi \in \{M, \eta, \chi_1, \chi_2\}$$

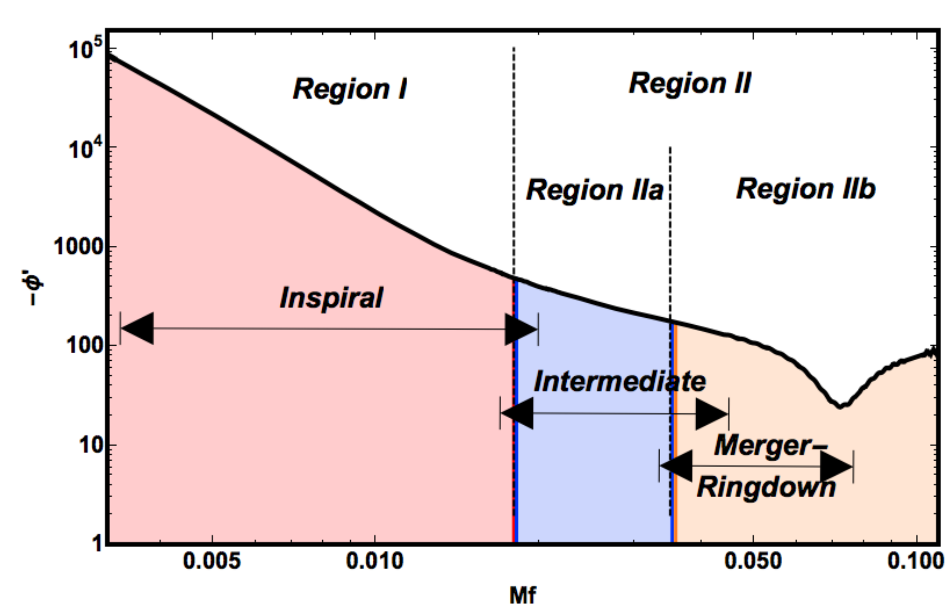
- Separately model the amplitude and phase as a function of frequency

$$\tilde{h}_{22}(f;\Xi) = A(f;\Xi)\; e^{-i\phi(f;\Xi)}$$

Frequency Regimes

- Adopt a modular approach
- Identify 3 separate frequency regimes: Inspiral, Intermediate, Ringdown





Model Construction: Inspiral

Hybrids

- NR Waveforms are too short!
- Target model that will be valid down to low frequencies ~ 10Hz
- Overcome by hybridising NR waveforms with SEOBNRv4

$$h(t) = \omega^{-}_{t_0, t_0+T} \, e^{i\varphi_0} \, h_{\rm PN}(t + \delta t) + \omega^{+}_{t_0, t_0+T} \, e^{i\varphi_0} \, h_{\rm NR}(t + \delta t)$$

Inspiral Phase

- Built on TaylorF2 phase calibrated with 3 *pseudo*-PN coefficients

$$\varphi_{\rm Ins} = \varphi_{\rm TF2}(Mf; \Xi) + \frac{1}{\eta}\left(\sigma_0 + \sigma_1 f + \frac{3}{4}\sigma_2 f^{3/4} + \frac{3}{5}\sigma_3 f^{5/3} + \frac{1}{2}\sigma_4 f^2\right)$$

$$A_{\rm Ins} = A_{\rm PN} + A_0 \sum_{i=1}^{3} \rho_i \, f^{(6+i)/3}$$

- *Improve* calibration by fitting to SEOBNRv4 (compared to v2 in PhenomD)
- *+3.5PN cubic-in-spin terms (Marsat 2015)*
- *+3PN quadratic-spin terms (Bohé et al 2015)*

Model Construction: Intermediate

Phase ansatz for intermediate (merger) regime:

- Freedom in *time* and *phase* shifts - use phase derivative!

$$\varphi(f) \rightarrow \varphi(f) + \varphi_0 + 2\pi t$$

- Conceptual shift from previous polynomial fits
- Opt for a *double* Lorentzian model (merger & ringdown)
- More robust - especially when including extreme mass ratio limit

$$\varphi'_{\mathrm{MR}} = \alpha_1 + \sum_{i=2}^{n} \alpha_n f^{-p_n} + \frac{a}{f_{\mathrm{damp}}^2 + (f - f_{\mathrm{ring}})^2}$$

Amplitude ansatz for intermediate (merger) regime:

- Use a fifth order polynomial with 2 free *collocation* points

$$A_{\mathrm{Int}} = A_0 \left(\delta_0 + \delta_1 f + \delta_2 f^2 + \delta_3 f^3 + \delta_4 f^4 + \delta_5 f^5\right)$$

Model Construction: Ringdown

Phase ansatz for ringdown:

- Lorentzian model as used in previous phenom models

$$\varphi'_{\mathrm{MR}} = \alpha_1 + \sum_{i=2}^{n} \alpha_n f^{-p_n} + \frac{a}{f_{\mathrm{damp}}^2 + (f - f_{\mathrm{ring}})^2}$$

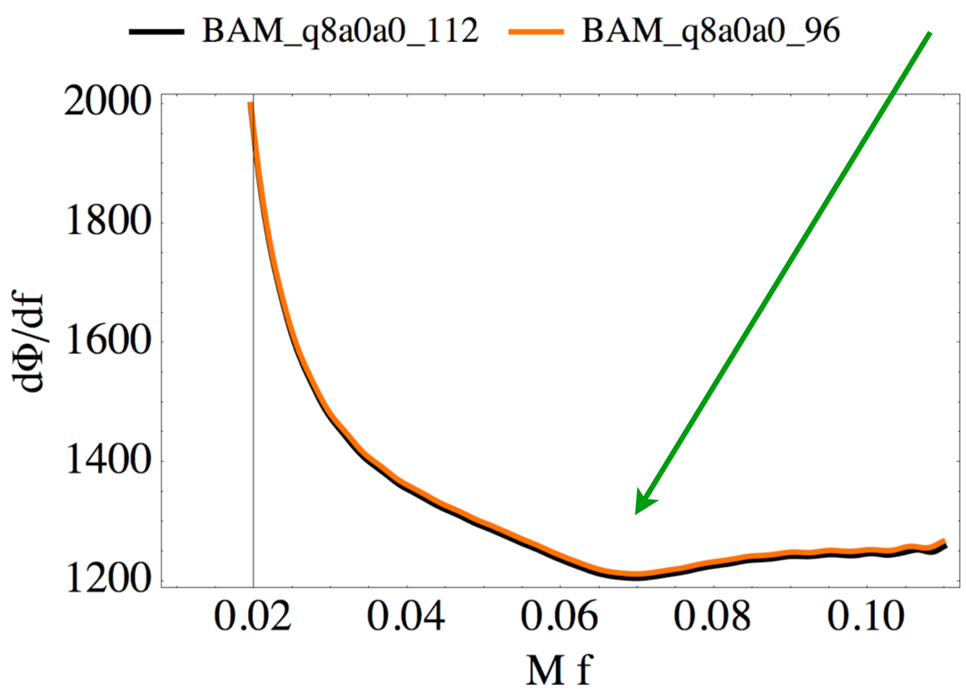
Amplitude ansatz for ringdown:

- As in PhenomD we use a deformed Lorentzian

$$h_{\mathrm{RD}} = \frac{a\, e^{-\lambda (f - f_{\mathrm{RD}})}}{(f - f_{\mathrm{RD}})^2 + \sigma^2}$$

- Peak ~ ringdown frequency
- Width ~ damping frequency
- High f falloff > polynomial

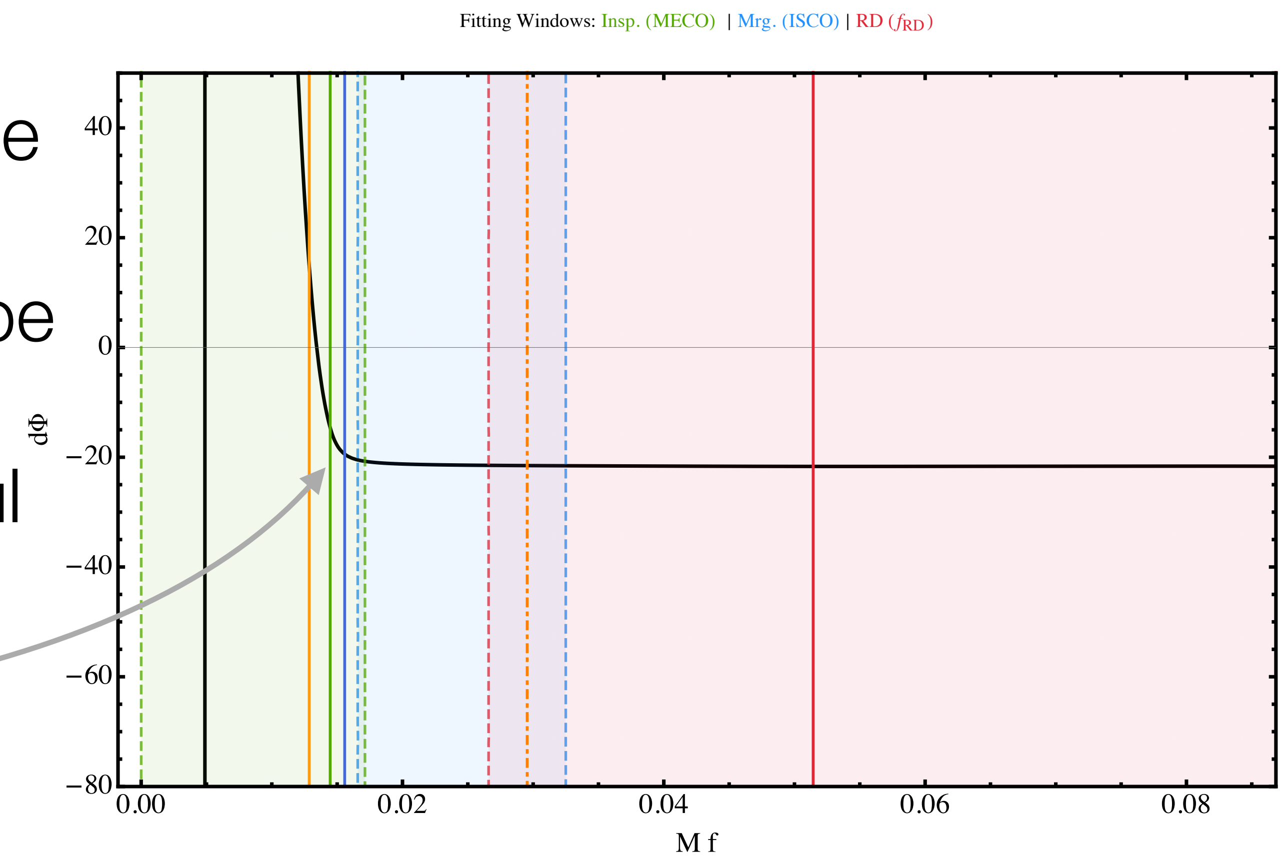
Ringdown frequency



Model Construction: Extreme Mass Ratio Limit

EMR limit:

- Features in waveform can become exaggerated
- Transition of inspiral/merger can be sharp
- Transition frequency needs careful choice
- A *better choice* is $f \sim f_{\rm MECO}$



Cabero+ PRD95 (2017)

Time domain waveforms used in EMR limit:

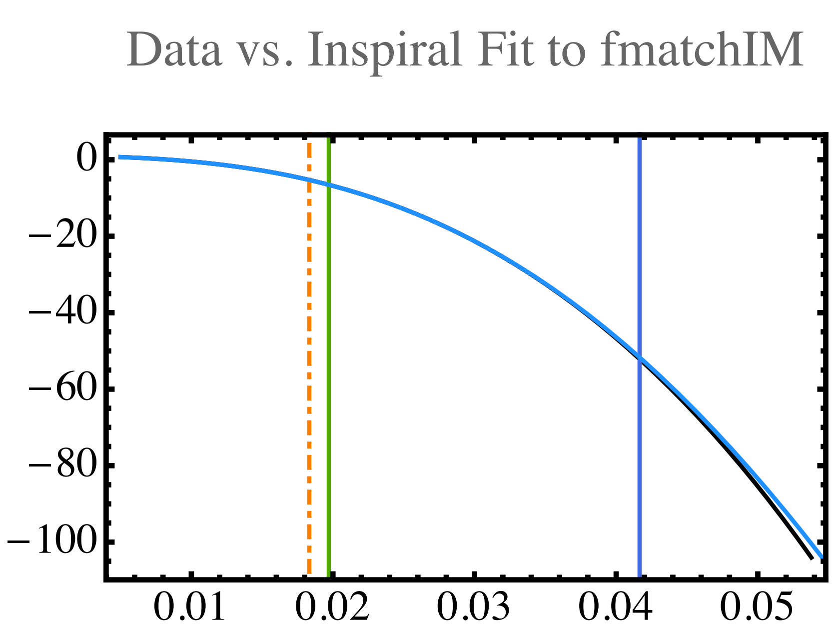
- Numerical approach + semi-analytic EOB test-mass dynamics (5.5PN)
- Conservative motion + linear-in-η radiation reaction
- Hyperboloidal coordinates - unambiguous waveforms at $\mathscr{I}^+$

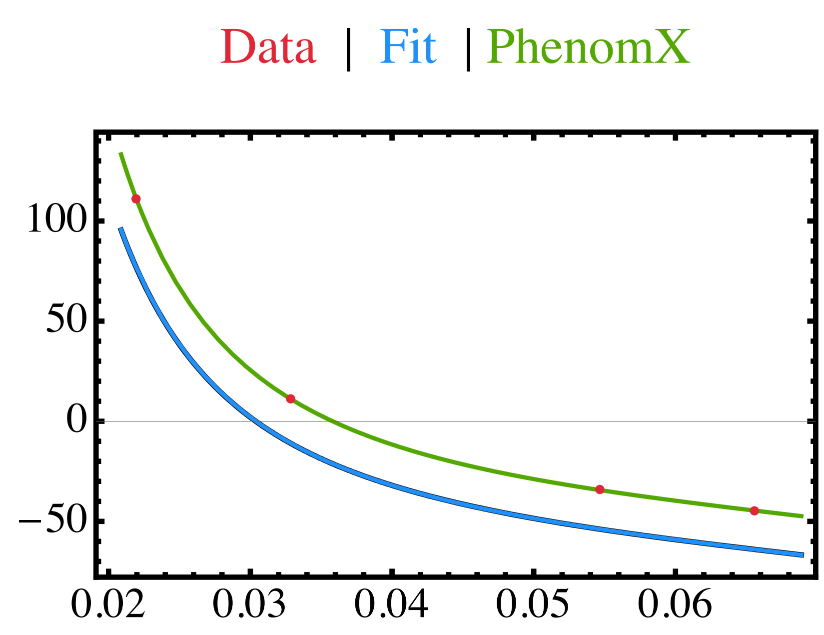
Bernuzzi+ PRD84 (2011), Harms+ CQG31 (2014)

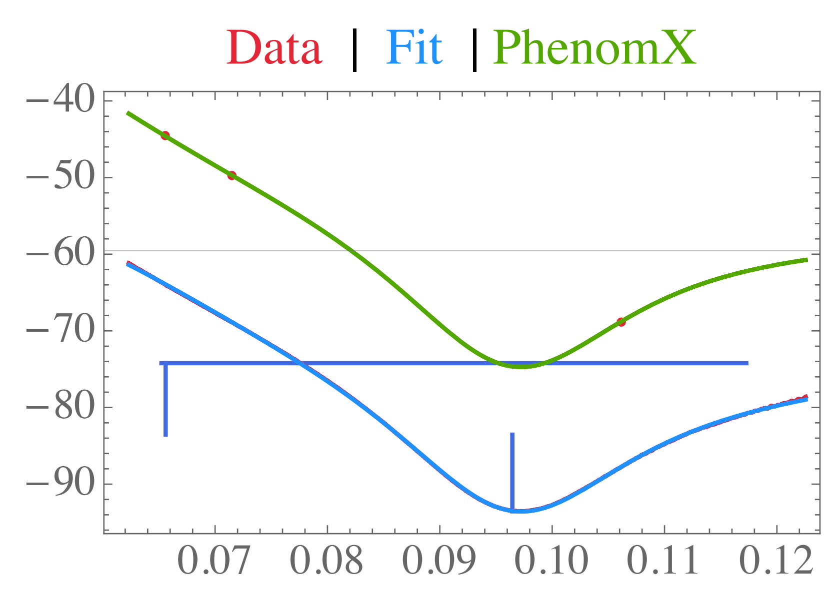
Hierarchical Fit: Collocation Points

Need to map from phenomenological coefficients to physical parameters

- Advocate use of collocation points
- *“Fitting for value at fixed frequencies and differences between values”*
- Find they are better conditioned than basis coefficients
- Accurate polynomial fits may require high order
- For ideal ansatz - may need only a few coefficients in some direction

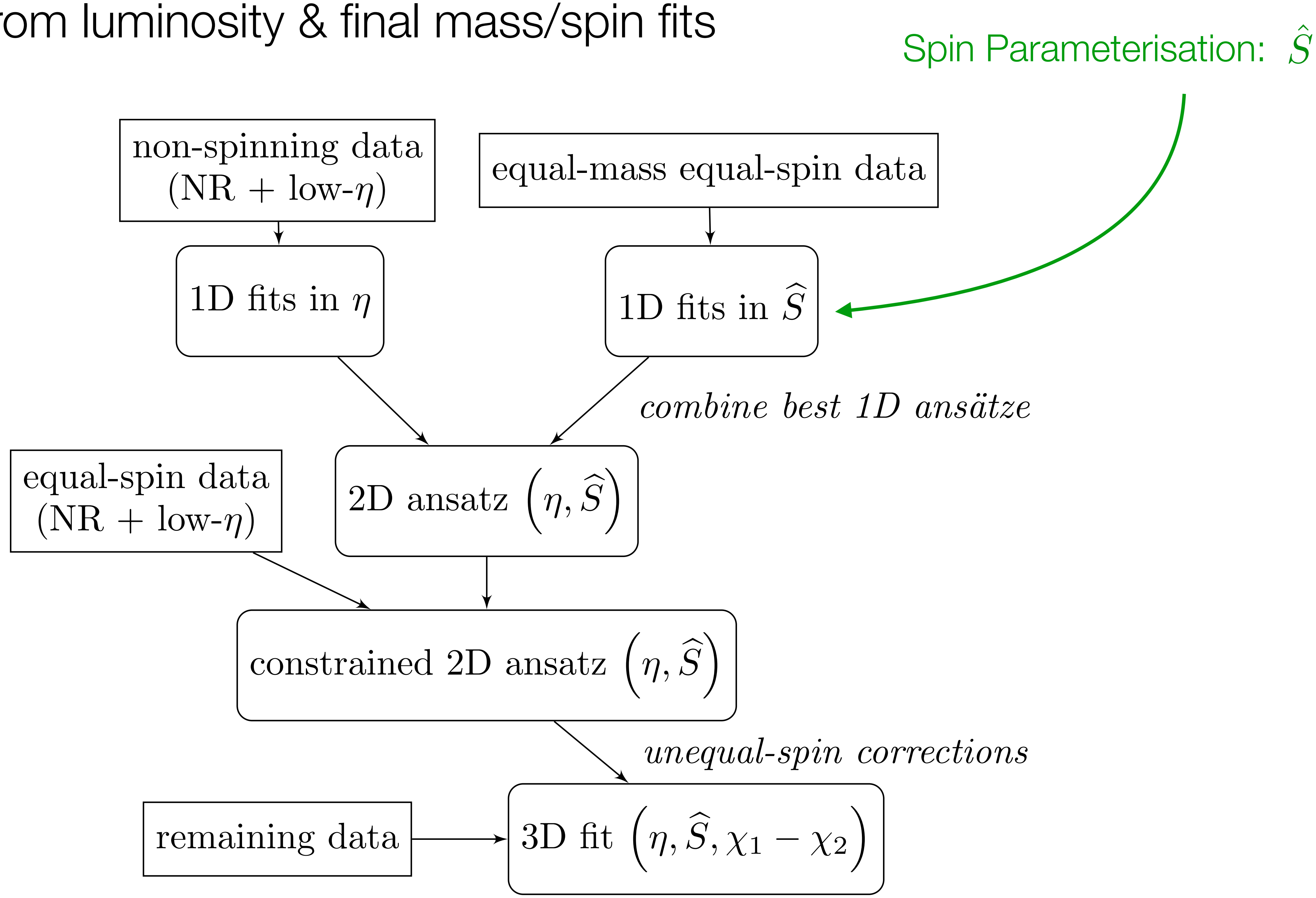






Hierarchical Fits: Workflow

- Insight from luminosity & final mass/spin fits

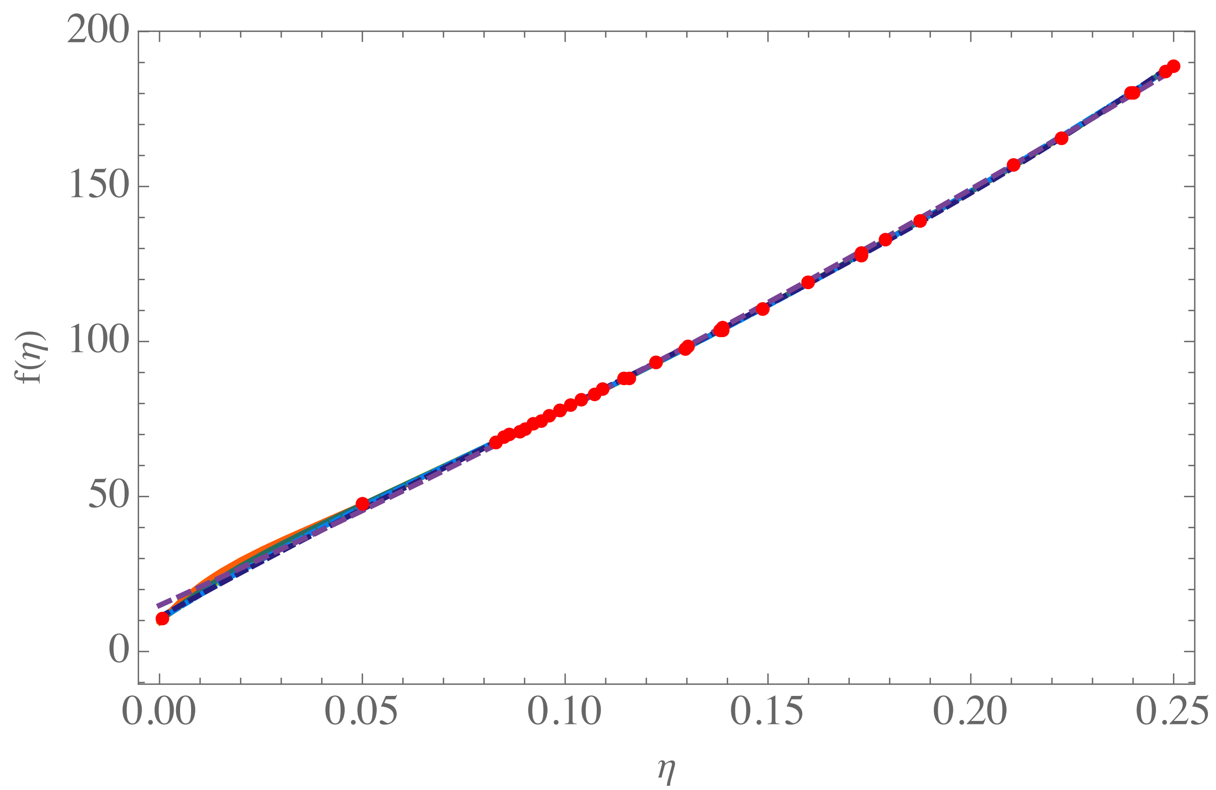


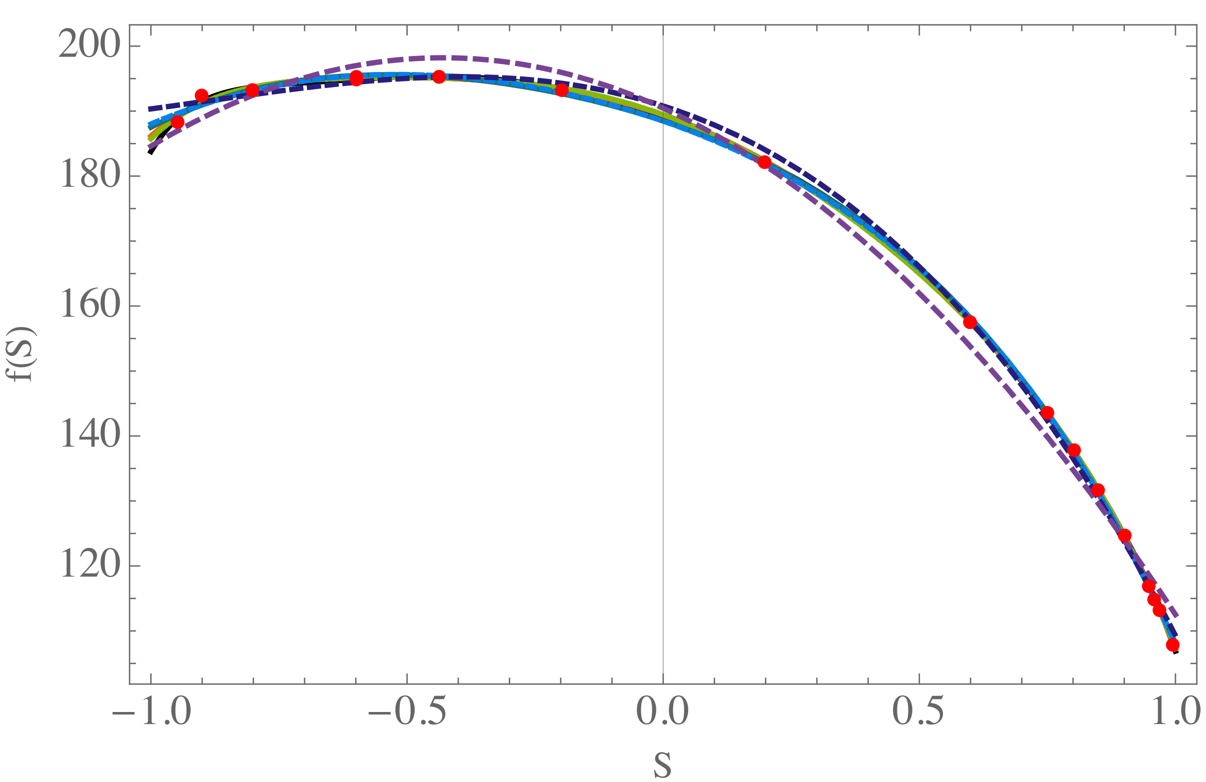
Jiménez+ (PRD2017), Keitel+ (arXiv:2016)

Hierarchical Fit: 1D Fits

Start by constructing 1D fits: non-spinning and equal mass, equal spin

- Parameter space is densely sampled by accurate NR simulations
- Can construct well conditioned and tightly constrained fits



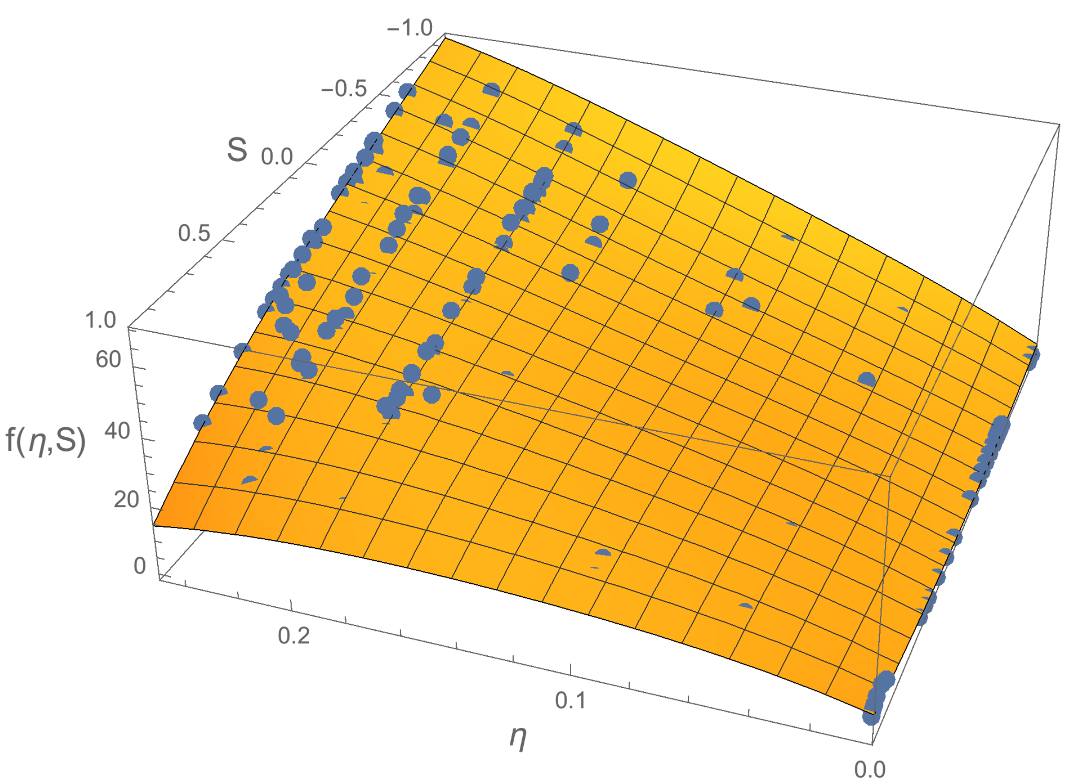


Hierarchical Fit: Equal Spin 2D Fit

- Fits informed using information criteria: BIC, AIC, AICc and residuals

$$\mathrm{BIC} = -2 \ln \mathcal{L}_{\mathrm{max}} + \mathrm{N}_{\mathrm{coeff}} \ln(\mathrm{N}_{\mathrm{data}})$$

- Start from 1D non-spinning $f(\eta)$ or equal spin $f(S) \to$ 2D $f(\eta, S)$



- Can choose spin parameterisation:

$$\hat{S} = \chi_{\mathrm{eff}} = \frac{m_1 \chi_{1L} + m_2 \chi_{2L}}{m_1 + m_2}$$

Works well for final-state

$$\hat{S} = \frac{m_1^2 \chi_{1L} + m_2^2 \chi_{2L}}{m_1^2 + m_2^2}$$

$$\hat{S} = \hat{\chi}_{\mathrm{PN}} = \left(1 - \frac{76\eta}{113}\right)^{-1} \left(\chi_{\mathrm{eff}} - \frac{38\eta}{113}(\chi_{1L} + \chi_{2L})\right)$$

Leading order spin effect on binary phasing - good for IMR

Hierarchical Fit: Unequal Spins

Unequal spin effects are a subdominant effect

- Functional form can be inferred from PN & BH perturbation theory

$$\mathcal{F}_{\mathrm{SO}}^{\mathrm{LO}} \propto \sqrt{1-4\eta}\left(-\frac{13}{16}+\frac{43}{4}\eta\right)\Sigma_\ell$$

- Motivates a general fit of the form

$$\lambda\left(\eta,\hat{S},\Delta\chi\right) = f_1(\eta)\Delta\chi + \underbrace{f_2(\eta)\Delta\chi^2 + f_3(\eta)\hat{S}\Delta\chi}_{\text{Next-to-Leading Order}}$$

$$f_1(\eta) = \sqrt{1-4\eta}P(\eta)$$

- In equal mass limit terms linear in $\Delta\chi$ must vanish due to $\sqrt{1-4\eta}$

$$\Sigma_\ell = m_2\,\vec{\chi}_2\cdot\vec{\ell} - m_1\,\vec{\chi}_1\cdot\vec{\ell}$$

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Hierarchical Fit: Unequal Spins

Fit residual of 2D surface with unequal spin effects

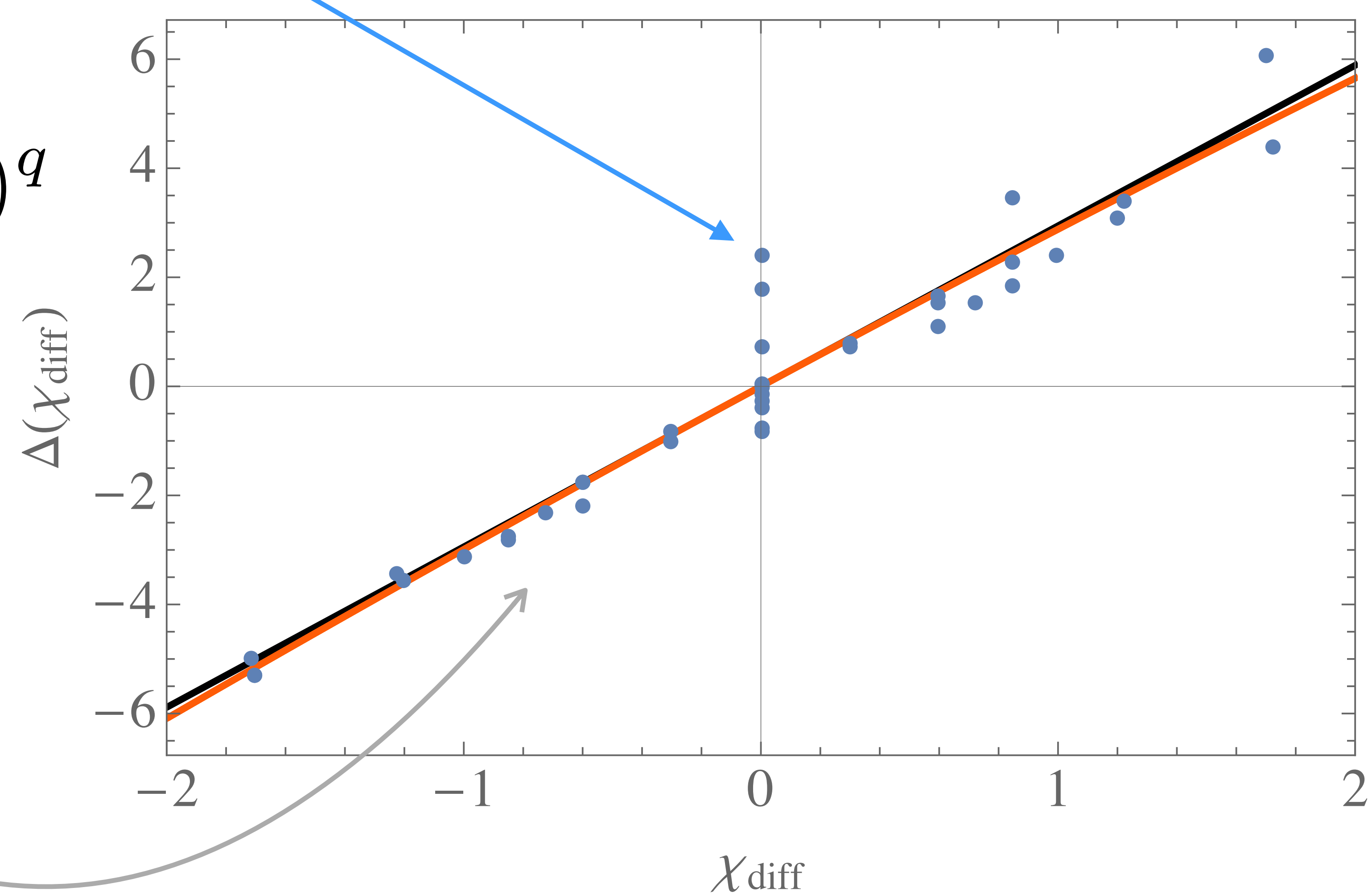
- Take dominant linear contribution to avoid overfitting noisy data

$$X = X_{Eq} + f(\eta)\,(\chi_1 - \chi_2) + \ldots$$

- General functional form

$$f(\eta) = a_0\,\eta^p(1-4\eta)^q$$

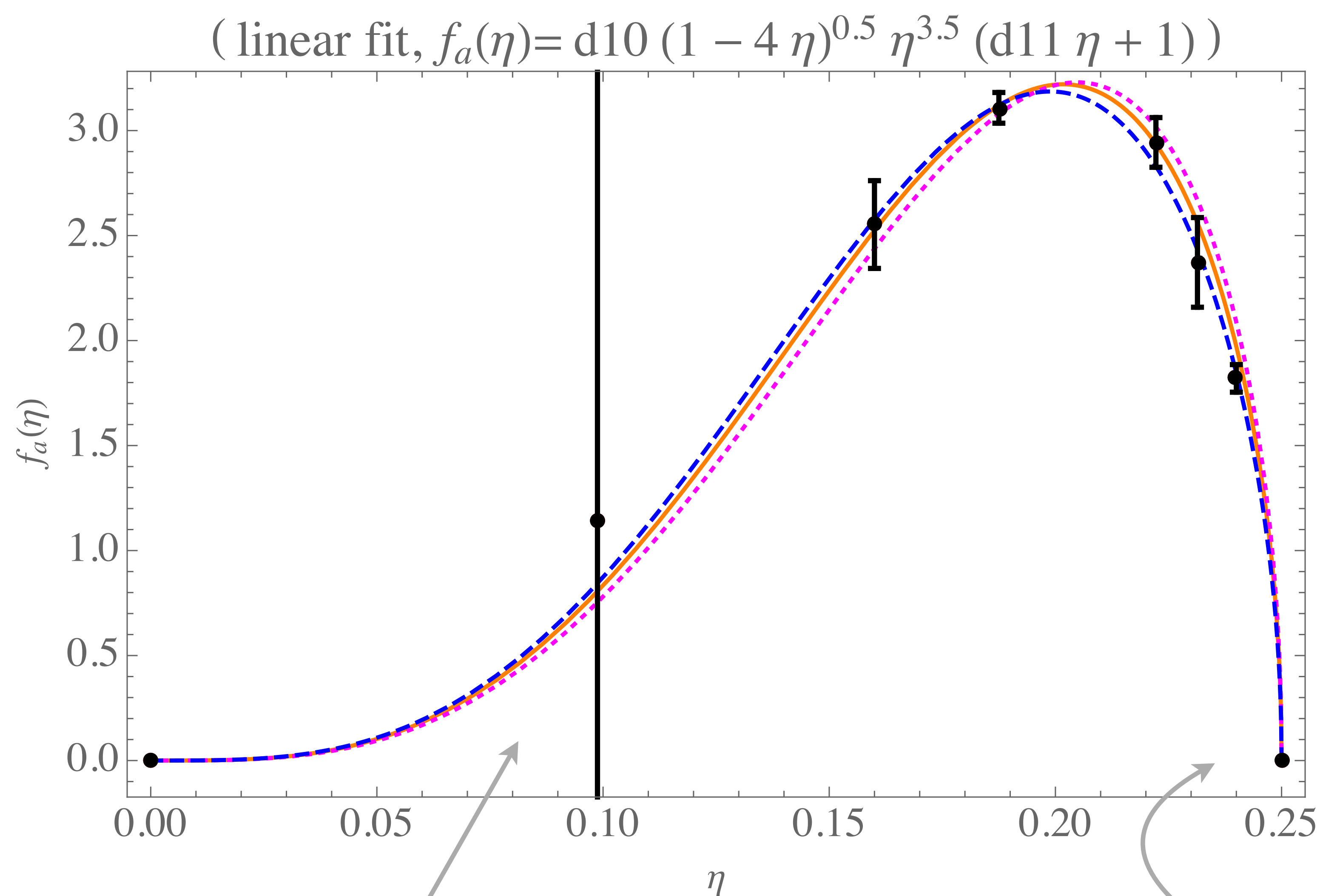
Outliers can be informative about data quality or poor ansatz



Fits per mass ratio: linear and quadratic

Hierarchical Fit: Unequal Spins

- We can now perform the same procedure to fit all phenomenological coefficients!
- More prominent in some coefficients - physical intuition behind this as well!



Per-η fits, residual fits and direct 3D fits should be consistent!

Error bars larger/unreliable in regions poorly sampled by NR

Vanishes in equal mass limit

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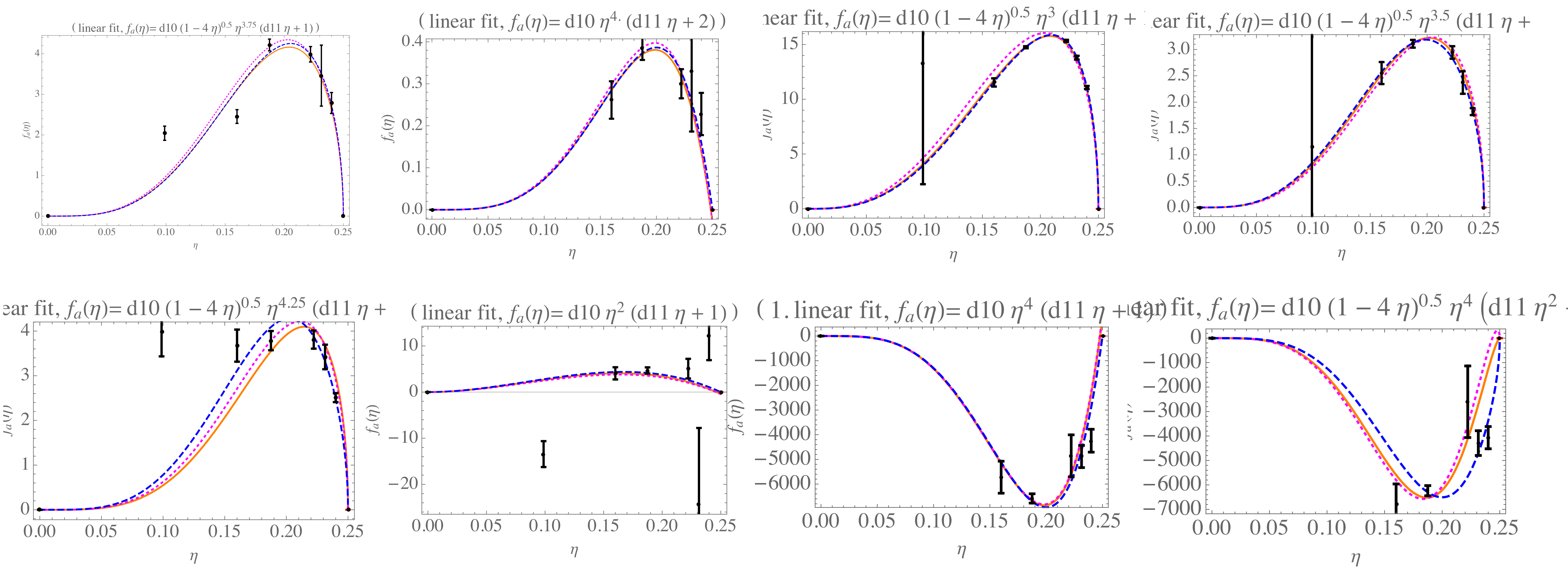
Hierarchical Fits: Unequal Spins, Outlook

- Rapid improvements with *more data* and better *control of systematics*



Model Validation: Preliminary Mismatches

Standard model validation tools:

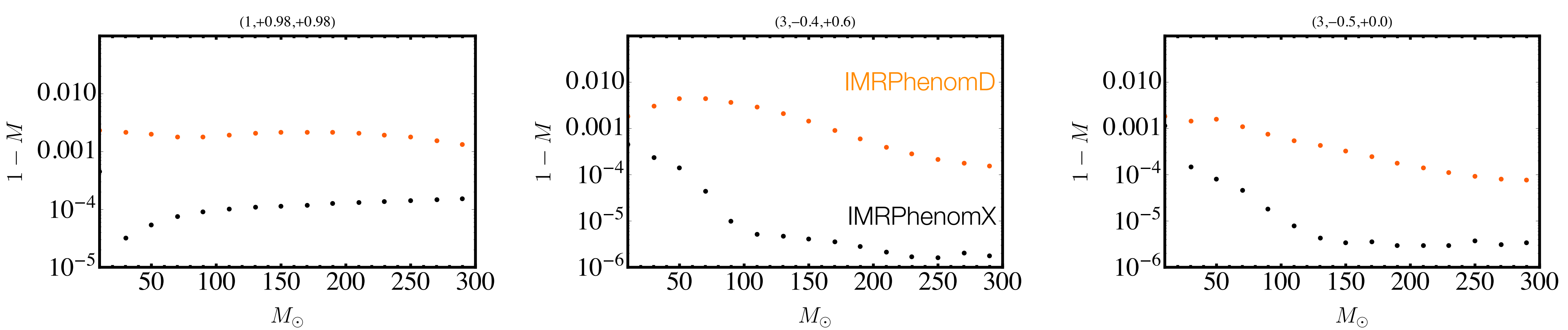
- The overlap is the noise weighted inner product

$$\langle h_1, h_2 \rangle = 4 \; \mathrm{Re} \int_{f_{\min}}^{f_{\max}} \frac{\tilde{h}_1(f) \; \tilde{h}_2^*(f)}{S_n(f)} \, df$$

- Calculate *mismatch* between *normalised* waveforms $\hat{h} = h / \sqrt{\langle h, h \rangle}$

$$\mathcal{M}(h_1, h_2) = 1 - \max_{t_0, \phi_0} \langle \hat{h}_1, \hat{h}_2 \rangle$$

- Indistinguishability $\mathcal{M} \lesssim (2\rho^2)^{-1} \leftarrow \rho \lesssim 25$ need $\mathcal{M} \lesssim 8 \times 10^{-4}$



Summary

- Focus has been on completing non-precessing 22-mode model (HM extensions discussed by Cecilio!)
  - *Need* high fidelity models for O3 and beyond (ET)!
  - Have robust method to fit to the non-precessing parameter space [Jiménez+ (PRD2017), Keitel+ (arXiv:2016)]
  - Have calibrated for the unequal spin effects
  - *Extreme mass ratio limit* constrains model - avoids pathological boundary conditions
  - Matching regions guided by *MECO*, *ISCO* and *ringdown* frequencies
  - Preliminary typical matches @ 20-200 solar masses ~ 10^{-4} to 10^{-6}
  - In Progress: Parameter estimation / Improved precessing models