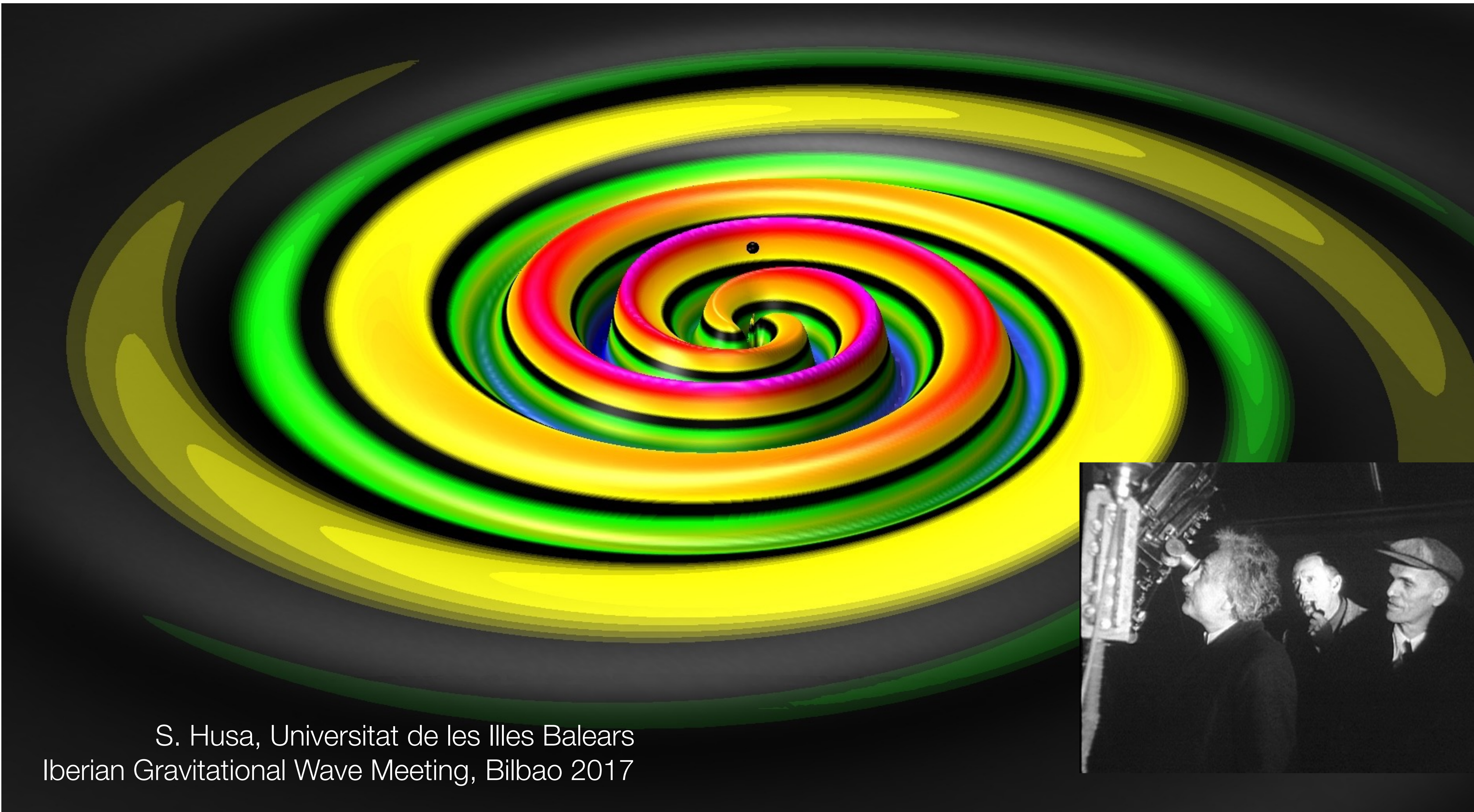
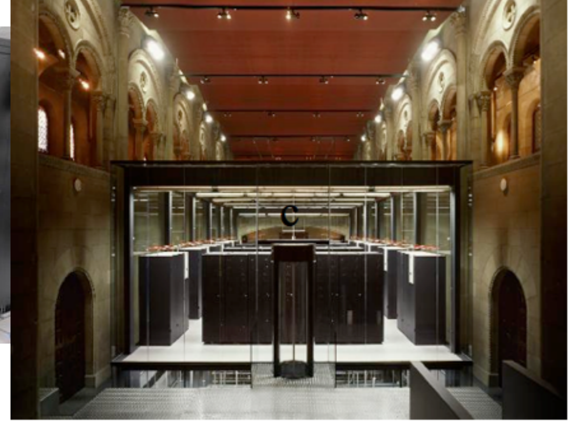
Modelling the dynamics and gravitational wave signal of coalescing black holes



S. Husa, Universitat de les Illes Balears
Iberian Gravitational Wave Meeting, Bilbao 2017



PRACE

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MultiDark
Multimessenger Approach for Dark Matter Detection

Thanks to collaborators in waveform modelling!

Contributors: Geraint Pratten, Marta Colleoni, Cecilio Garcia, Antoni Ramos, Michael Pürrer, Mark Hannam, Sebastian Khan, Frank Ohme, Alejandro Bohé, Francisco Jimenez, Juan Calderon, Alicia Sintes, Bernd Brügmann, Nathan Johnson-McDaniel, Patricia Schmidt, Marcus Thierfelder, Vijay Varma, Parameswaran Ajith, Daniela Paredes, Daniel Ruiz Reynes, Alfred Castro, David Keitel, Leila Haegel, Marcus Thierfelder, Wolfgang Tichy.

UIB Universitat de les Illes Balears

RES
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Ingenio 2010

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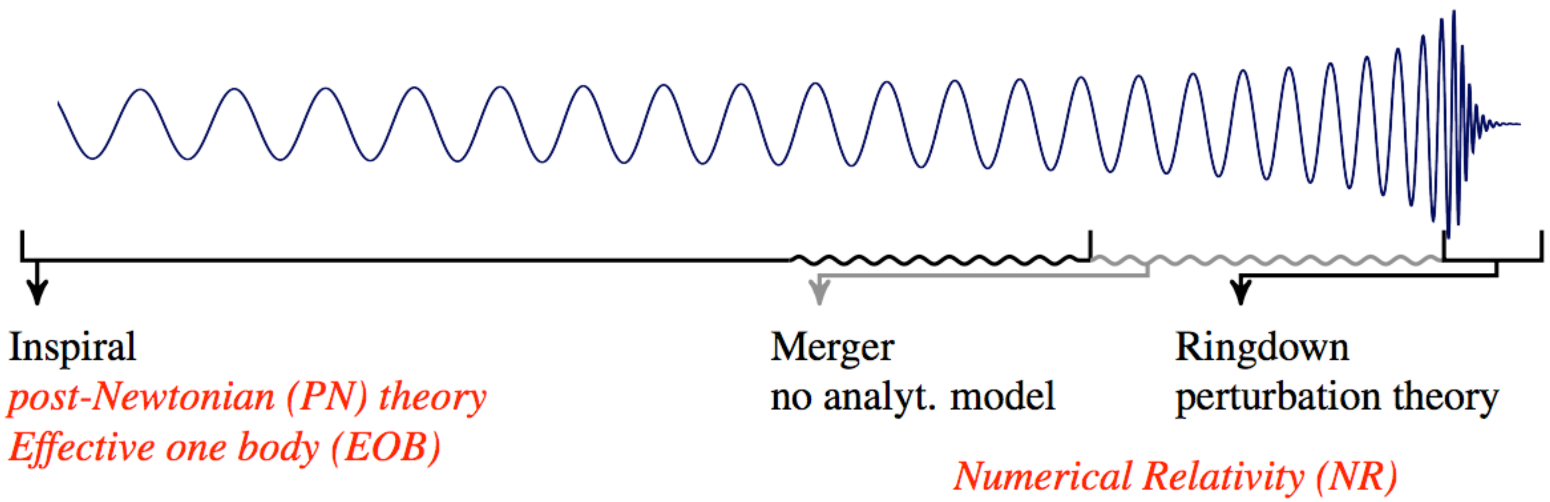
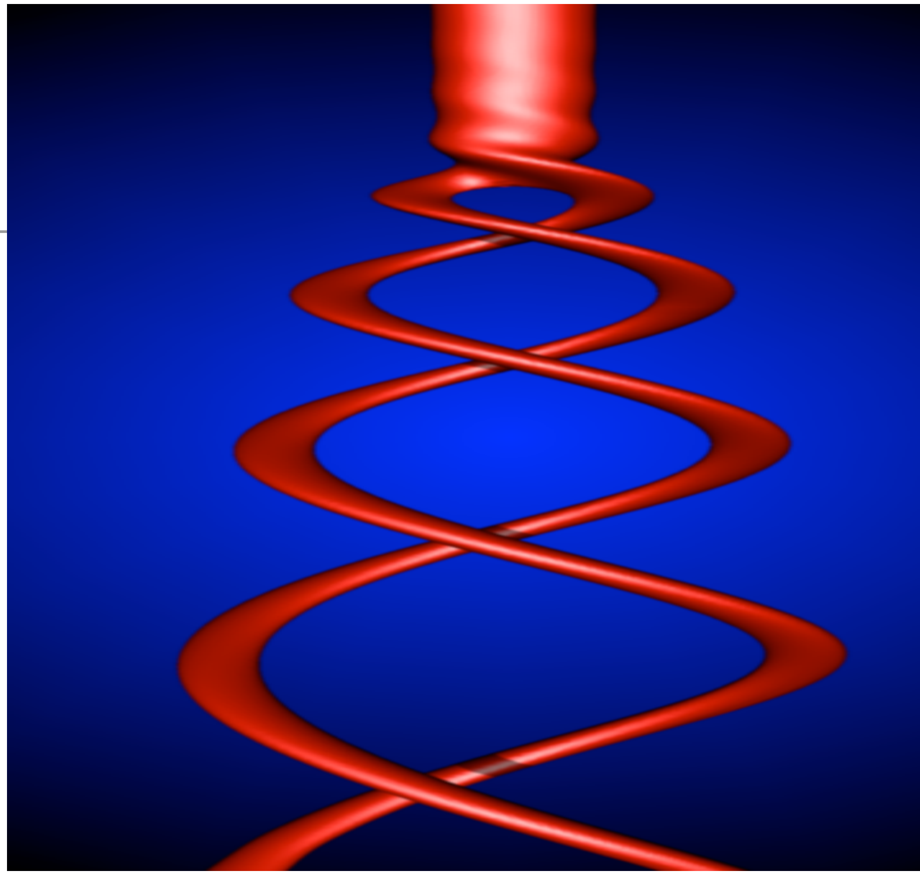
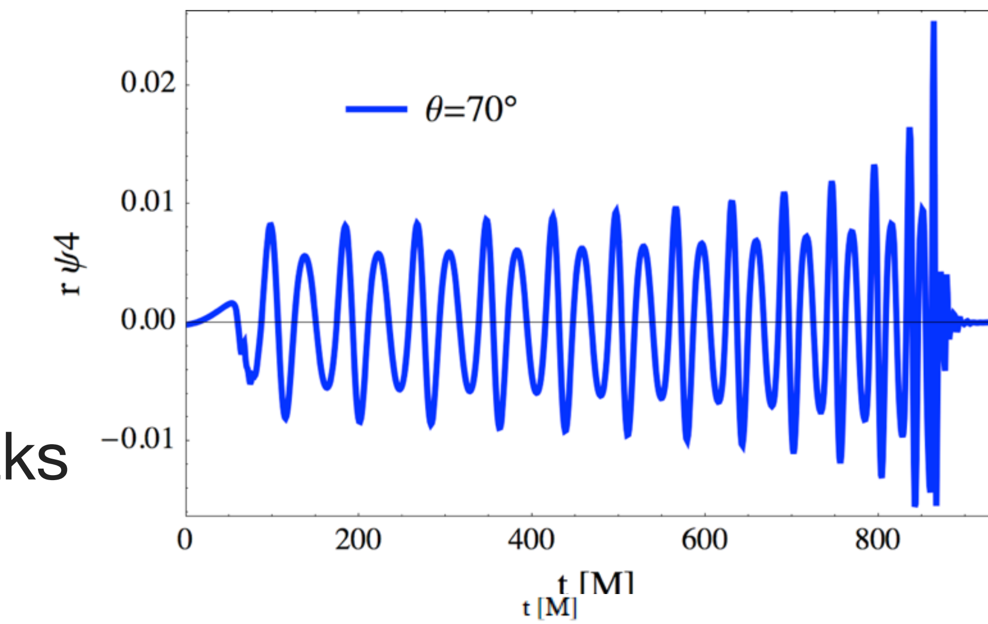
ICTS INTERNATIONAL CENTRE for THEORETICAL SCIENCES
TATA INSTITUTE OF FUNDAMENTAL RESEARCH

Anatomy of BH coalescence

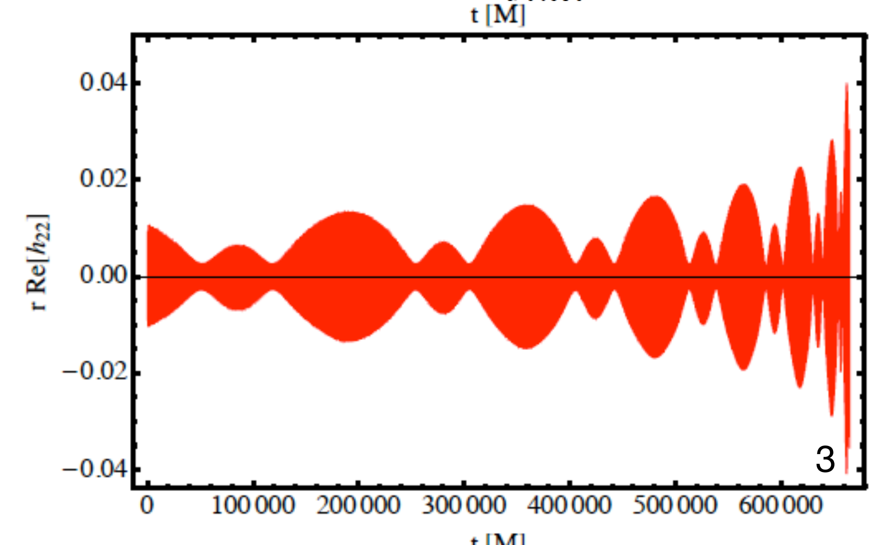




- Inspiral: energy loss to GWs leads to adiabatic inspiral, well described by post-Newtonian perturbation theory.



- Late inspiral & merger: post-Newtonian expansion breaks
    - solve full Einstein equations numerically as PDEs, “match” to post-Newtonian inspiral.
    - Most of the energy released (< 12 % of the mass).
- Ringdown: superposition of damped harmonics, frequencies known from perturbation theory.



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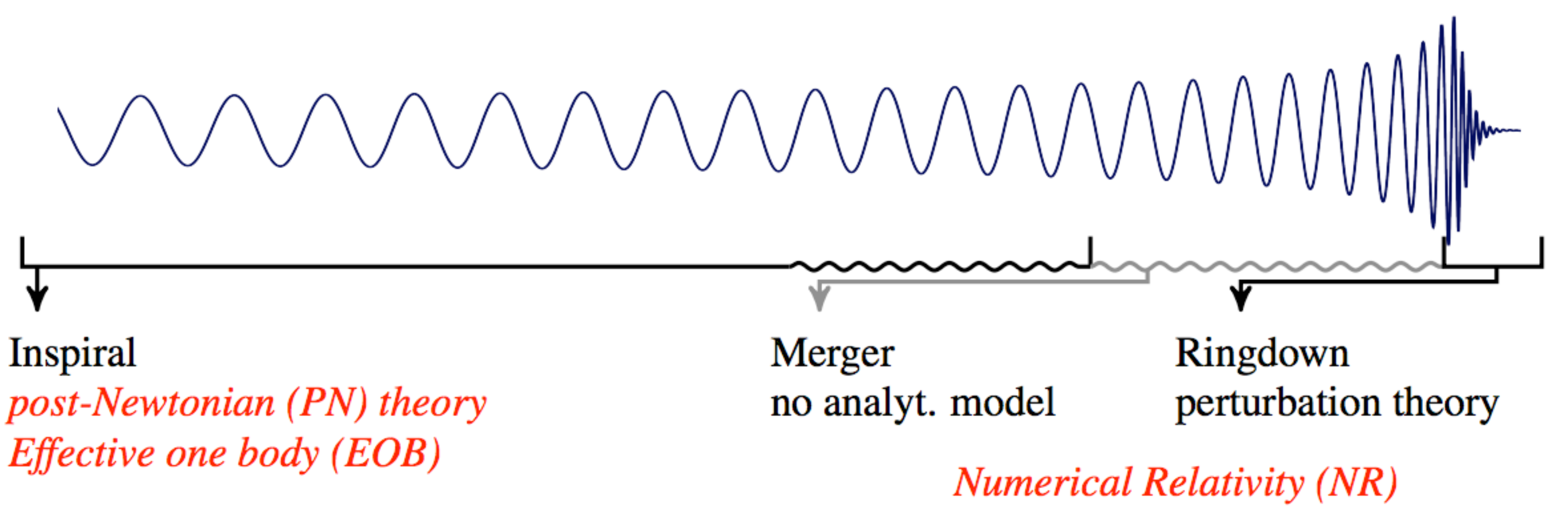

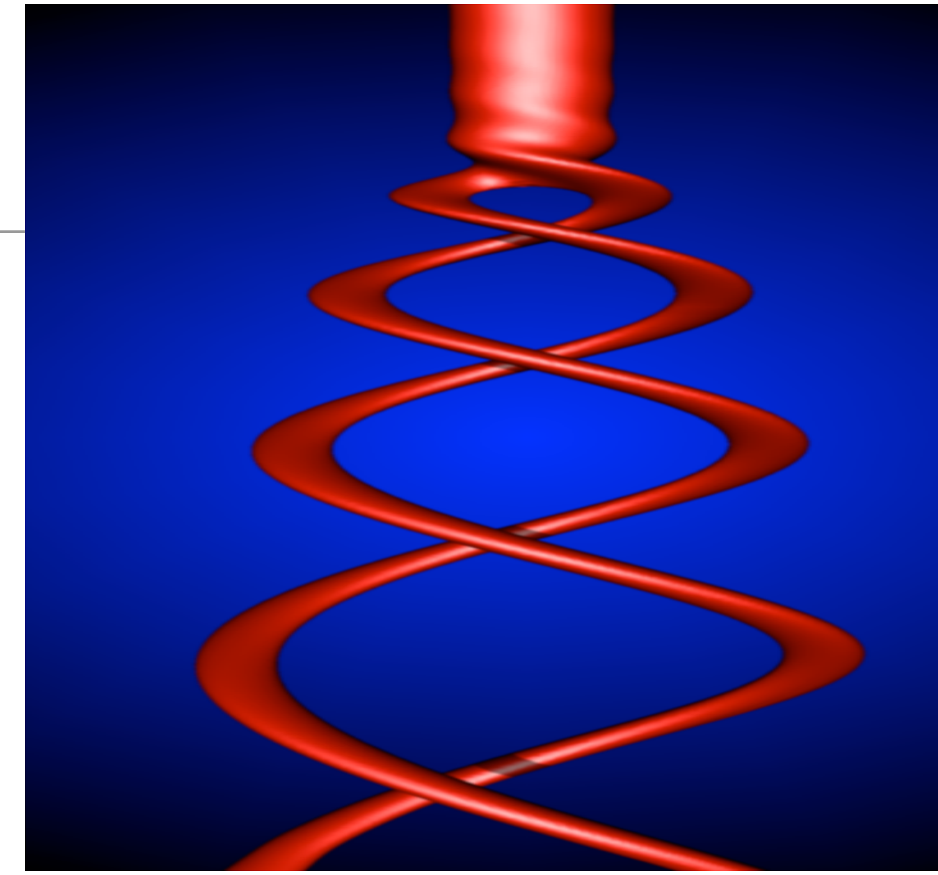

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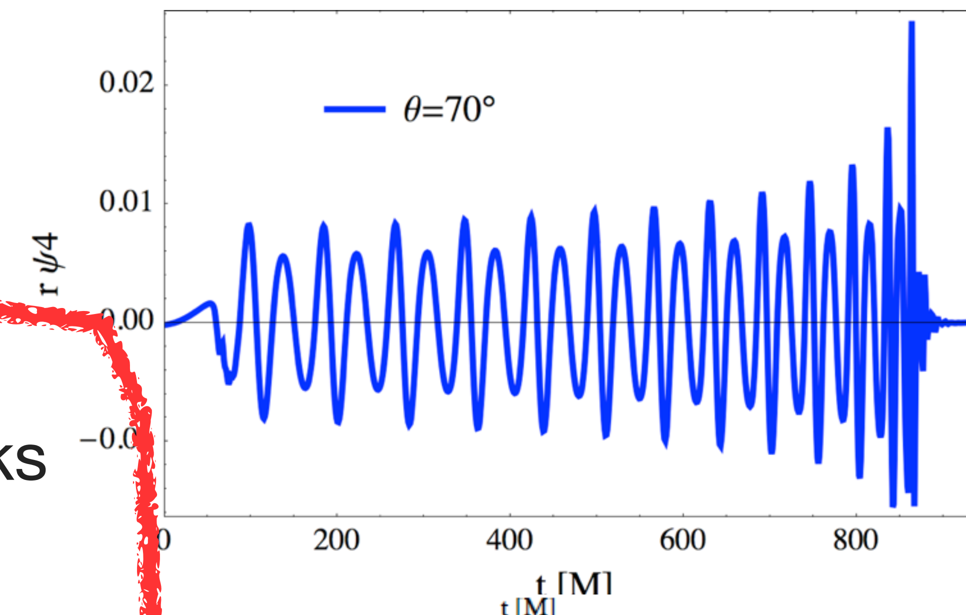

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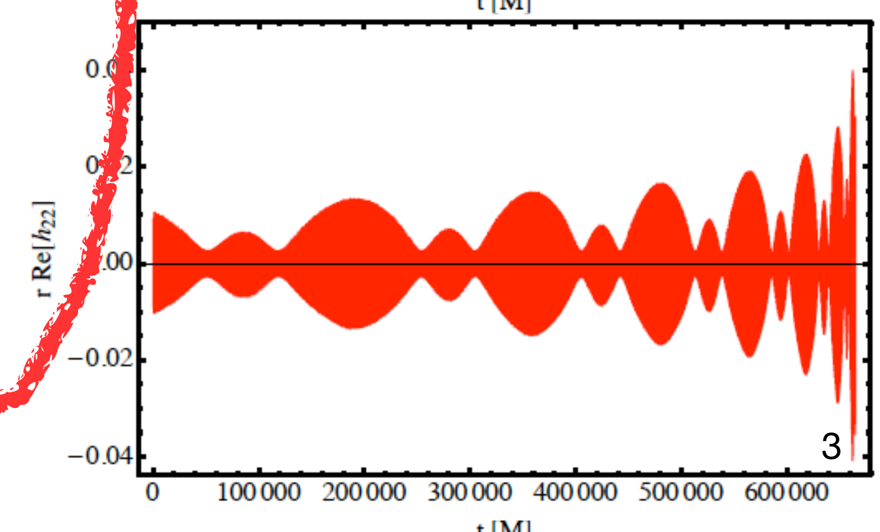
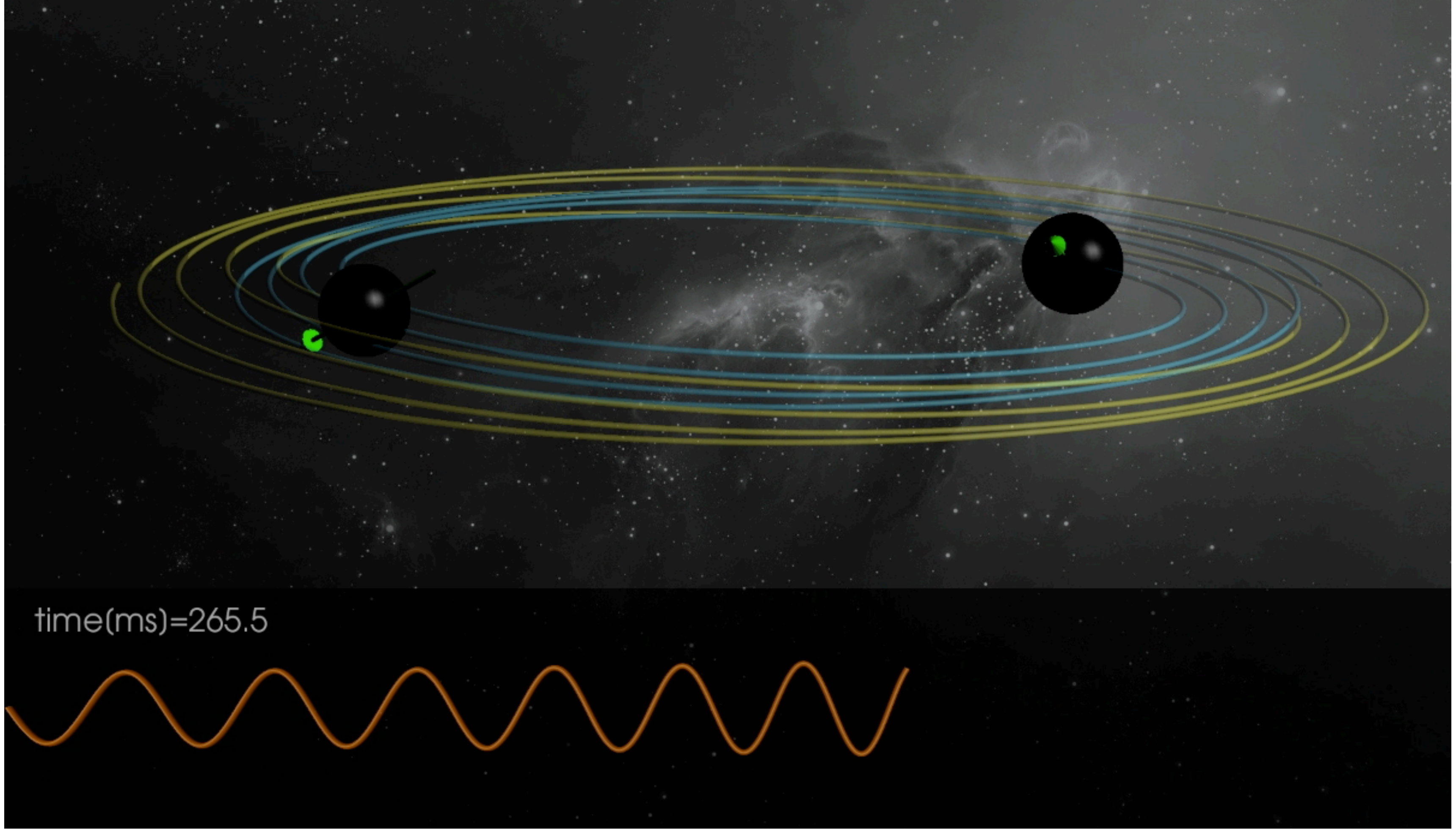
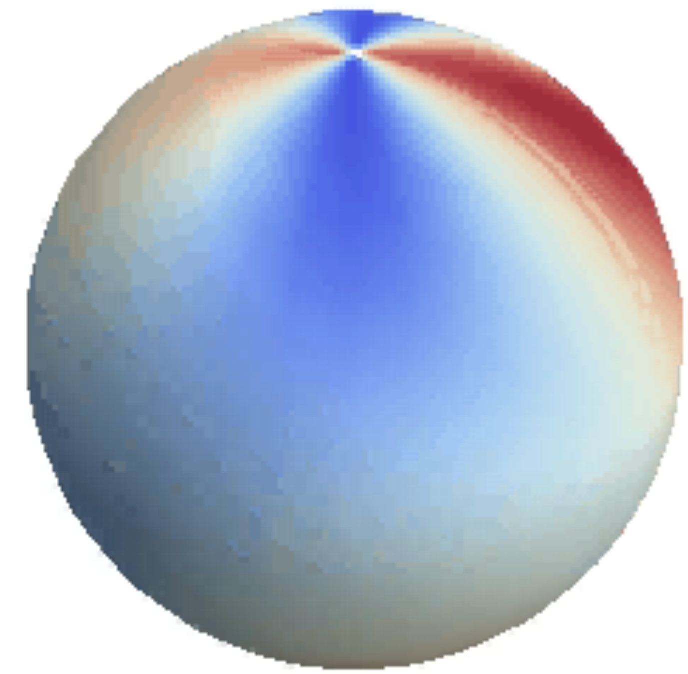




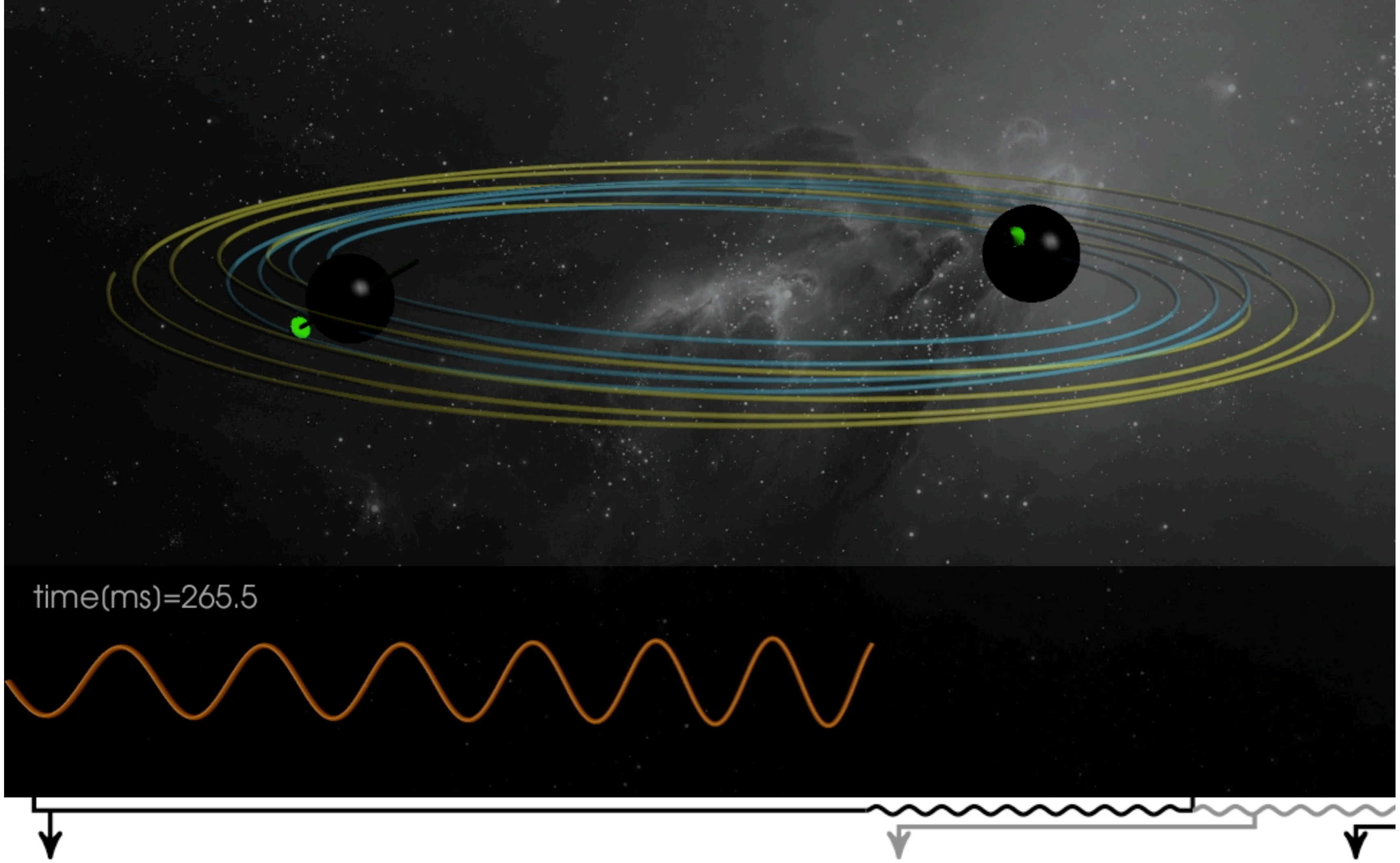


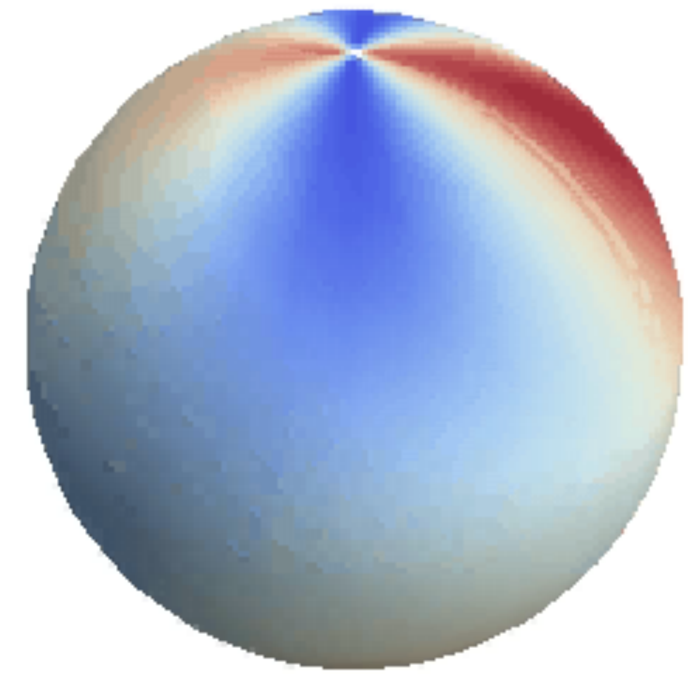
Goal:

- Synthesise model of the complete waveform across the parameter space from PN/EOB, BH perturbation theory, self-force, numerical relativity, …
- model: surrogate for true waveform, approximate “formula” that interpolates parameter space and maps physical parameters to a waveform.

- understand errors; ensure sufficient accuracy for LIGO, Virgo, 3rd Generation, LISA, …

- understand what can be measured: individual spins? test GR?





Inspiral
post-Newtonian (PN) theory
Effective one body (EOB)

Merger
no analyt. model

Ringdown
perturbation theory

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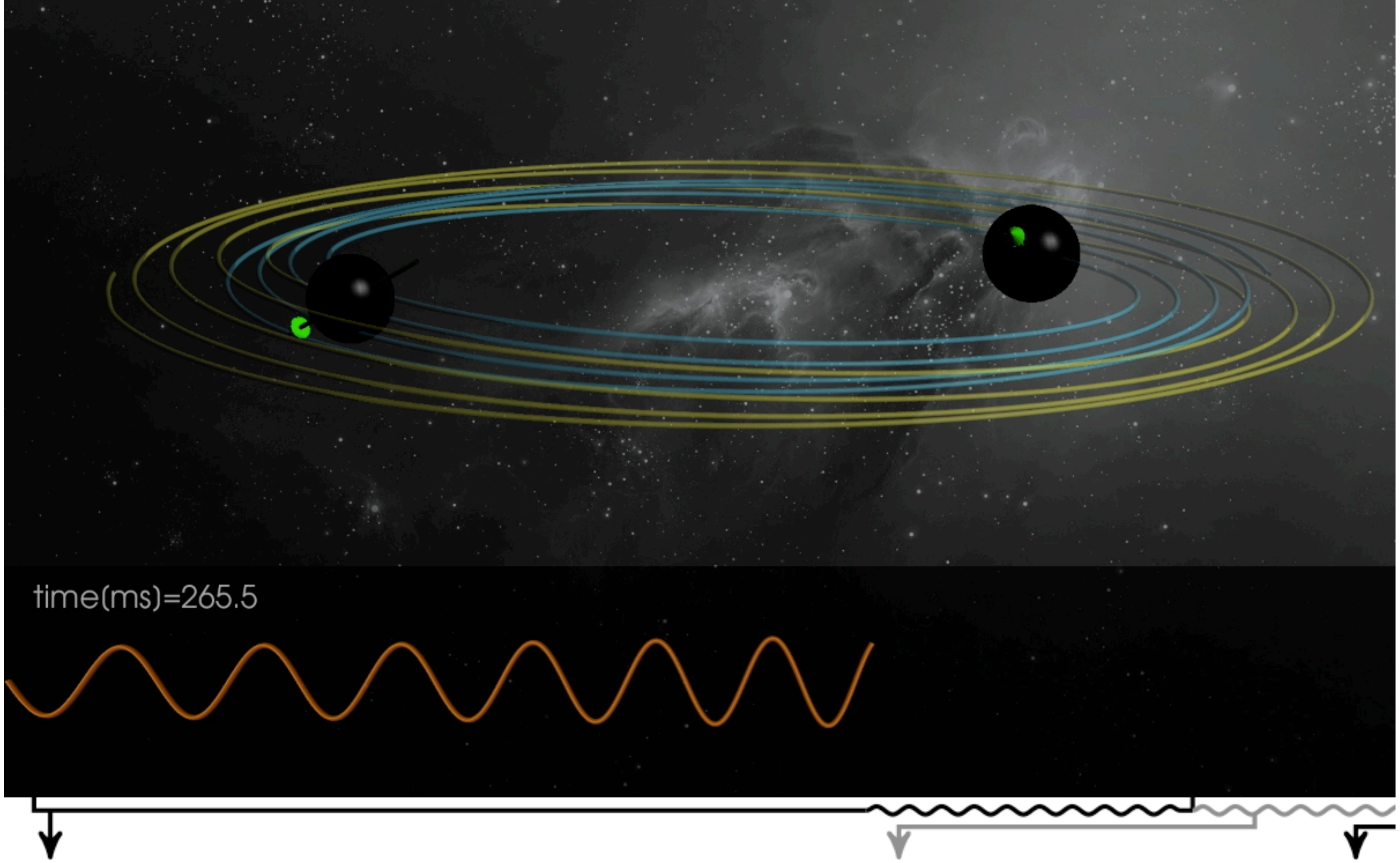

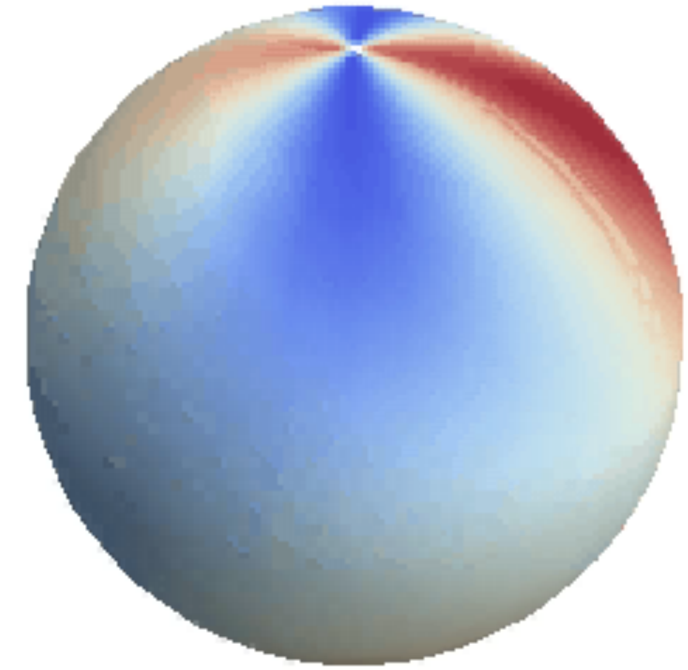

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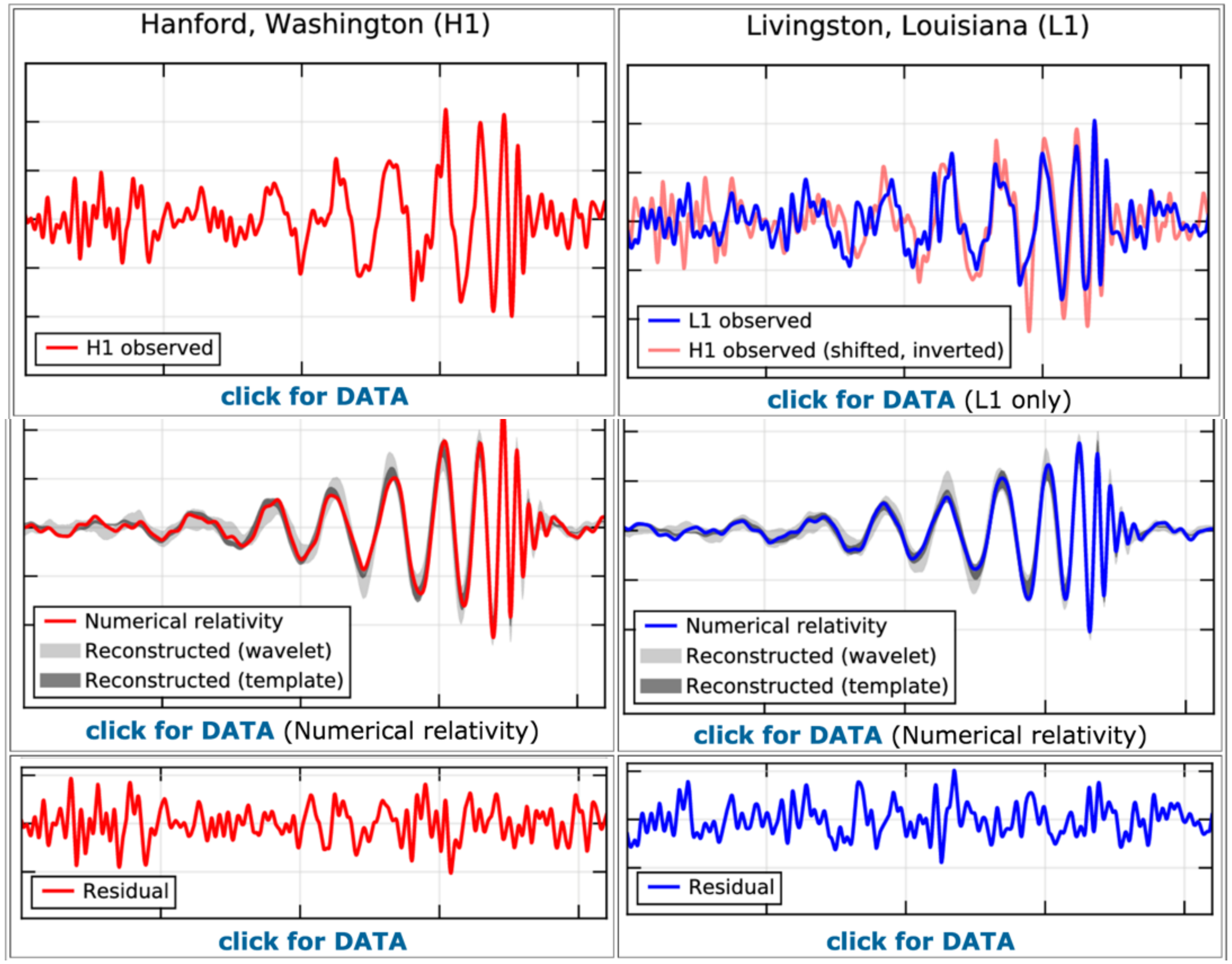
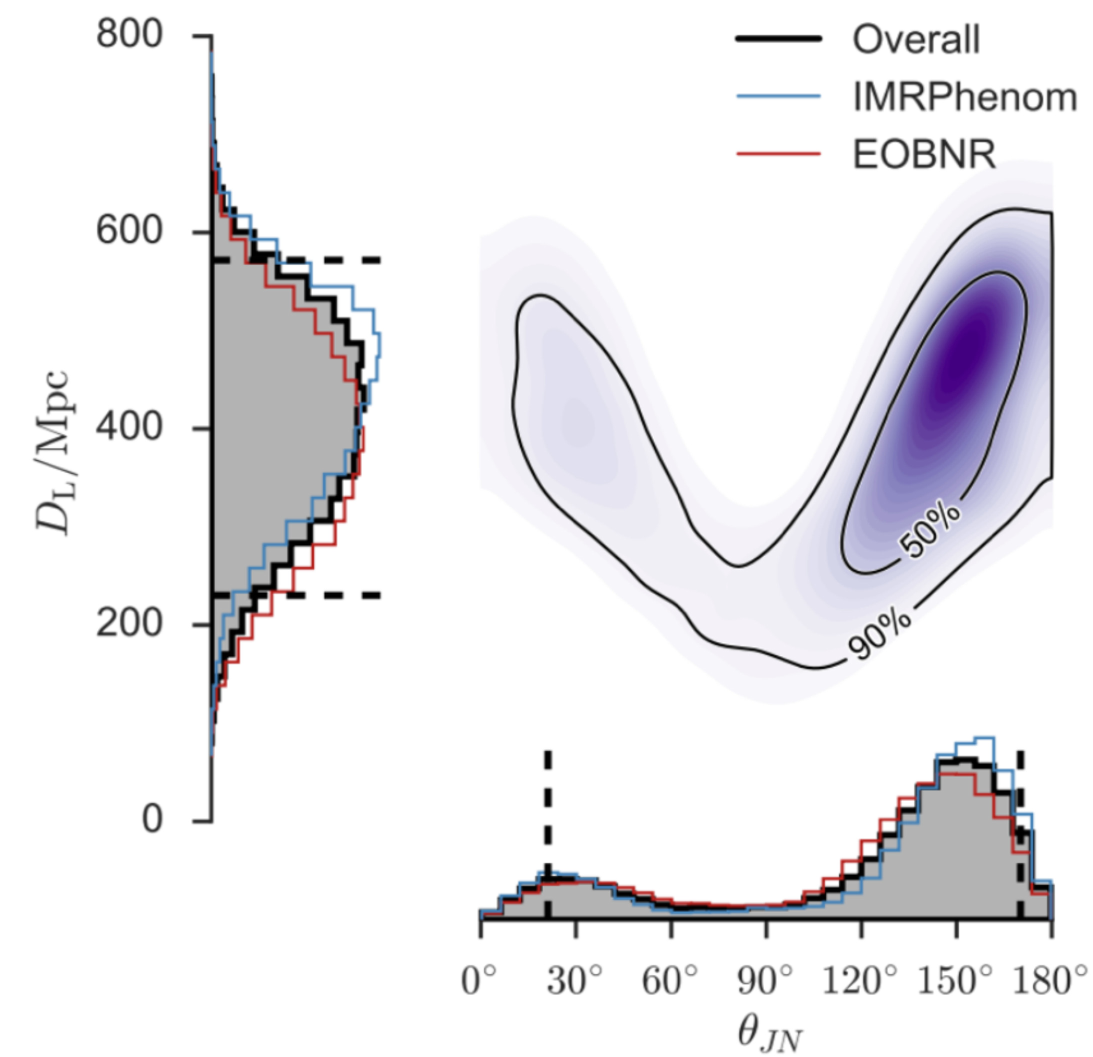
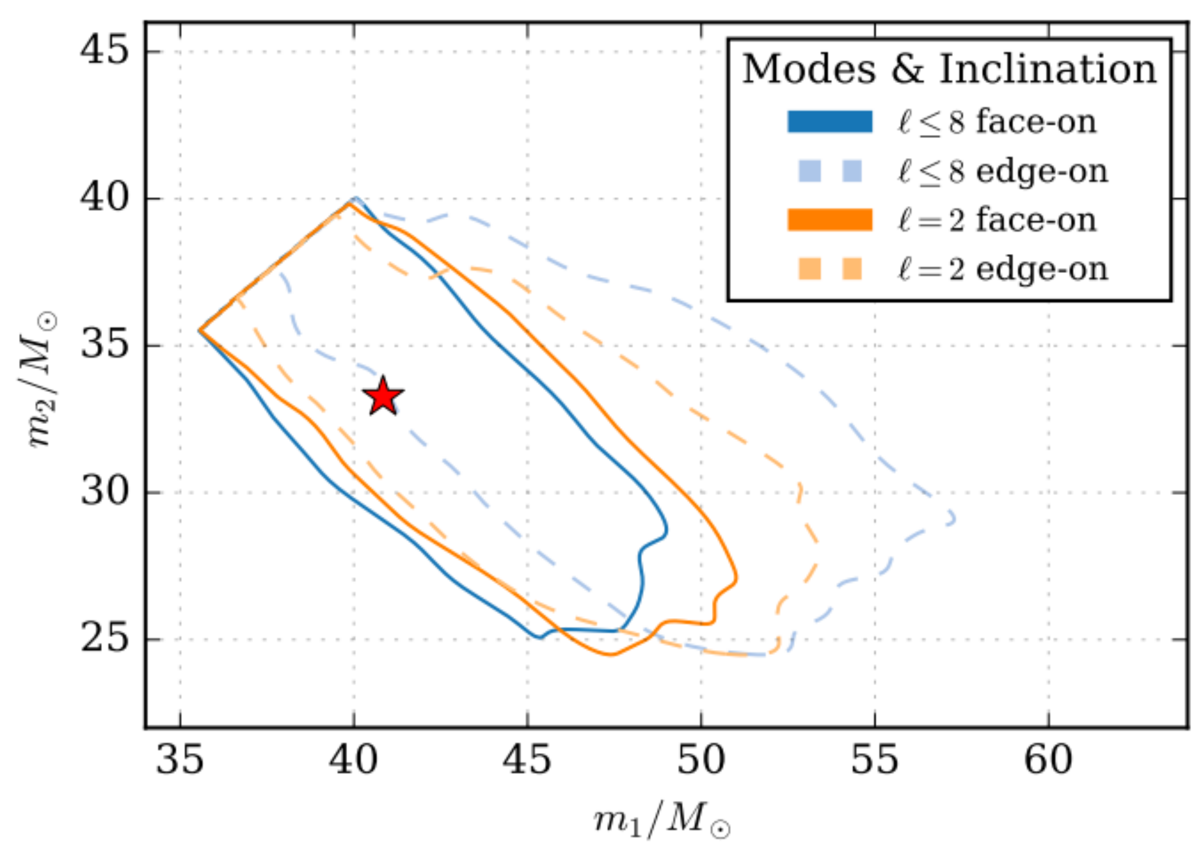




EOB [Buonanno+Damour 1998] & Phenomenological waveform model families have been essential in identifying the sources of the first GW detections.



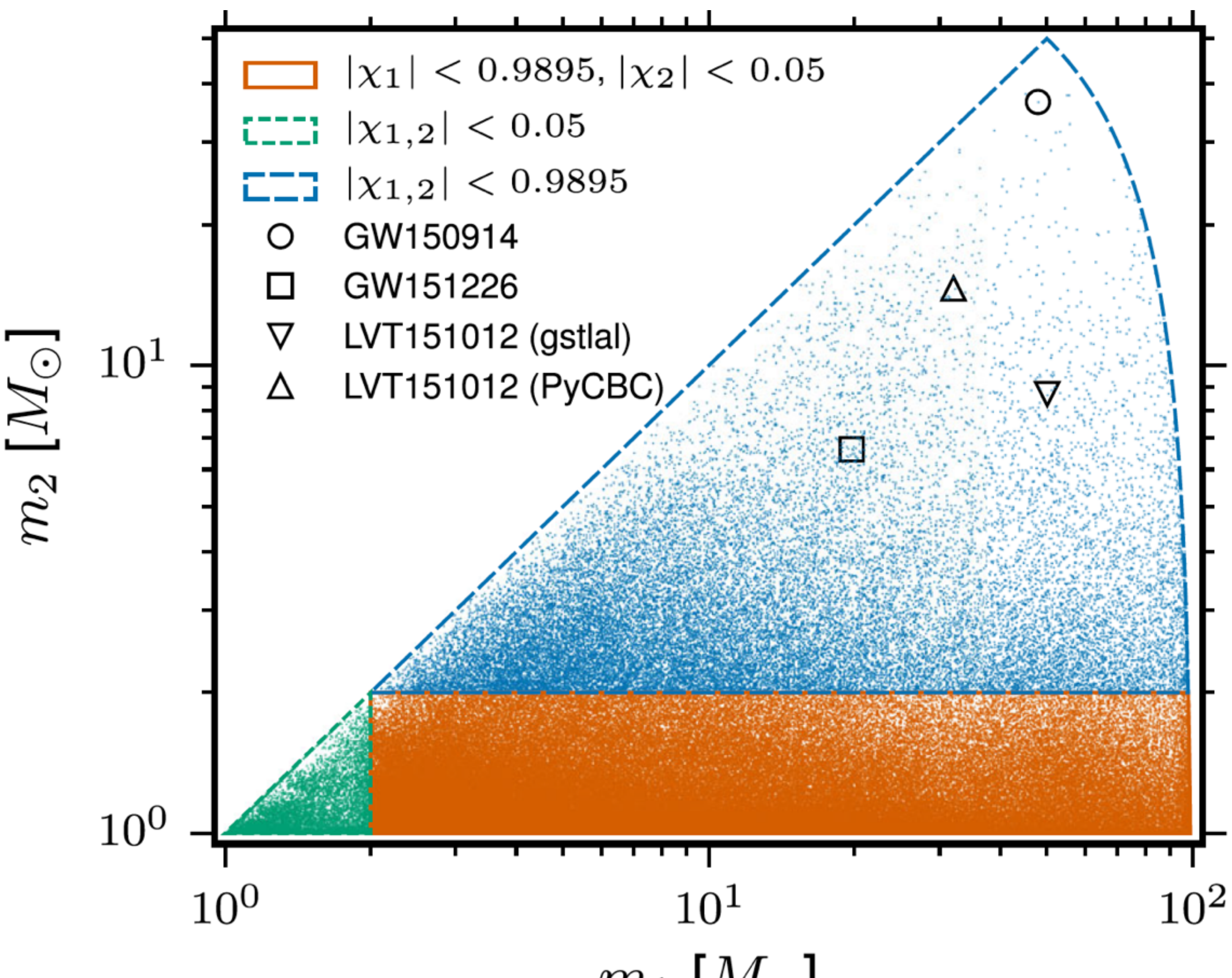
The models have shortcomings, but they do not significantly affect the first detections:

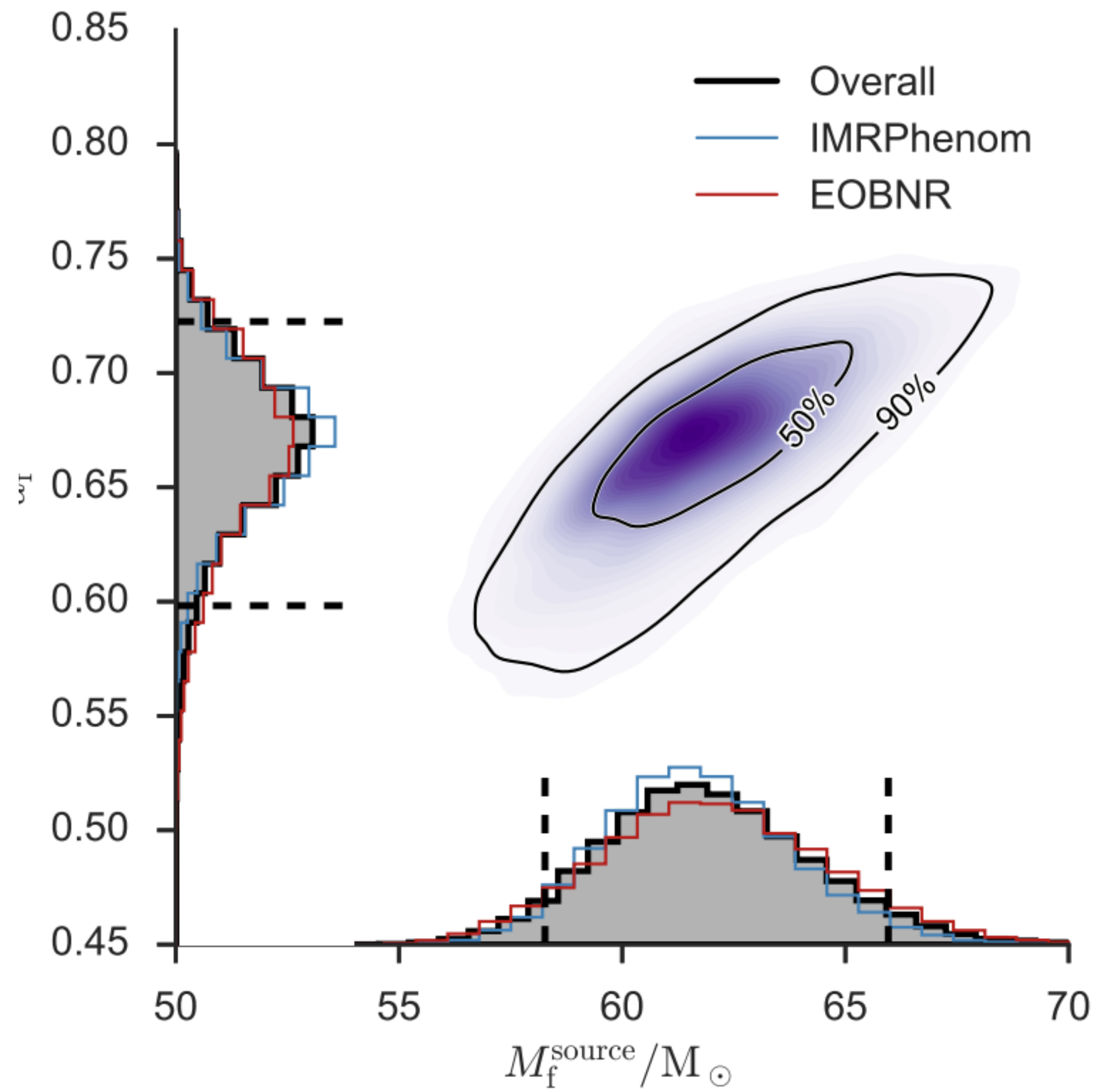
“Effects of waveform model systematics on the interpretation of GW150914”, CQG 34 (2017) 104002 [LIGO+Virgo]

Searches vs. parameter estimation/model selection

- Compact binary coalescence (LIGO/Virgo CBC group) workflow: split data analysis into 2 parts:

- detection: what is the statistical evidence of seeing a signal above background, fixed template bank [rough parameter estimation].
- Bayesian parameter estimation: vary templates with random walks in parameter space, using MCMC etc., test consistency in waveform models.





Are Waveforms good enough?

WF error in matched filter context is naturally defined in terms of overlap:

$$\langle h_1, h_2 \rangle = \max_{\phi_0, t_0} 4\Re \int_{f_1}^{f_2} \frac{\tilde{h}_1(f)\, \tilde{h}_2^*(f)}{S_n(f)}\, df$$

$$||\Delta h||^2 = \langle h_1 - h_2, h_1 - h_2 \rangle$$

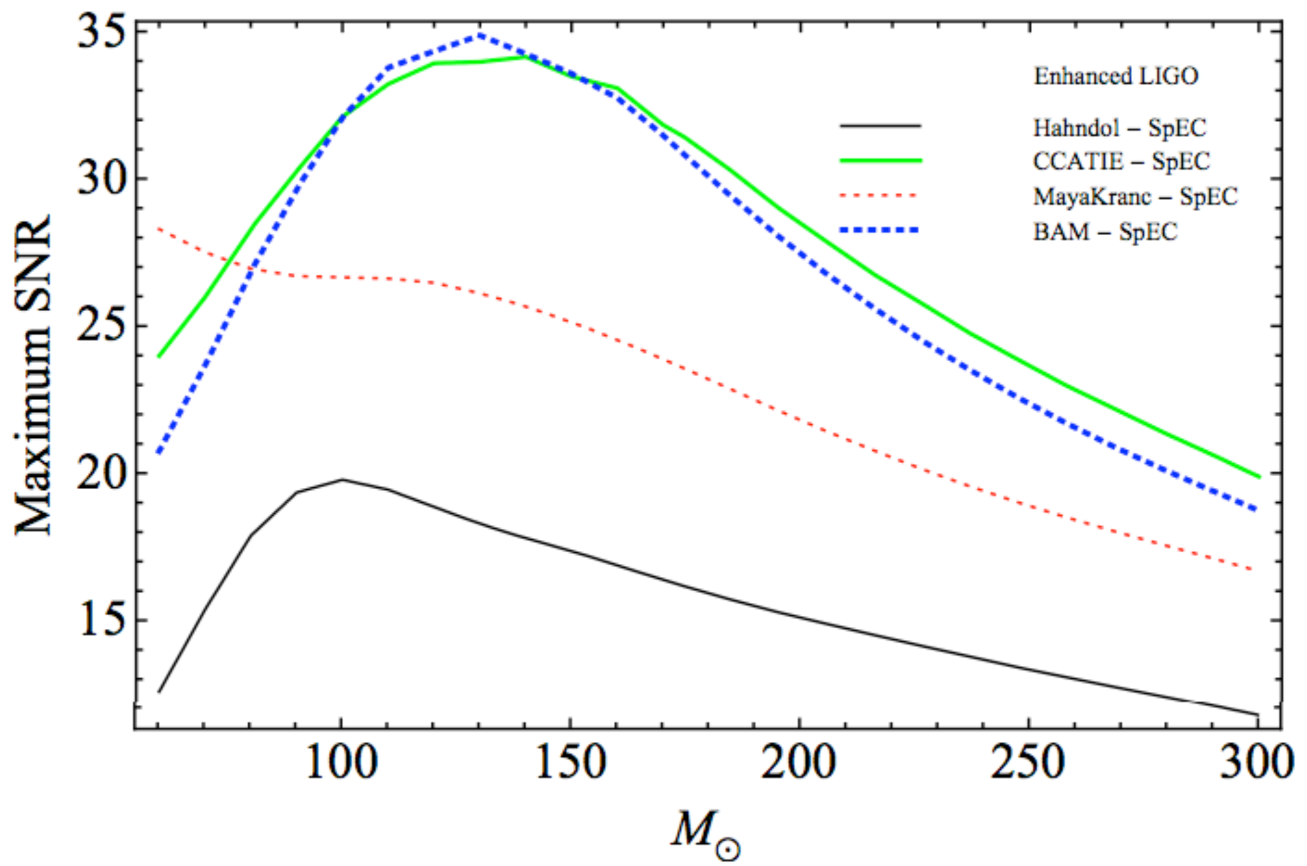
$$\mathcal{M} = 1 - \langle h_1, h_2 \rangle / (||h_1||\, ||h_2||)$$

SNR: $\rho = ||h||$

indistinguishable: $||\Delta h||^2 < 1$ Fairhurst, KITP 2008

$$||h_1|| \approx ||h_2|| \Rightarrow ||\Delta h||^2 = 2\rho^2 \mathcal{M}$$

Samurai: Hannam+ '09



M=3% ≅ **10 %** signal loss, **M=0.5 %, 5x10^{-5}, 5x10^{-7}** undistinguishable @ **SNR 10,100, 1000**

- Searches & parameter estimation use WF families - maximize over mass, spins, ...
- Computing M with fixed physical parameters can drastically overestimate accuracy requirements: small bias in physical parameters may have large effect on match.

From model to LIGO/Virgo data analysis

- So you have a new waveform model that resolves current shortcomings (higher modes, better precession, beyond GR, ….)!

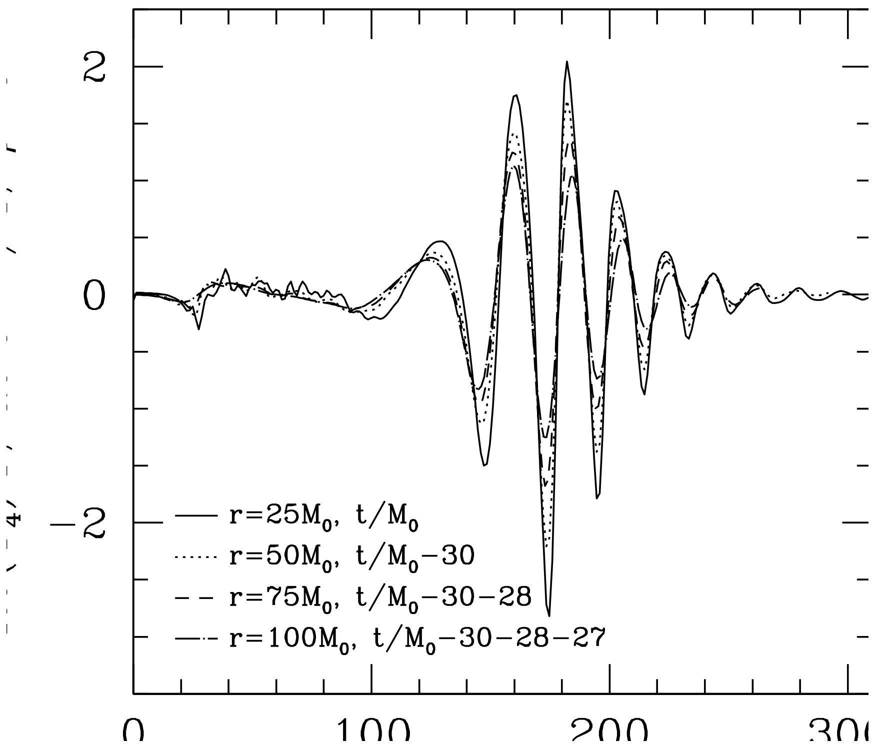
- Will it be used in data analysis? PROBABLY NOT

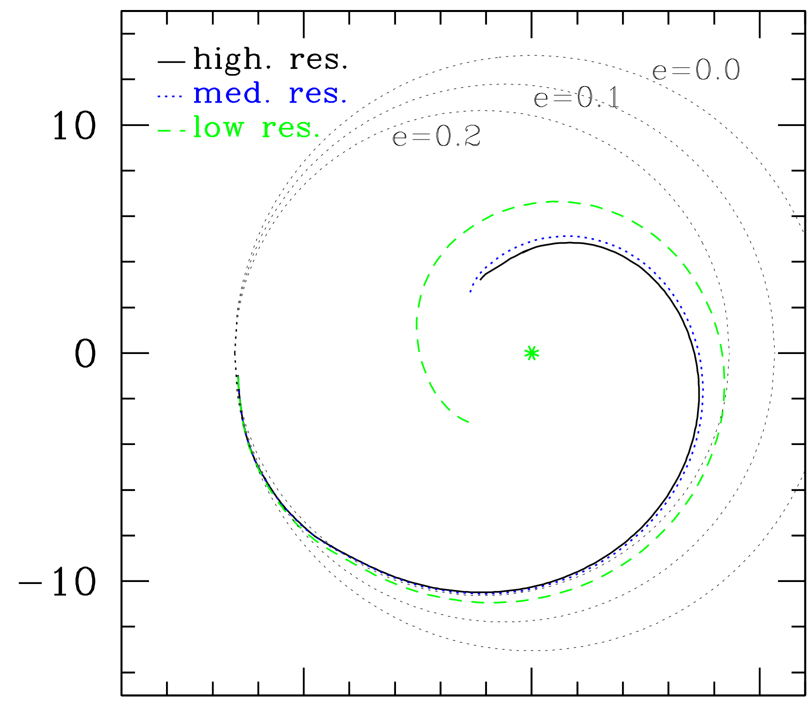
- Unless:
  - You make sure it is coded up in LAL (LIGO Algorithms Library)
  - You make sure the LAL code passes LIGO-Virgo review
  - Advertise in the CBC (compact binary coalescence) working group

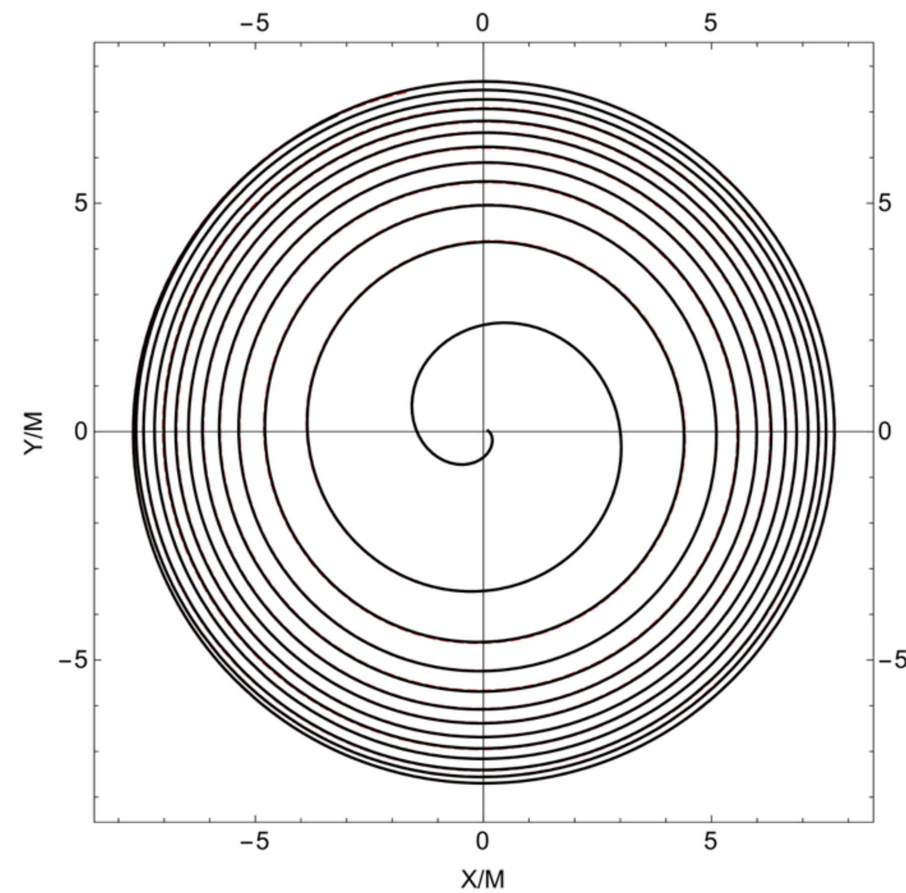
We have only learned how to evolve BHs a decade ago!

- First orbit + GWs: 2005 (Pretorius) - surprise breakthrough overcoming 4 decades of struggle with unstable algorithms.

- ~ 3 PhD theses from first NR waveforms to the inspiral-merger-ringdown models used during O1.







m1/m2=18, S=0.4

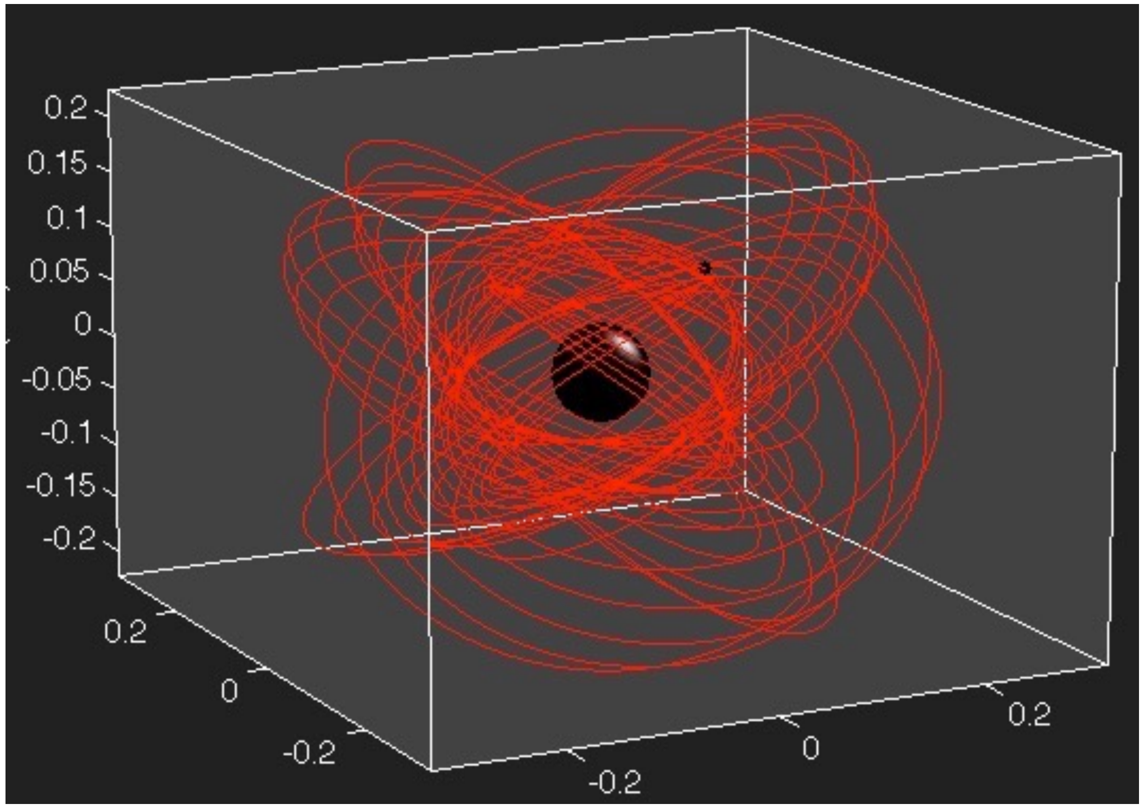
- BBH breakthrough happened at the same time as S4 science run: data analysis methods for compact binaries had to be built based on PN inspiral information until NINJA project (2008).

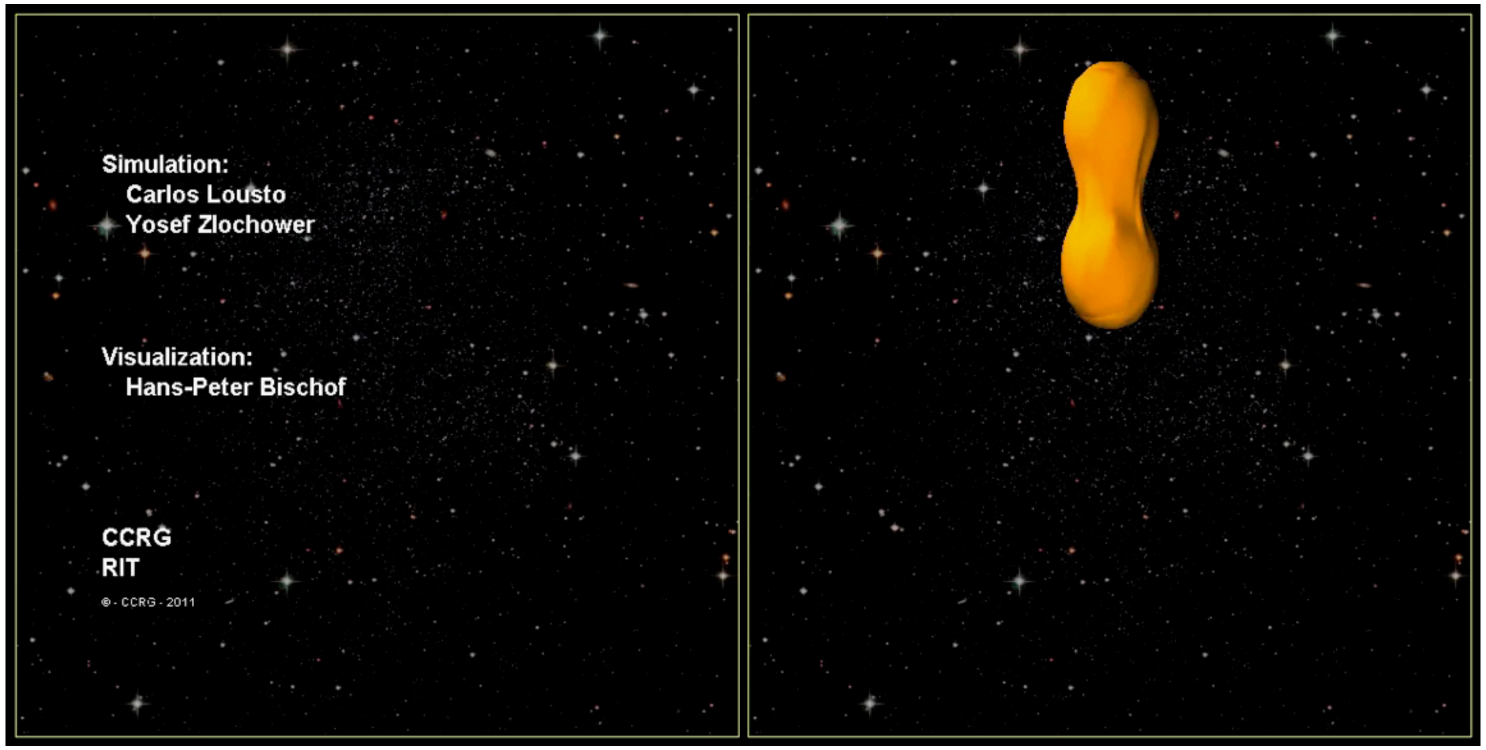
The Binary BH parameter space

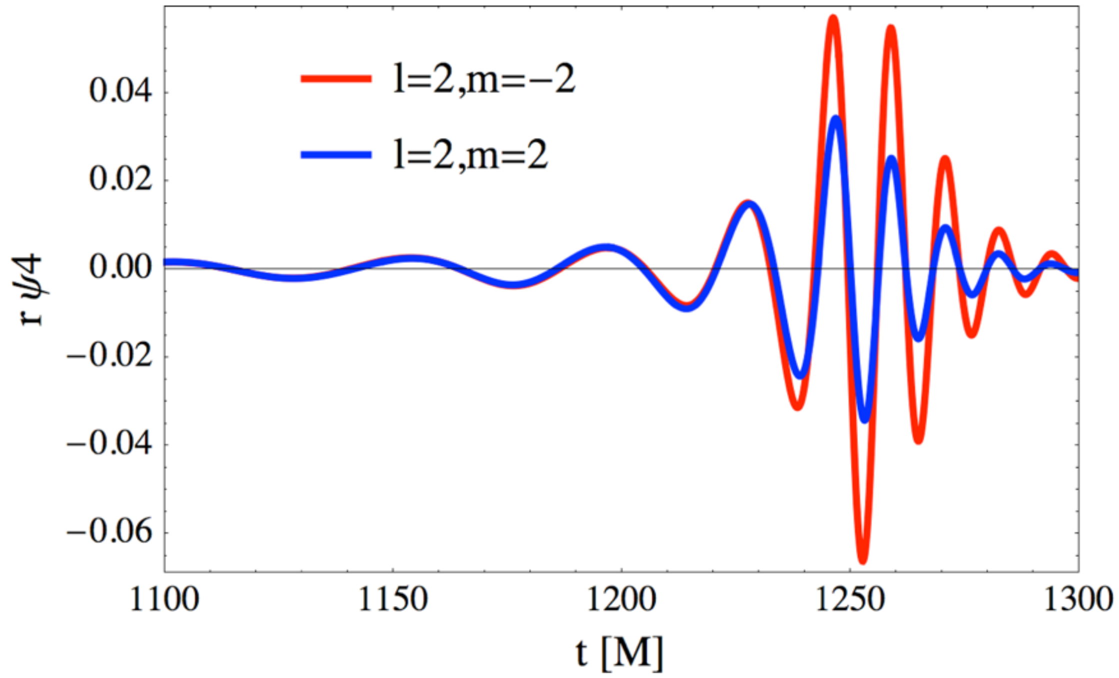
- Results scale with mass: 7-dim for models (mass ratio + 6 spin components) not counting eccentricity (ground based: possibly negligible).

- Simple 3-dim subspace: Spins aligned with orbital angular momentum: no precession, orbital plane is preserved.

- Spins in the orbital plane break the symmetry between dominant l=2 m=2 & m=-2 modes: large recoils possible with precessing spins.







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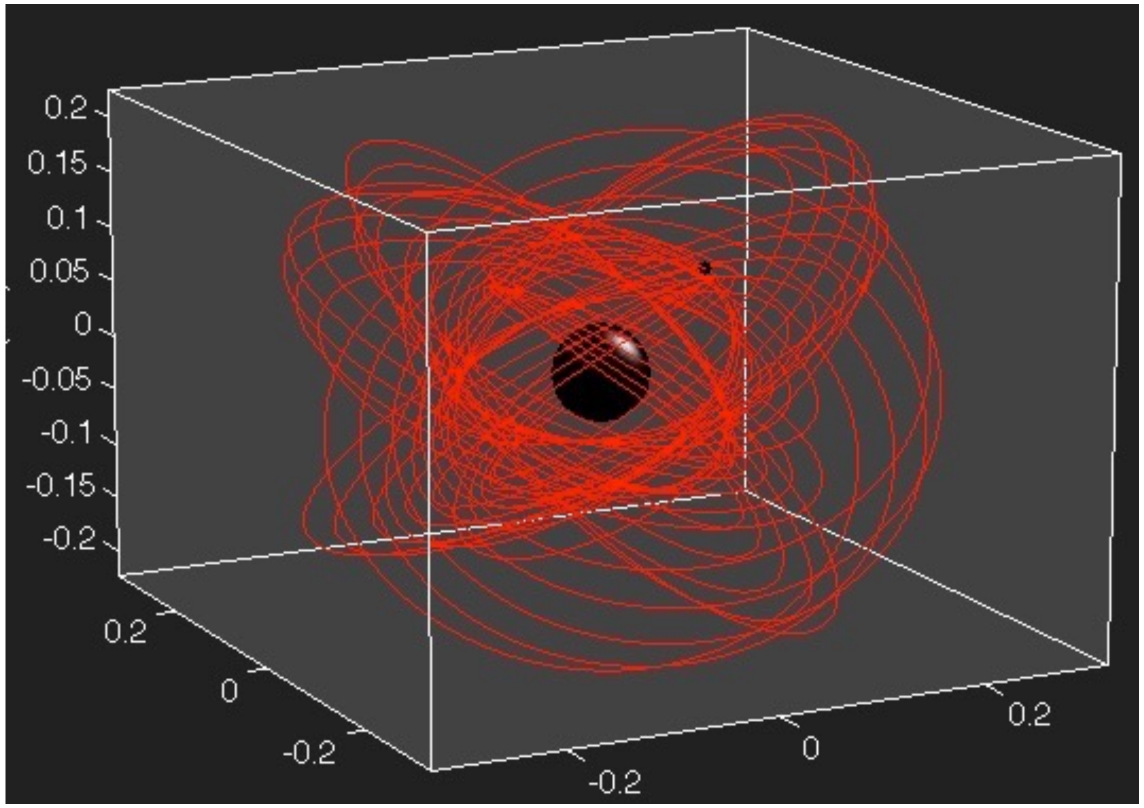
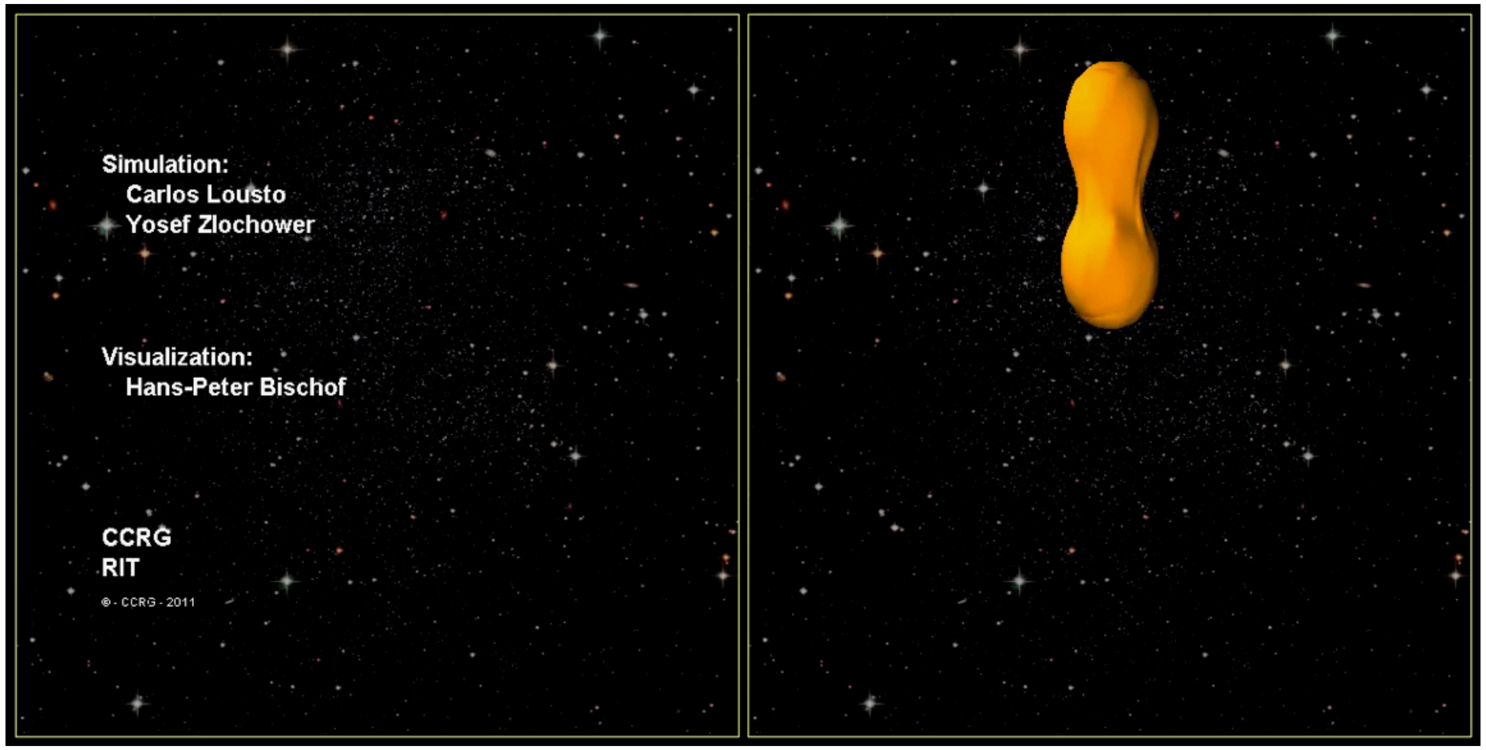
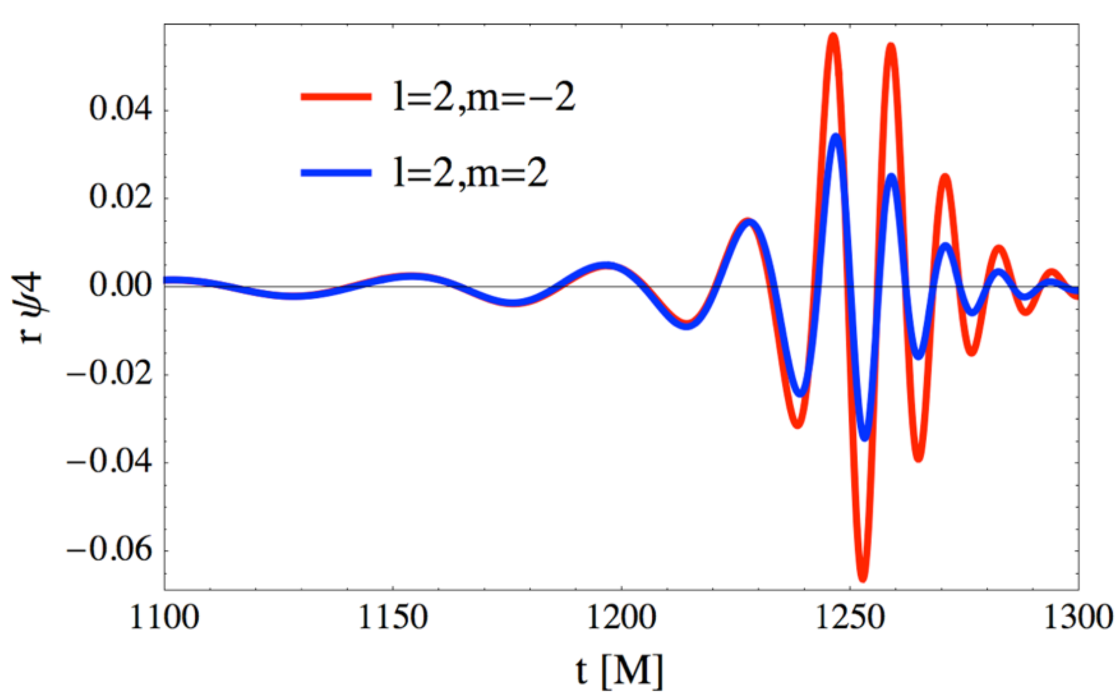
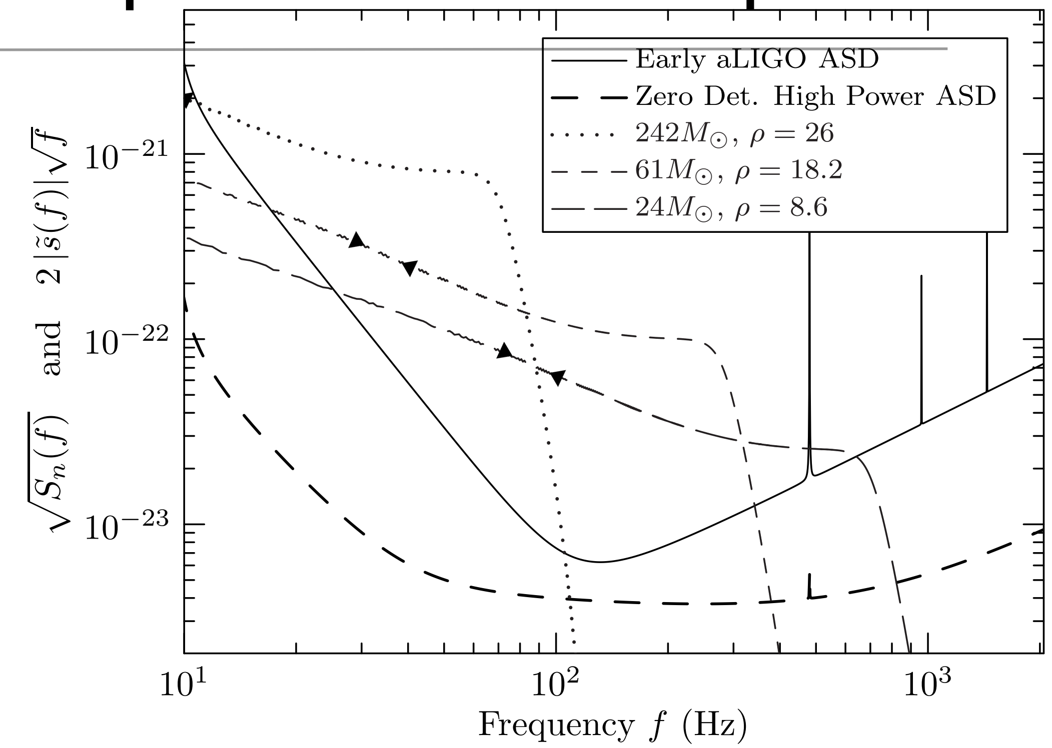






NR exploration of the Binary BH parameter space

- Parameter space exploration is expensive:
  simulations ~several 10^5 CPU hours.
- Extending full NR to low frequencies is very expensive!



$$T_{coalescence} \approx \eta^{-1} f_{initial}^{-8/3} \qquad \eta = \frac{m_1 m_2}{(m_1 + m_2)^2}$$

- How many waveforms do we need?
  - ~ 10s without precession, 0-100's with precession?
- Are waveforms good enough? For detection? Parameter estimation?
  - Quality of NR waveforms is a question of cost - how accurate/densely sampled do we need them? How high is the SNR we expect?

How hard is it to fit the parameter space?

First large NR parameter study: Recoil, Jena Group, 2007

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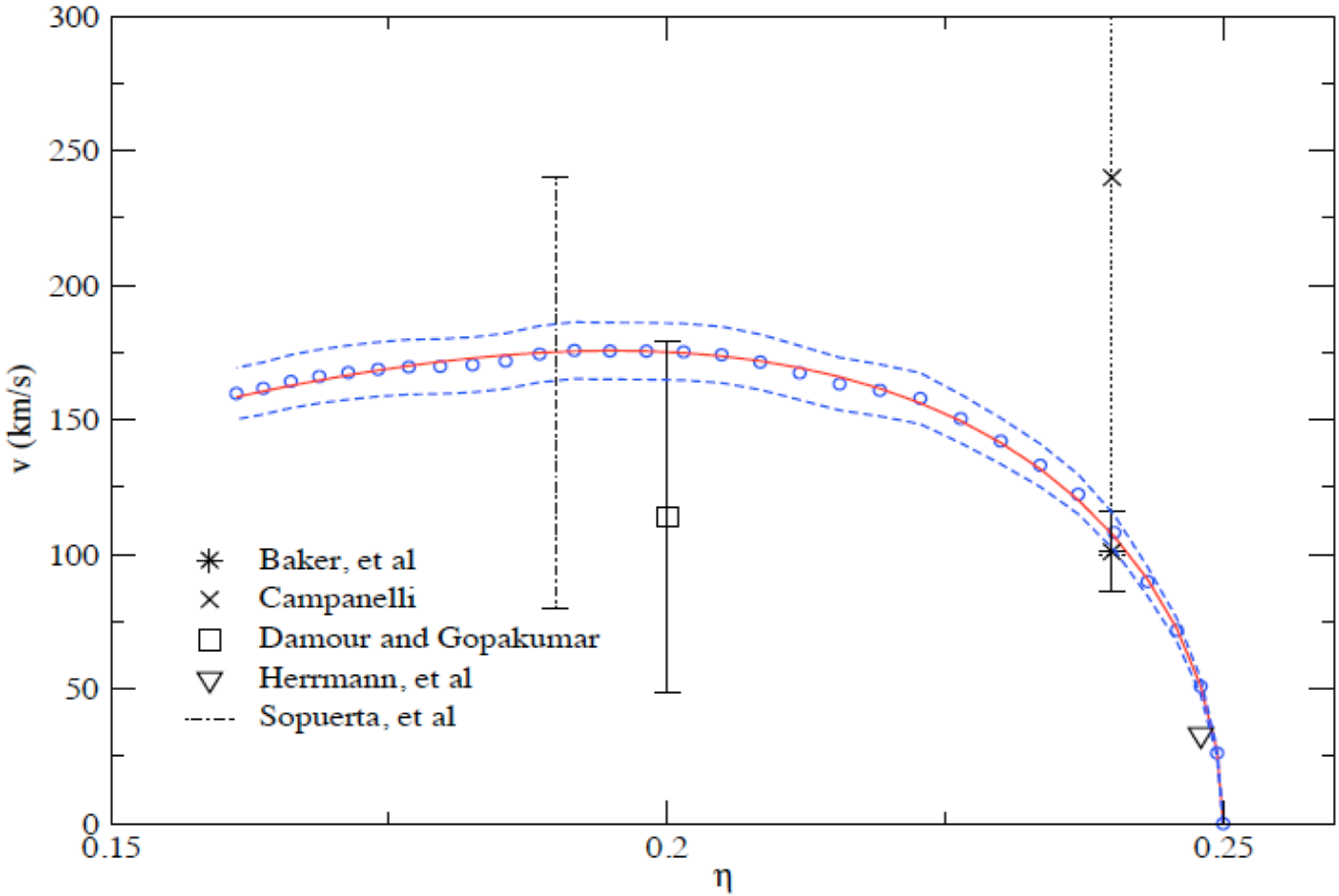
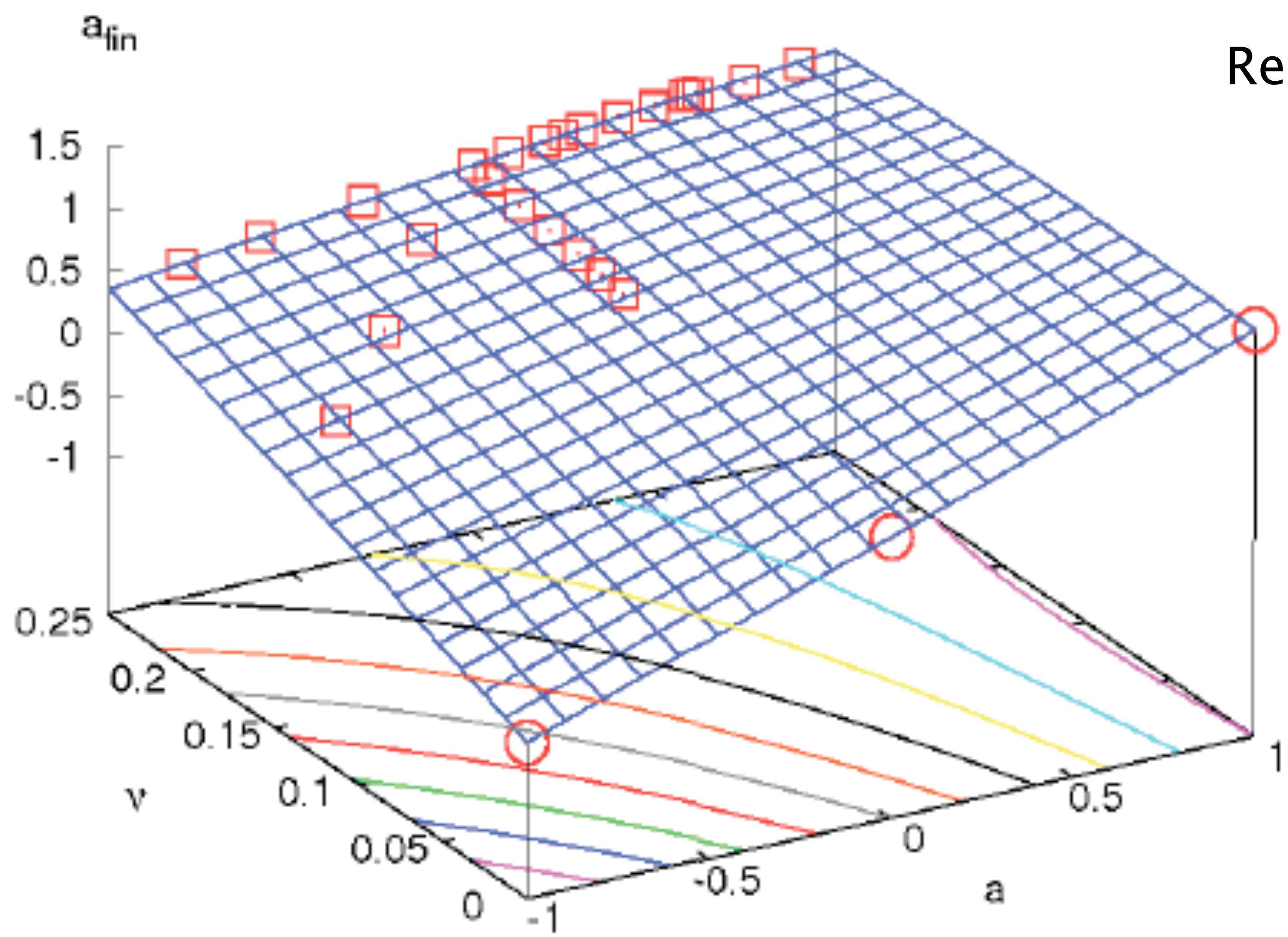


First large NR parameter study: Recoil, Jena Group, 2007

Good news: Dominant parameter dependencies are not very hard to fit, at least without precession.



Rezzolla et al., Astrophys.J.674:L29-L32,2008

Shows 2 dominant dependencies in non-precessing parameter space: symmetric mass ratio & “total spin”.

What about subdominant effects, e.g. spin difference effects?

Standard fitting approaches for full 3D non-precessing parameter space:

- Choose good or “best” effective spin -> not optimal to extract maximum information
- Taylor series up to a certain order, drop insignificant terms -> prone to overfitting

Modelling strategies

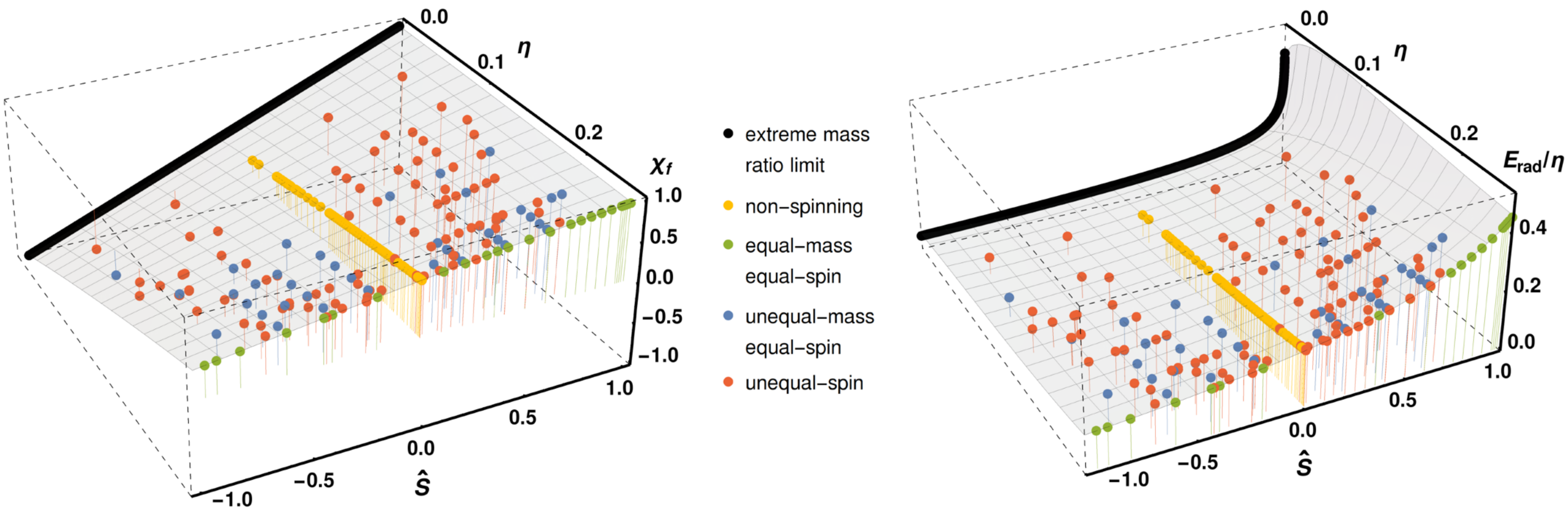
- best approximation to solutions of EE for finite set of cases: e.g. NR+PN hybrid
- First choice: model the original equations, or the resulting waveforms?
- Effective One Body approach (Damour, Buonanno, Nagar, ….): model the energy and flux of a particle inspiral in an effective metric, then integrate ODEs numerically.
    - Re-summation of post-Newtonian results: assume a functional form (e.g. for the Hamiltonian as a function of momentum and separation), then Taylor expand and match free coefficients to known post-Newtonian expansion coefficients.
    - Tuning to Numerical Relativity can be incorporated by adding unable coefficients that are fitted to NR instead of matched to PN.
    - Slow - need a fast model of the phenomenological EOB model.
- Phenomenological waveform models: Make a physically motivated ansatz for the waveform in terms of suitable parameters, fit to each waveform, then fit coefficients across parameter space.
- Construct direct interpolation for a set of waveforms, without intermediate phenomenological model, can use the same methods as for fast evaluation of EOB.

Problems to address & and an example approach

- Model simple functions: -> split WF into amplitude and phase, or model Hamiltonian/Flux/…
- Frequency or time domain: time domain naturally suited for EOB, otherwise frequency domain for data analysis.
- High dimensionality:
  - Physical parameter space (7+2): need to understand complicated phenomenology/deal with performance of brute force methods in high dimensions.
  - model functions: reduce to ansatz coefficients, or grid up (e.g. ROM).
- Avoid overfitting to noise: -> present example data driven hierarchical approach
- Example:
  - grid up amplitude and phase with ~ 30 frequency data points
  - interpolate values in parameter space with polynomials
  - reconstruct WF as spline

Spin & final state: hierarchical fitting approach

- Kerr BH perturbation theory:
    - final mass & final spin-> complex frequencies of spheroidal harmonic QNMs.

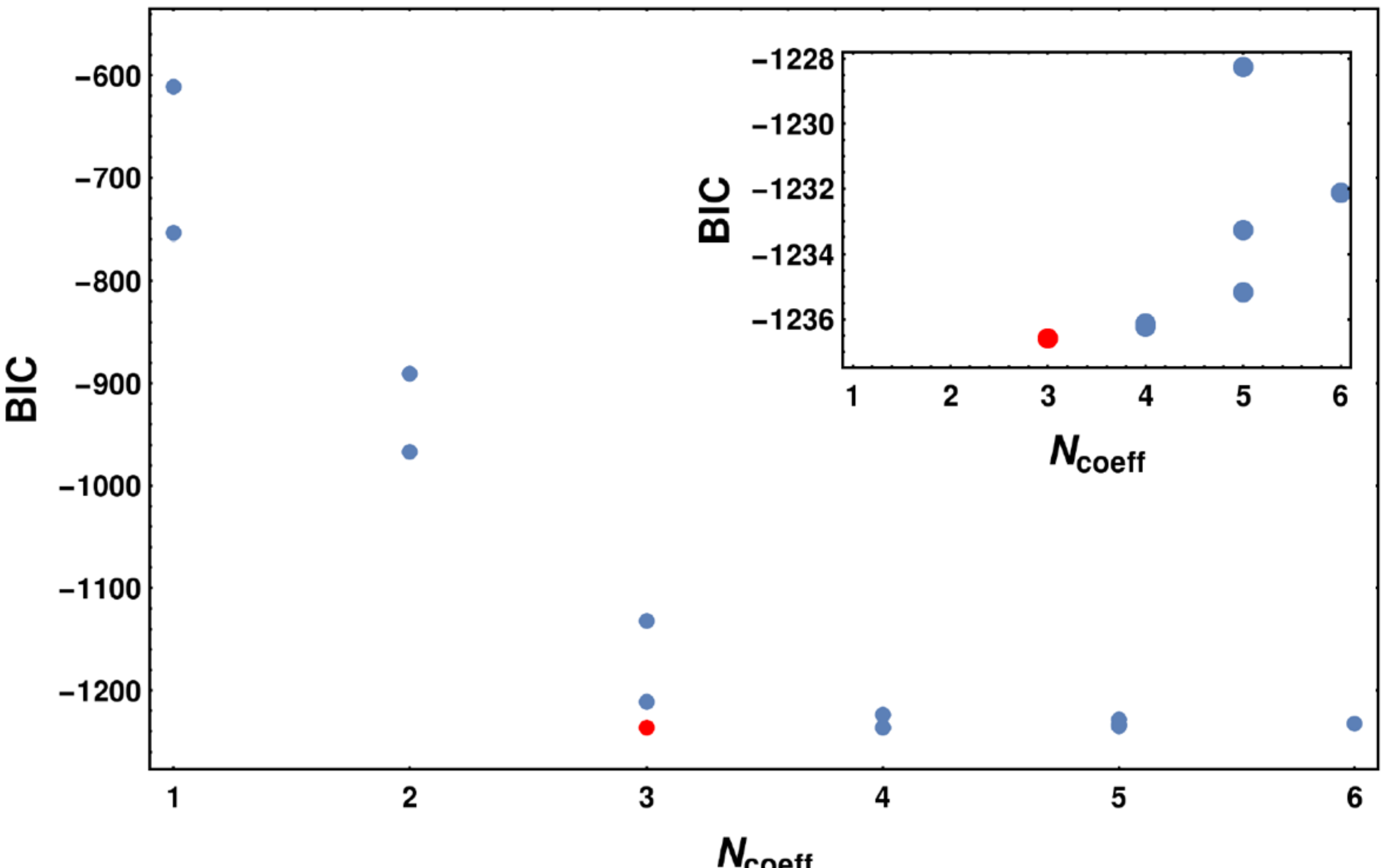


- Extreme mass ratio limit:
    - energy & angular momentum of particle @ ISCO (Bardeen+ 1972)

Ranking of fits

Minimising RMSE error alone will lead to overfitting.

Use BIC/AIC/AICc information criteria (better: full Bayesian analysis):
penalise models with too many free coefficients.



BIC for non-spinning final spin fit.

$$\mathrm{AIC} = -2\ln\mathcal{L}_{\mathrm{max}} + 2N_{\mathrm{coeffs}}$$

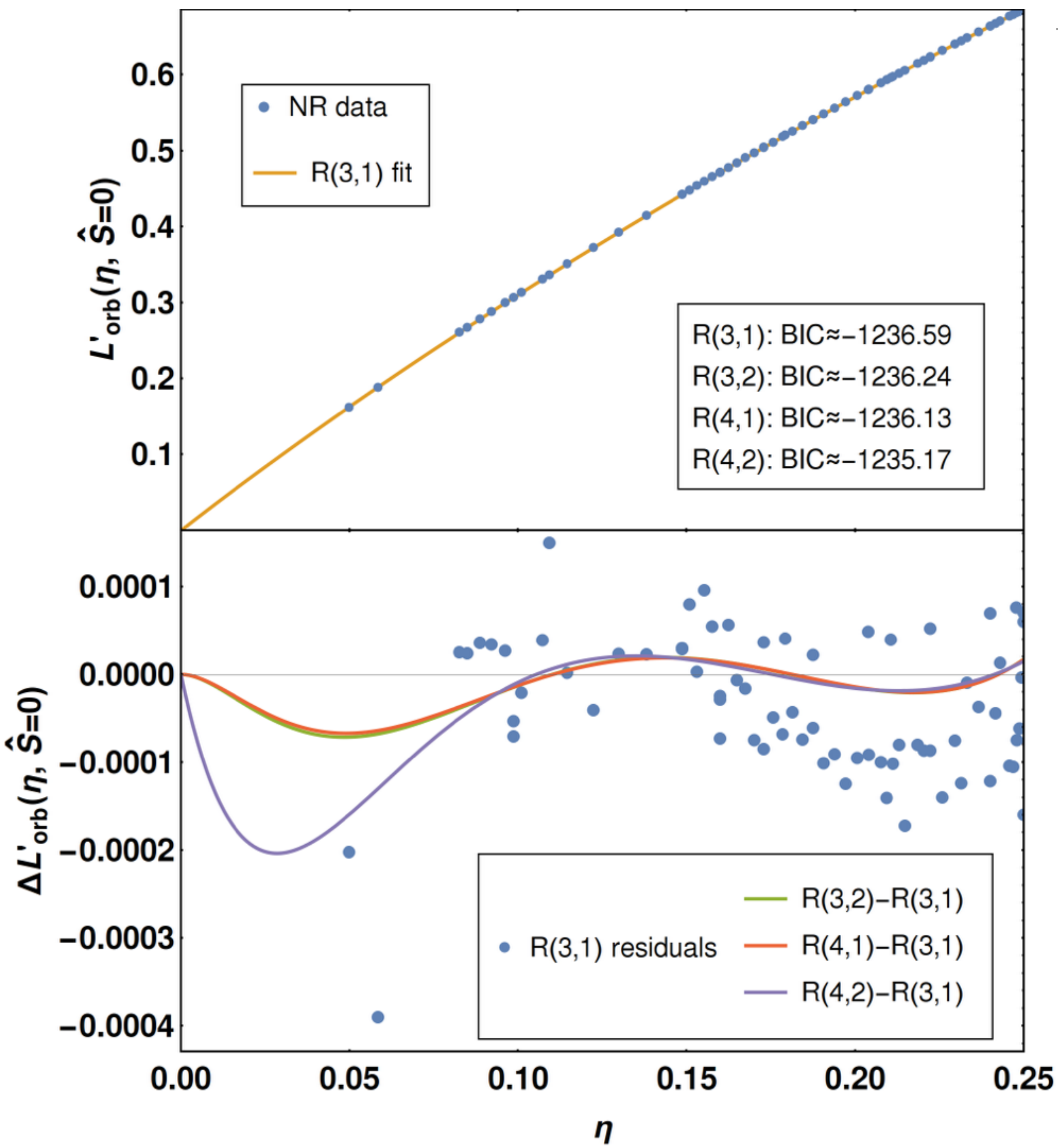
$$\mathrm{AICc} = \mathrm{AIC} + \frac{2N_{\mathrm{coeffs}}(N_{\mathrm{coeffs}}+1)}{N_{\mathrm{data}} - N_{\mathrm{coeffs}} - 1}$$

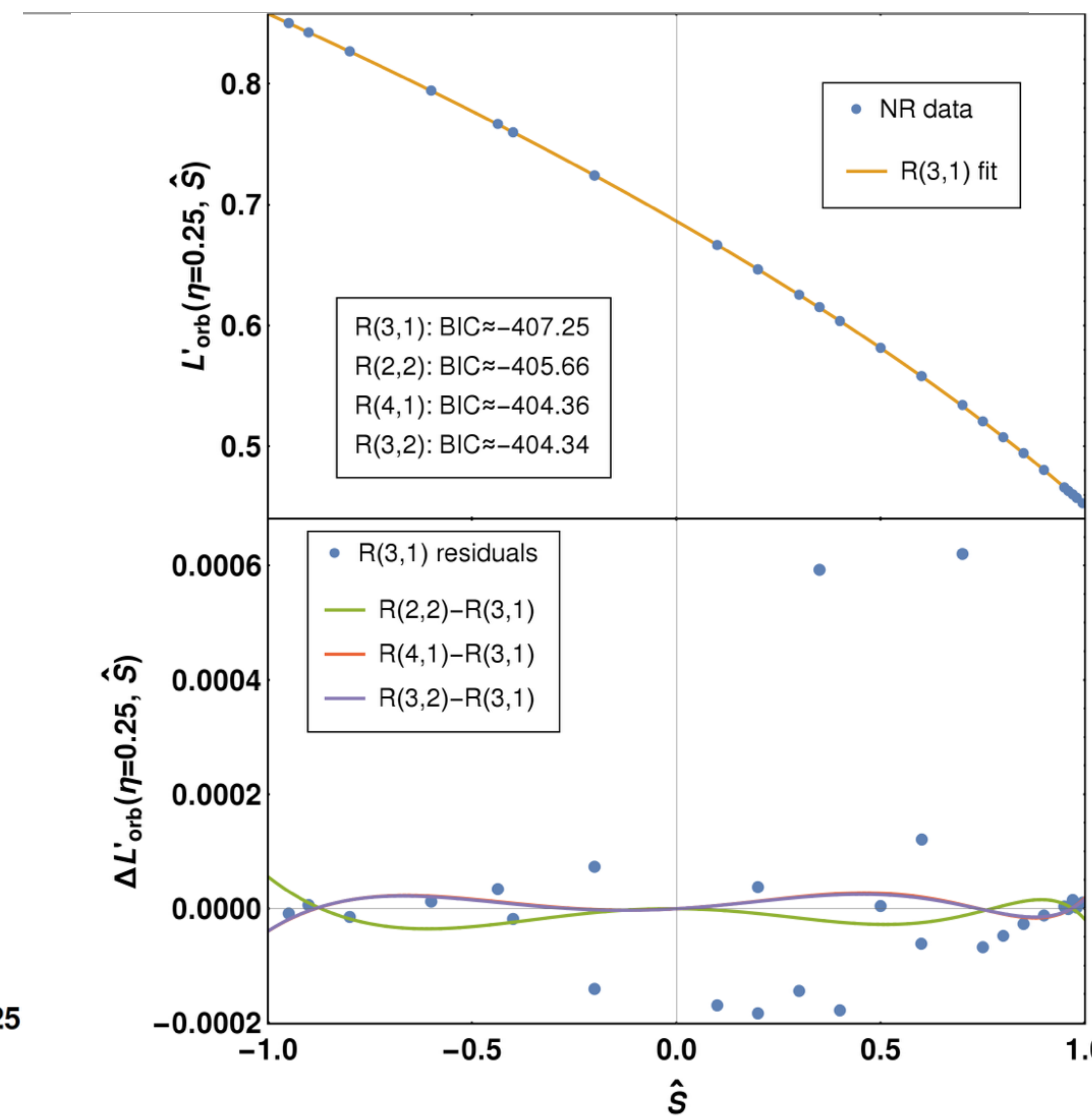
$$\mathrm{BIC} = -2\ln\mathcal{L}_{\mathrm{max}} + N_{\mathrm{coeffs}}\ln(N_{\mathrm{data}})$$

Polynomial fits tend to “level off” at orders > 5.

=> Polynomials not enough, use rational f.

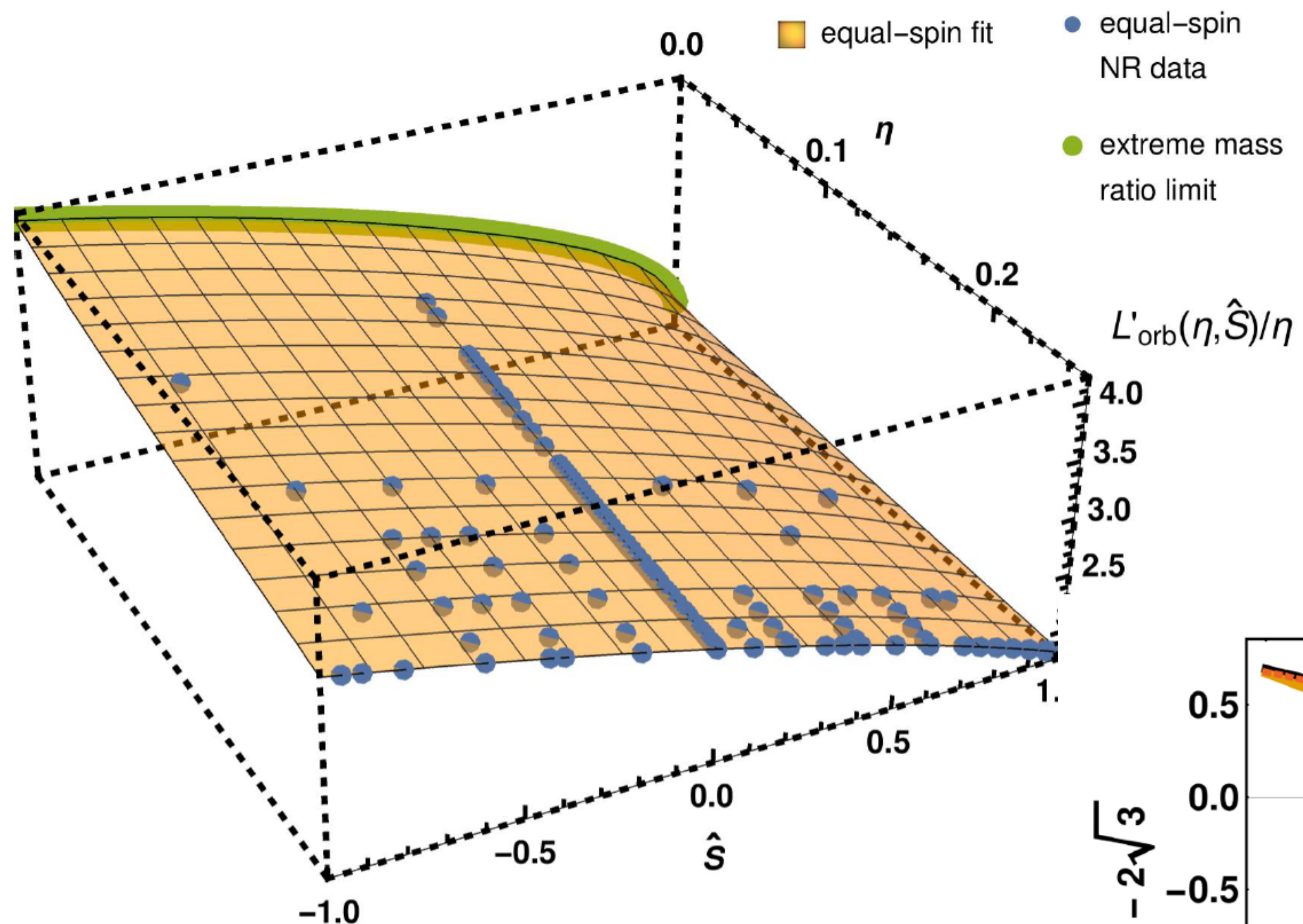
Continue simple:
non-spinning and equal BH 1D problems





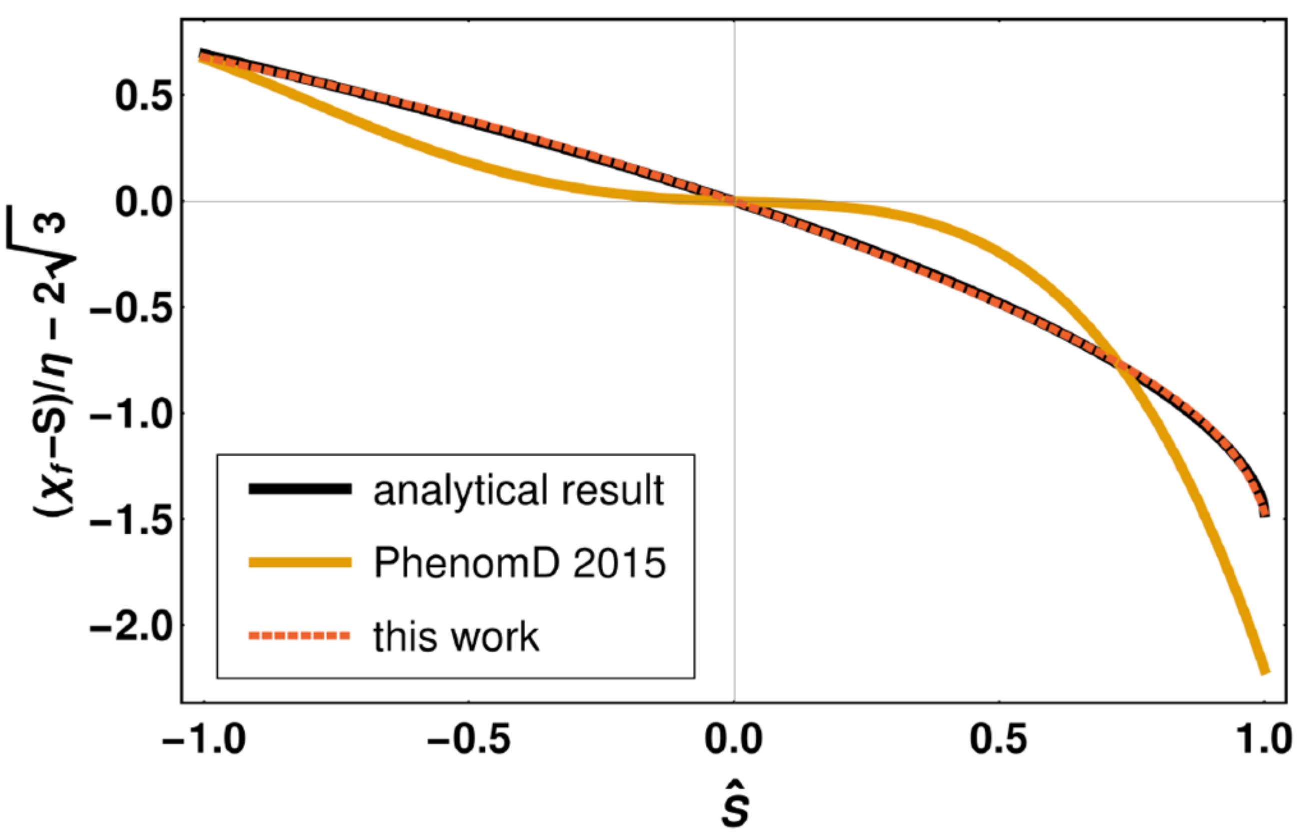
Rank fits within a wide class of fit expressions: choose rational functions

Next step 2D: equal spins, include EMR limit



Choosing an appropriate effective spin parameter, we get a reasonable fit across parameter space!

Including many data points with highly unequal spins in the fit will decrease the fit quality!

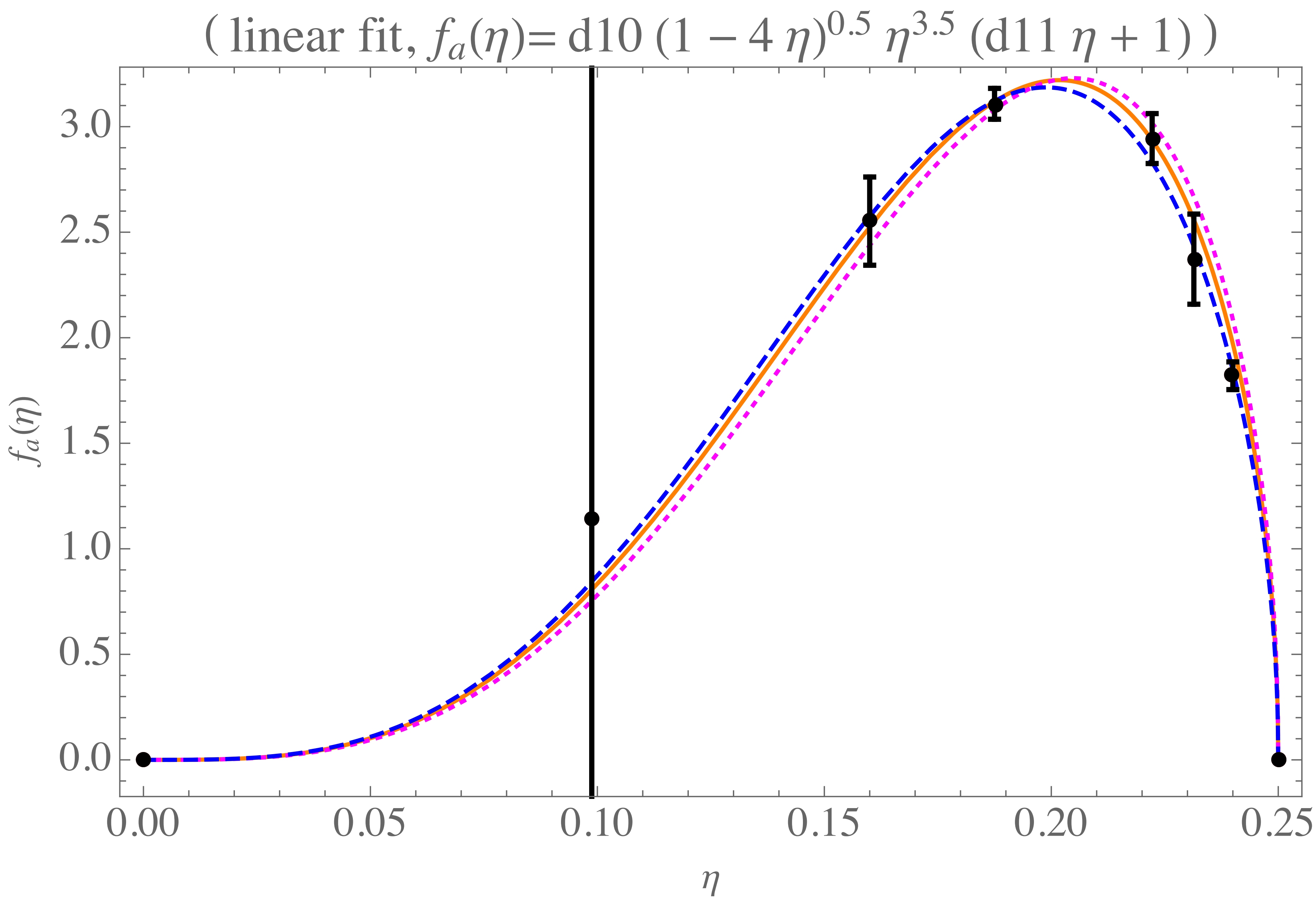


Parameter space fitting errors tend to be dominated by boundary effects.

EMR limit: geodesics on Kerr spacetime

Sub-dominant contribution: linear in spin difference

$$a_f = a_f^{Eq} + f(\eta)(\chi_1 - \chi_2)$$



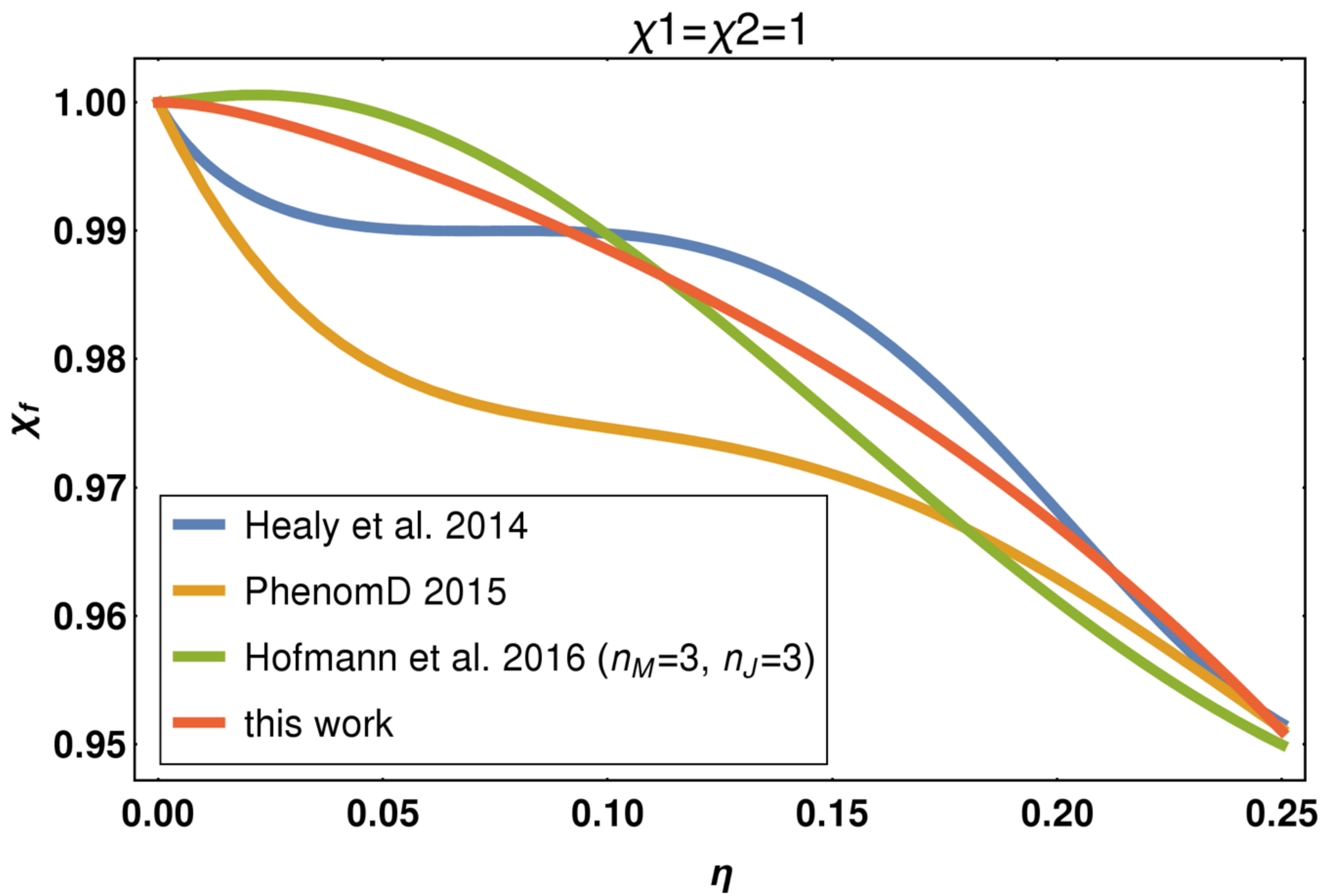
Per-η fits, residual fits and direct 3D fits should be consistent!

Fit behaviour for extreme spins of individual BHs

- Correct extreme mass ratio limit needs to take into account radiated energy (was neglected in early fits for simplicity).

$$\chi_f = \frac{S_f}{M_f^2}$$

- Fits should not overshoot extreme Kerr limit.



Phenomenological modelling of IMR waveforms

- Key “design” ideas:
- “phenomenological”: minimal assumptions - look at waveforms and describe what we see.
  - Frequency domain: matched filter calculations in Freq. domain
  - Explicit expression in terms of elementary functions -> fast, simple
    - inspiral: PN + higher order “pseudo-PN” terms

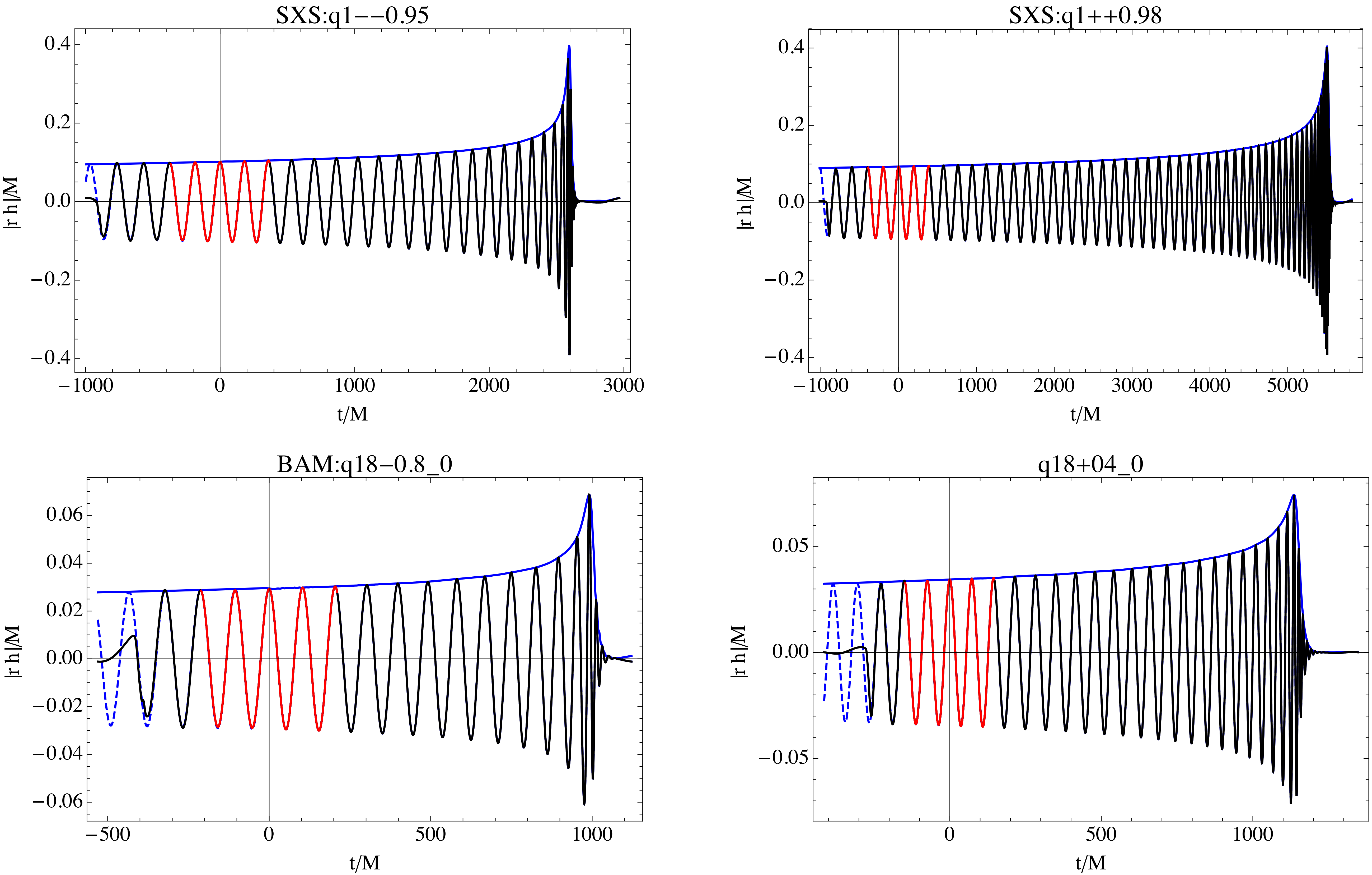
$$\varphi'_{\mathrm{MR}} = \alpha_1 + \sum_{i=2}^{n} \alpha_n f^{-p_n} + \frac{a}{f_{\mathrm{damp}}^2 + (f - f_{\mathrm{ring}})^2}$$

$$h_{\mathrm{RD}} = \frac{a\, e^{-\lambda(f - f_{\mathrm{RD}})}}{(f - f_{\mathrm{RD}})^2 + \sigma^2}$$

Hierarchical Strategy

- Start with l=|m|=2 spherical harmonic mode.
- Model directions in parameter space in order of importance.
  - Non-spinning (PhenomA)
  - Aligned spins: single effective spin, no spin difference effects (B/C).
  - **Aligned spins: more than one effective spin (PhenomD)**
  - **Leading precession effects via PN, no NR calibration: PhenomP**
  - Full 3D aligned spin parameter space:
    - **final state and peak luminosity**
    - full model -> Geraint's talk (PhenomX)
  - Aligned spin higher modes: Cecilio's talk
  - NR calibration of leading precession effects: before O3?
  - Go beyond leading precession effects: before design sensitivity?

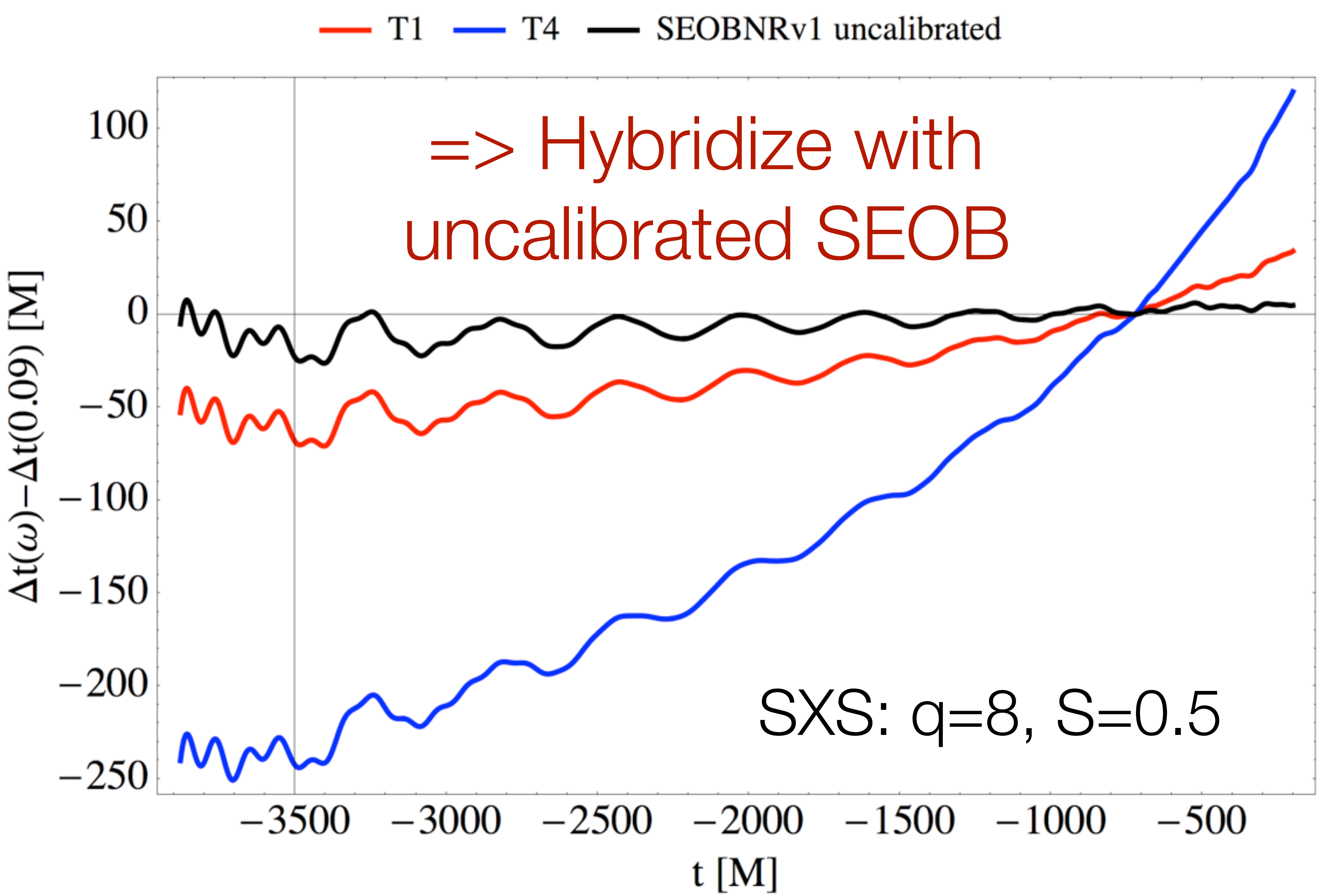
Hybrid waveforms: corner cases



Choice of inspiral approximant

- Compare required time-shifts in hybridization procedure for PN approximants: flatter curve is better.

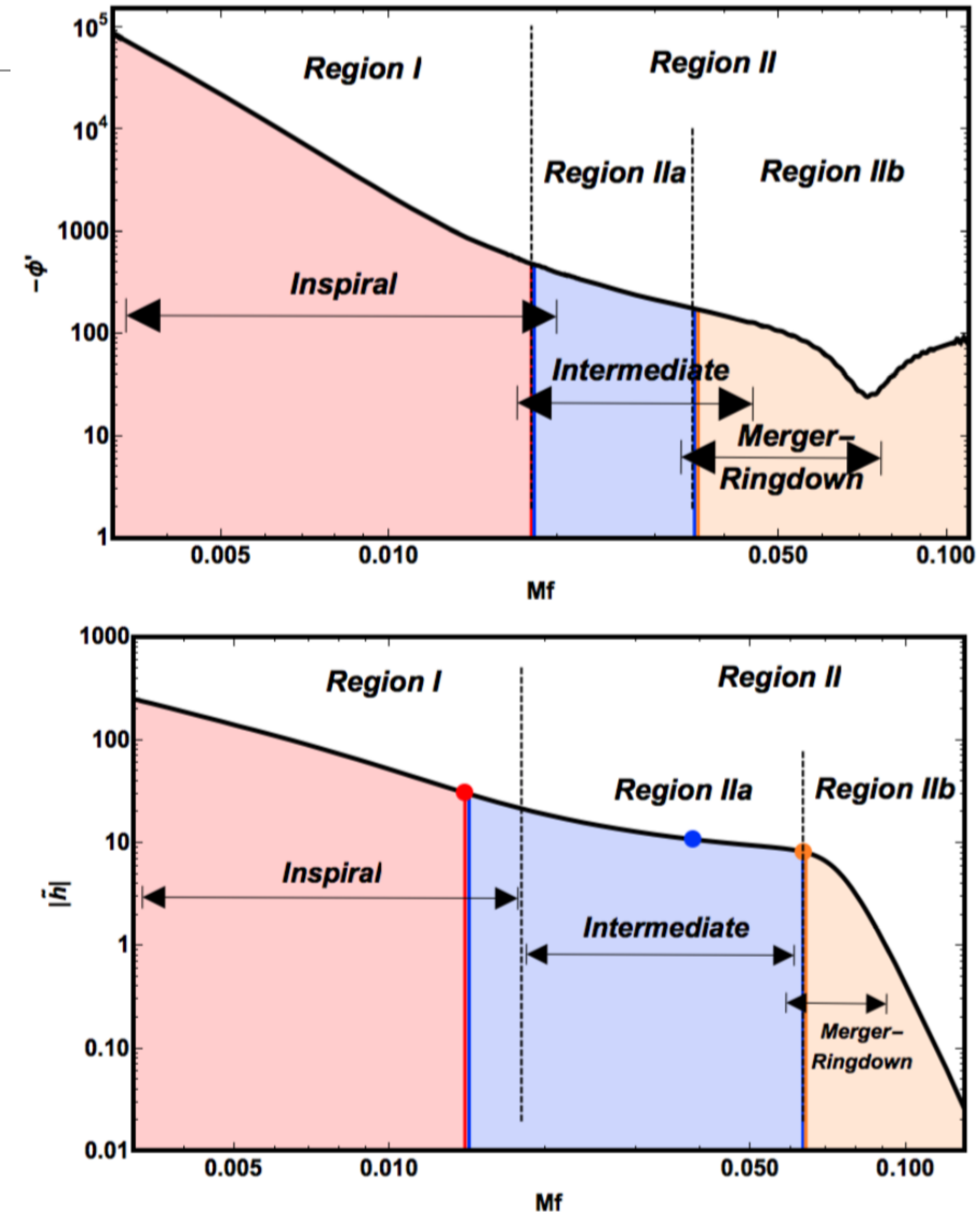
-> decide for uncalibrated SEOBNRv2 for PhenomD,
now: SEOBNRv4_opt



Splitting into amplitude/phase & frequency regions

Divide and conquer:

- Split waveform into amplitude and phase, model simple non-oscillatory functions.
- Simplicity of modelling increases with the number of frequency-regions.
- Simplest: tens of points, cubic spline.
- Our choice - 3 regions:
  - inspiral (use PN intuition)
  - merger-ringdown (use QNM intuition)
  - intermediate: starts @ MECO

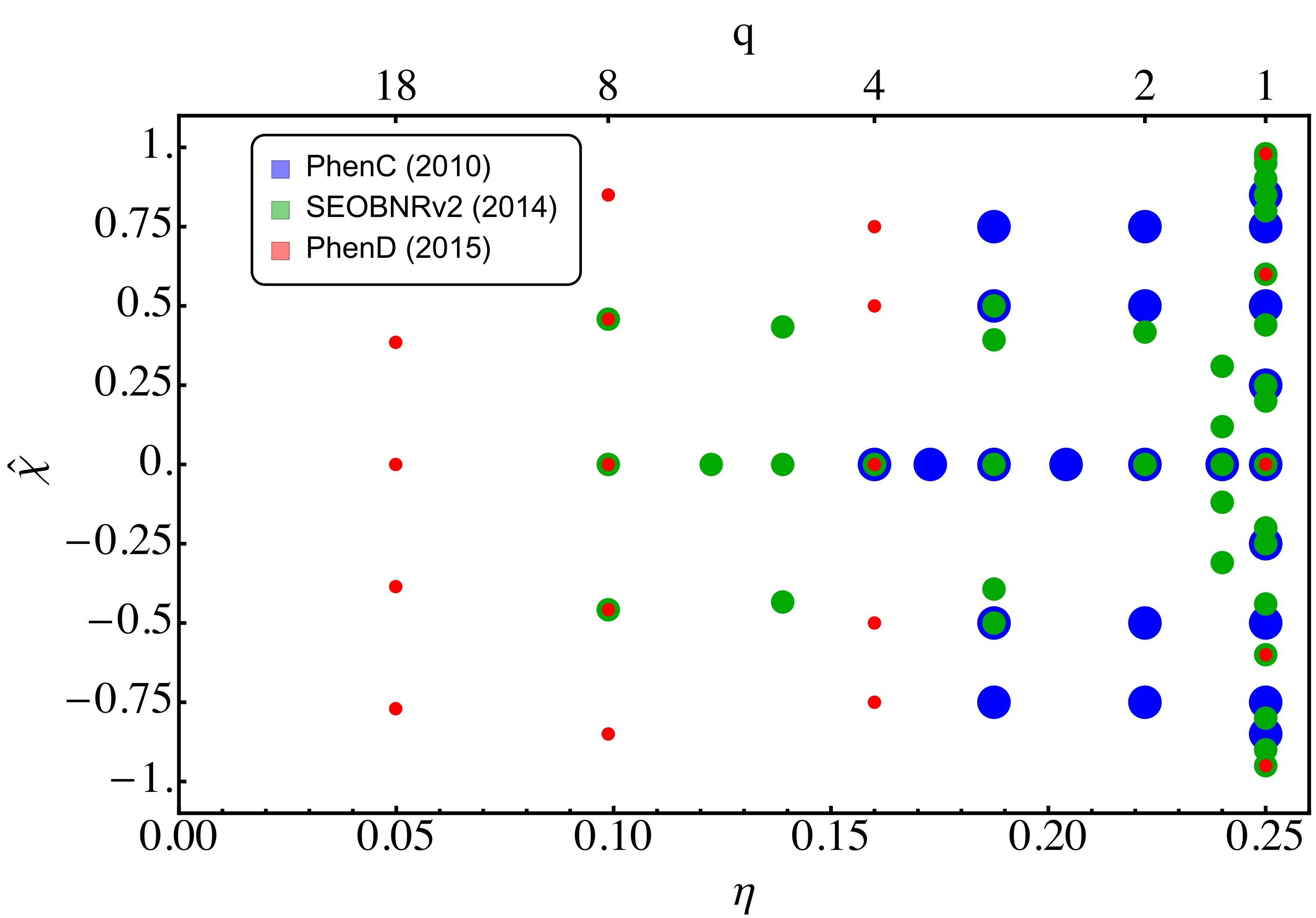


- Phenom* are modular, e.g. inspiral and MRD can be tuned from different waveform sets, variations of Phen* models easy to generate.

Calibration data sets: PhenomD/PhenomX

Use NR Waveforms from 2 collaborations/codes:

- SpEC code/SXS collaboration: pseudo spectral code, based on BH excision, generalised harmonic formulation of Einstein Equations
- BAM code: “moving puncture” (singularity avoiding gauge) finite difference mesh refinement, BSSN formulation of Einstein Equations



PhenomD:
calibrated to 19 SXS+BAM WFs

PhenomX: ~ 260 SXS+BAM
~30 Teuk Code
(Harms, Nagar, Bernuzzi)

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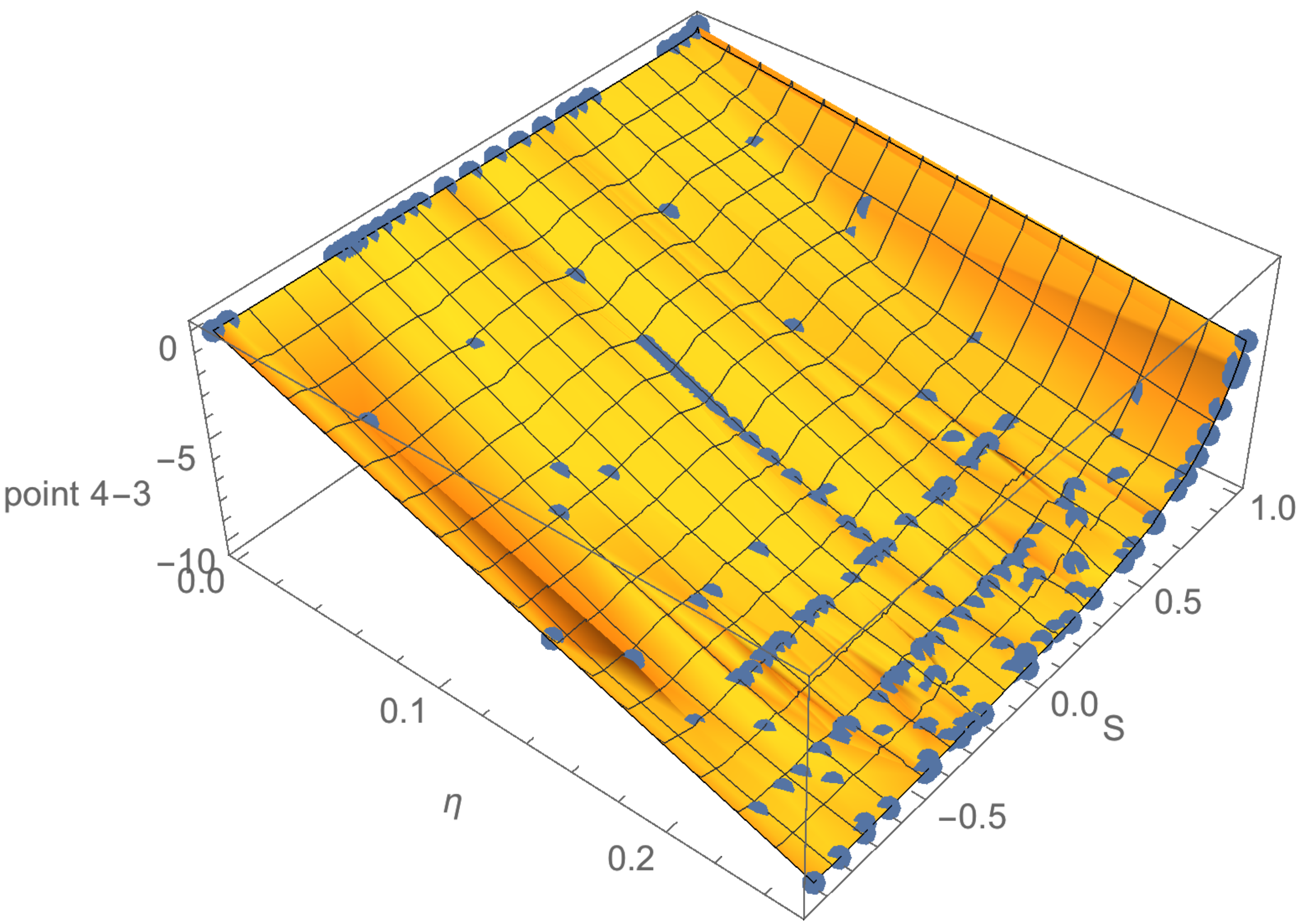

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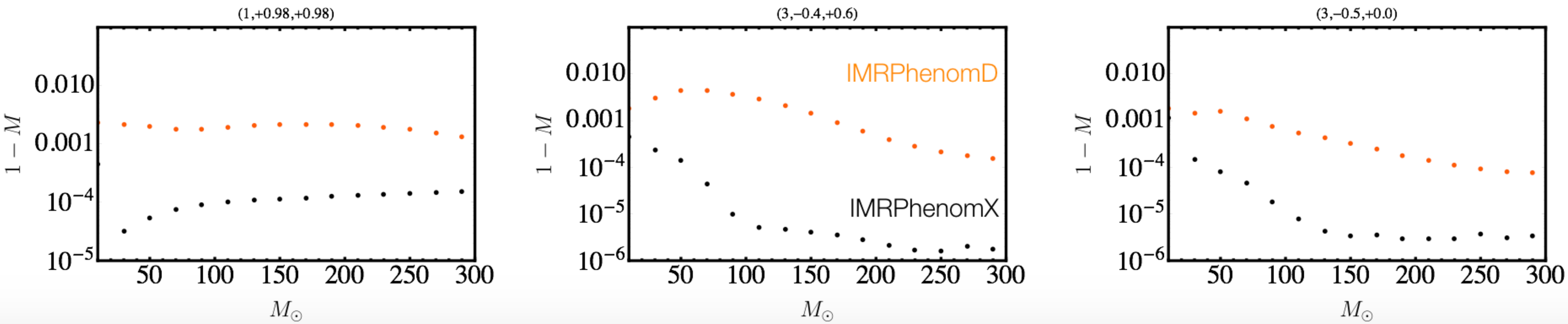
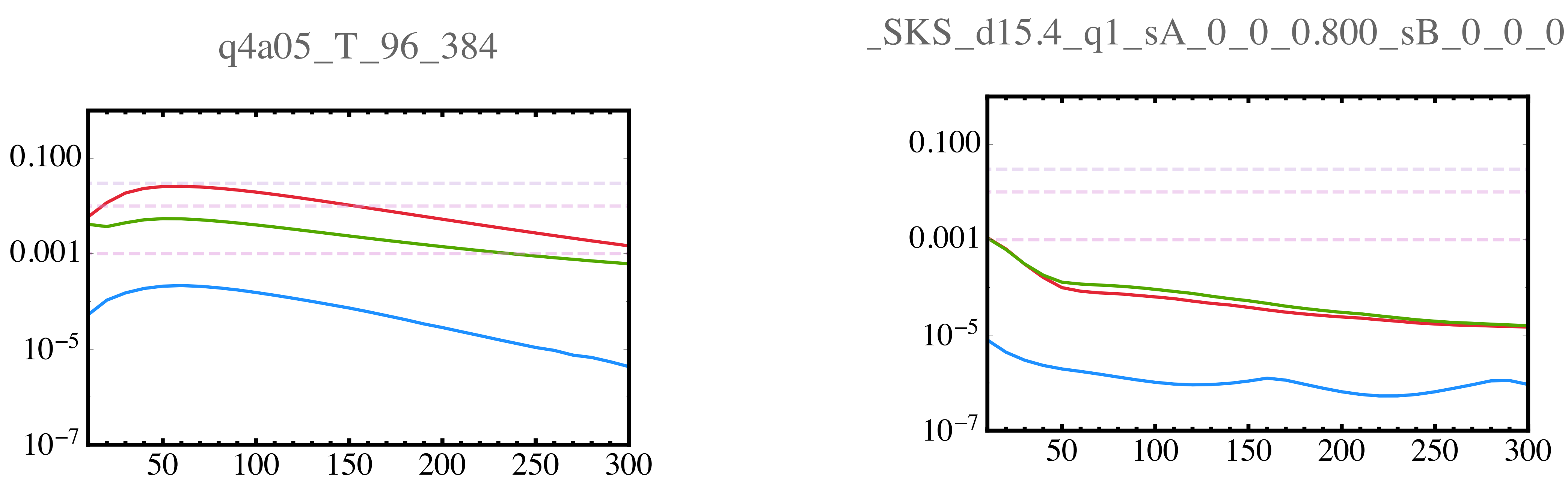
Mismatches: early aLIGO noise curve, low freq. cutoff @ 20 Hz





Kumar+, arXiv:1601.05396

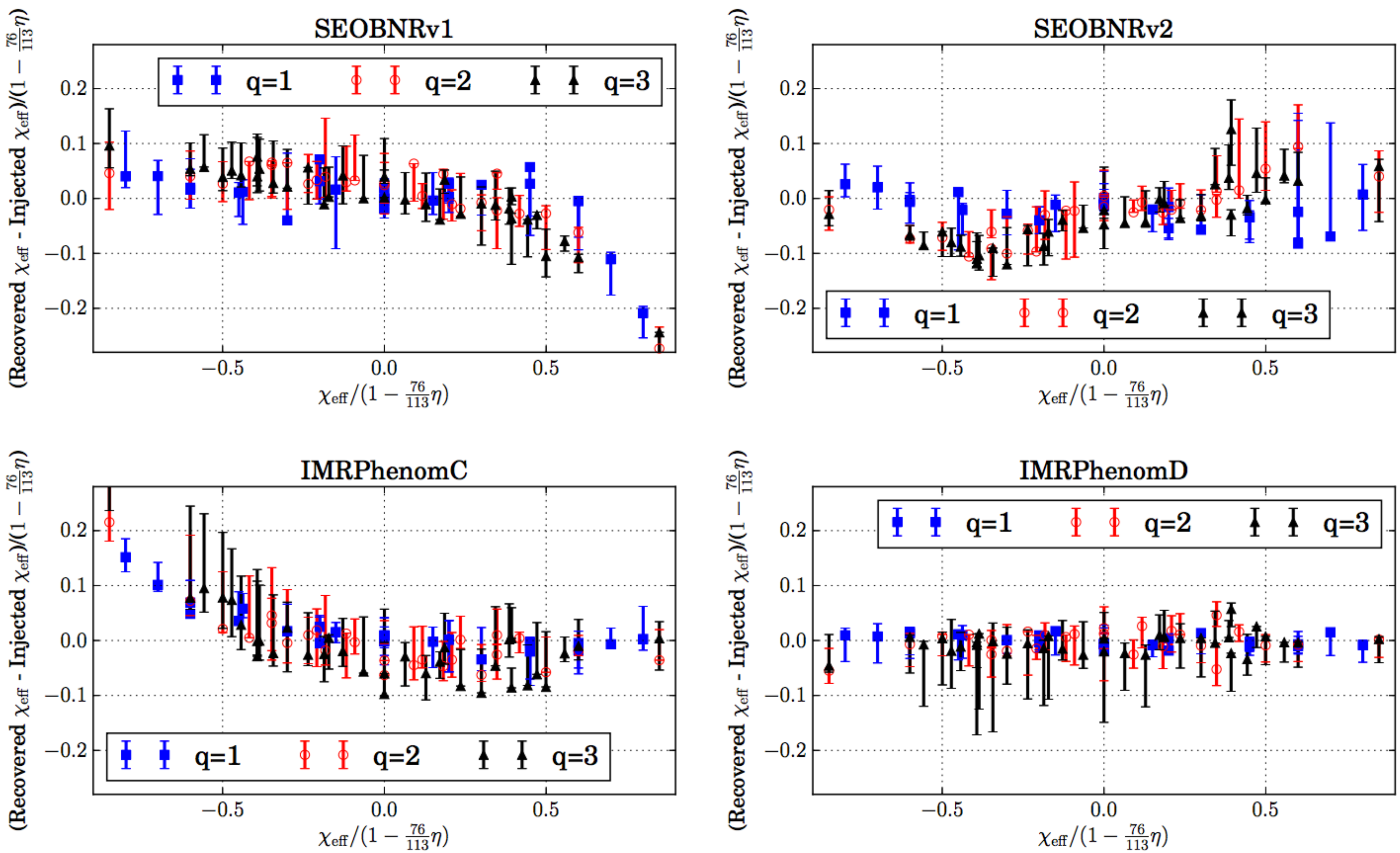
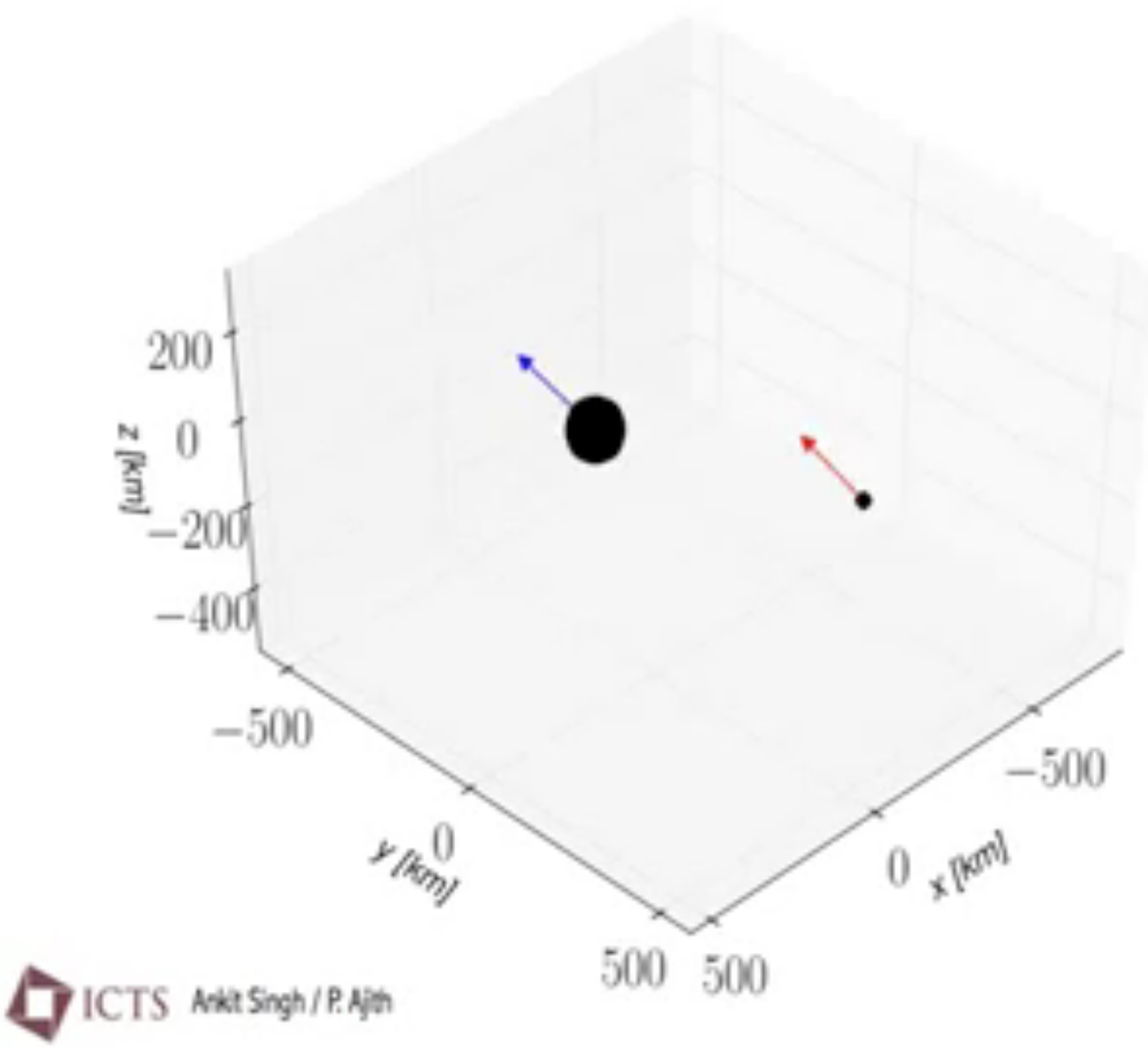
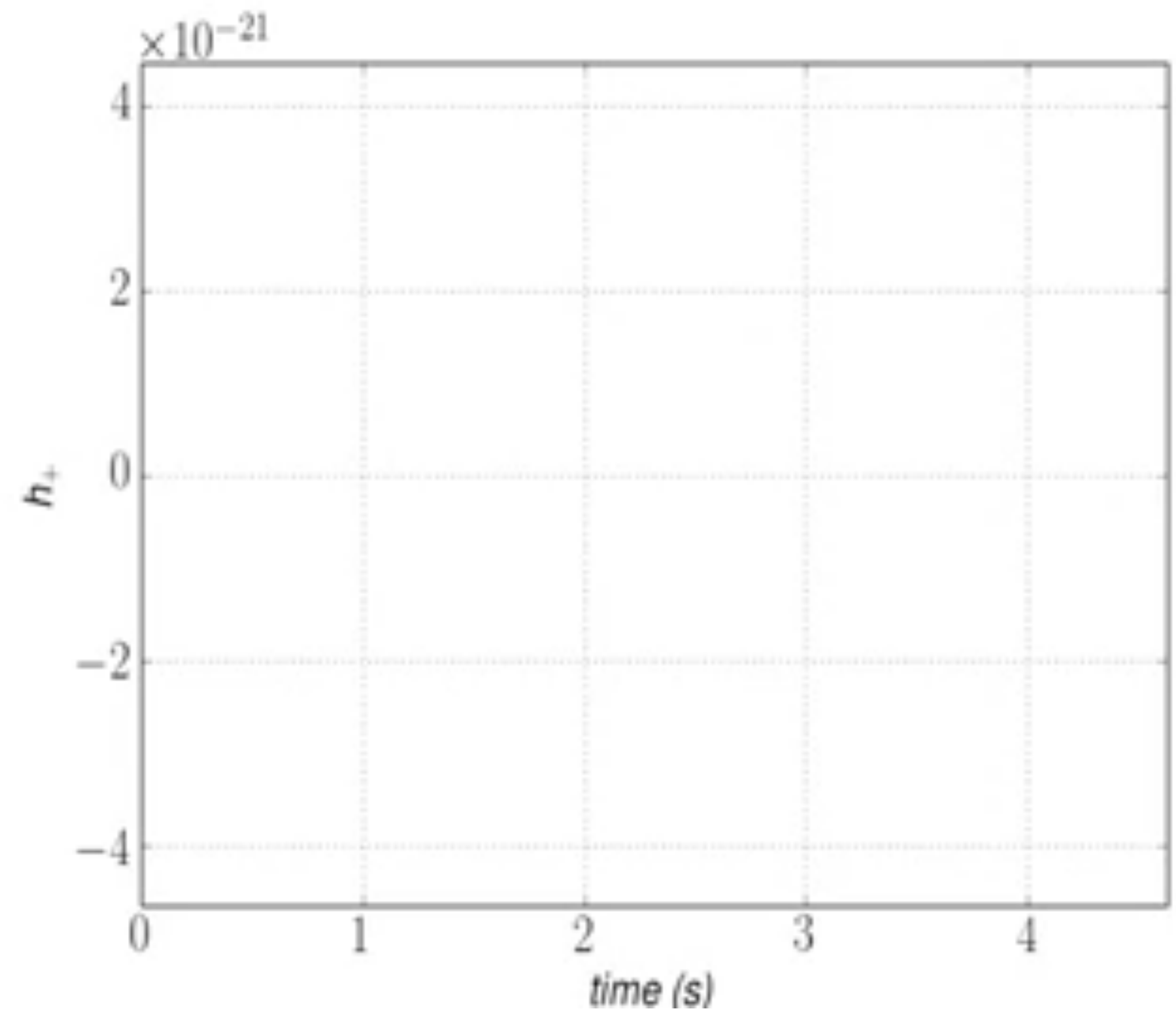


FIG. 10. Systematic bias in the recovery of the effective spin parameter $\chi_{\rm eff}$, as a function of the normalized effective spin of the NR waveforms. The plot-markers show the recovered $\chi_{\rm eff}$ for a binary with total mass fixed at $80M_\odot$, while the "error-bars" show the range spanned by the recovered q as the injected binary mass is varied between its lowest allowed value and $150M_\odot$.

Spin & Precession

- Spin components in the orbital plane (orthogonal to orbital angular momentum) -> Precession: orbital plane precession modulates the amplitude.
  - precession time scale >> orbital time scale





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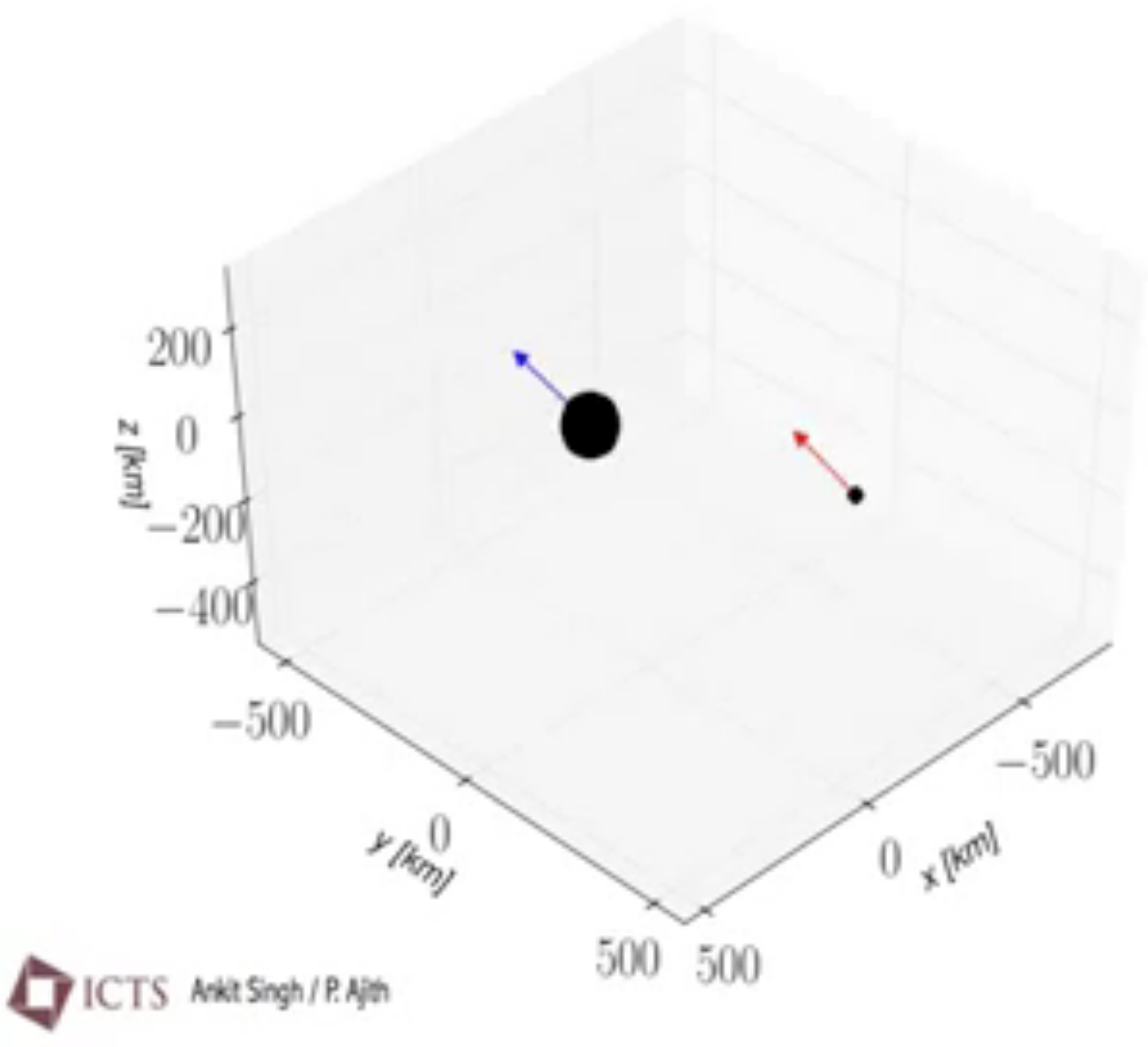

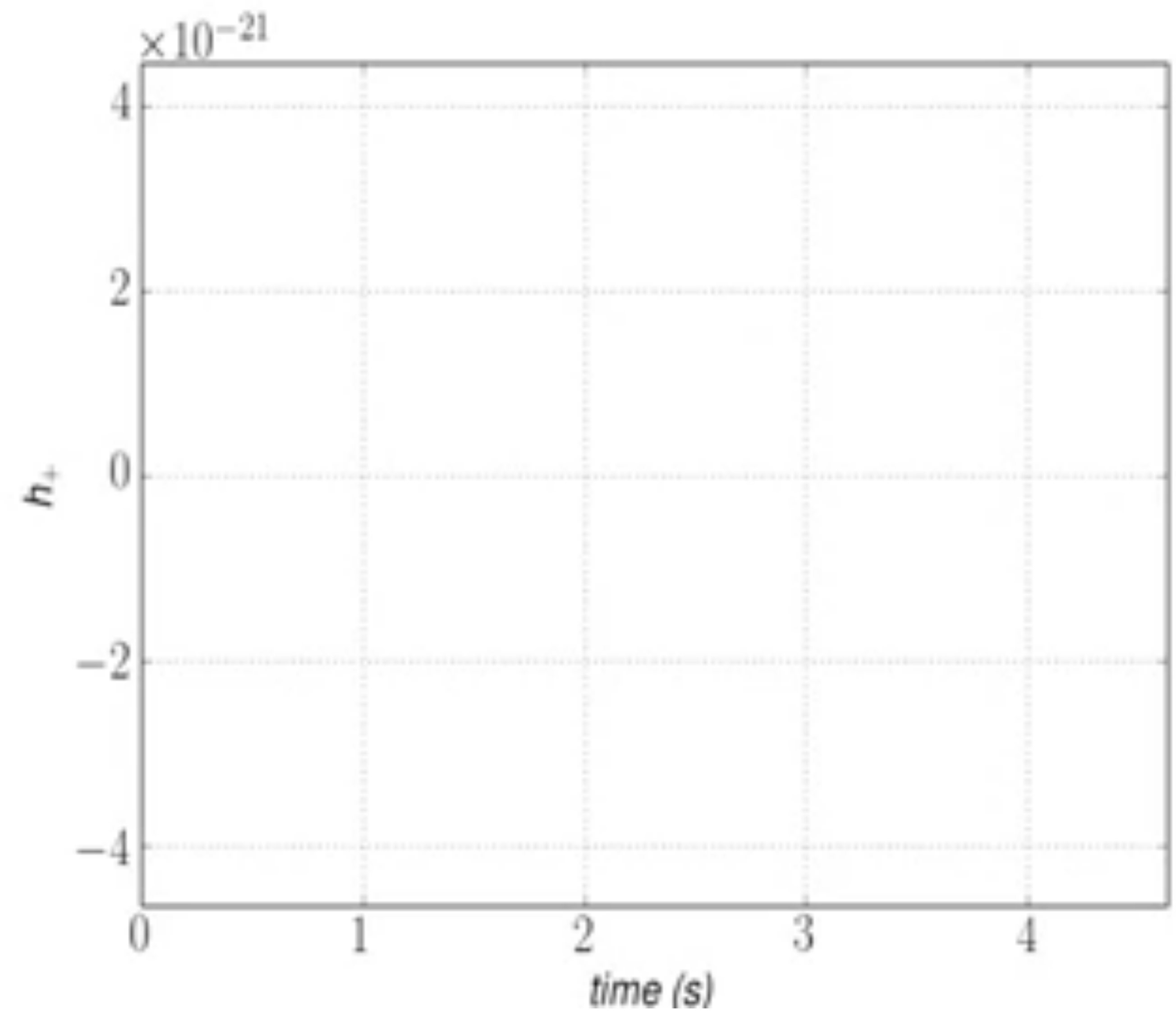

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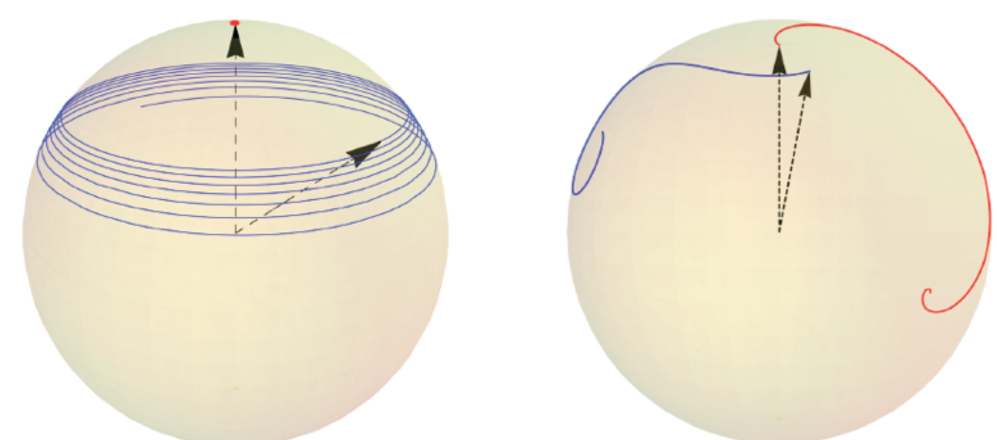
$$\vec{J} = \vec{L} + \vec{S}_1 + \vec{S}_2$$

$$\{|\vec{S}_i|, |\vec{S}_i \cdot \vec{L}|, |\vec{S}_{\perp i}|\}$$

- Co-rotating frame: radiated $\vec{L}$, energy & phasing essentially unaffected by precession -> dominated by $\vec{S}_i \cdot \vec{L}$
  - Final spin/mass approximations.
  - Approximately factor WF into precession x (aligned spin)?



$\vec{J}$ $\vec{L}$

Spin & Inspiral

- Leading order PN (inspiral) spin effect: spin-orbit Hamiltonian (+ GW flux)

$$H_{SO} = 2\frac{\vec{S}_{\text{eff}} \cdot \vec{L}}{r^3} \qquad \dot{S} = -2\frac{\vec{S}_{\text{eff}} \times \vec{L}}{r^3} \qquad \vec{S}_{\text{eff}} = \left(1 + \frac{3}{4}\frac{m_2}{m_1}\vec{S}_1\right) + \left(1 + \frac{3}{4}\frac{m_1}{m_2}\vec{S}_2\right)$$

- Aligned spins: repulsive force; anti-aligned: attractive force.
- Precession: orbital plane precession modulates the amplitude.
- Approximately preserved: $\{|\vec{S}_i|, |\vec{S}_i \cdot \vec{L}|, |\vec{S}_{\perp i}|\}$

  over most of the parameter space also direction of $\vec{J} = \vec{L} + \vec{S}_1 + \vec{S}_2$

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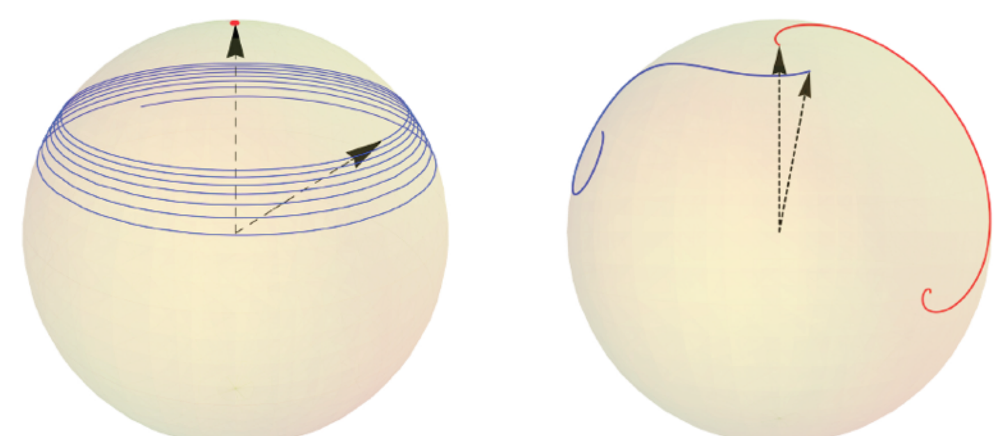
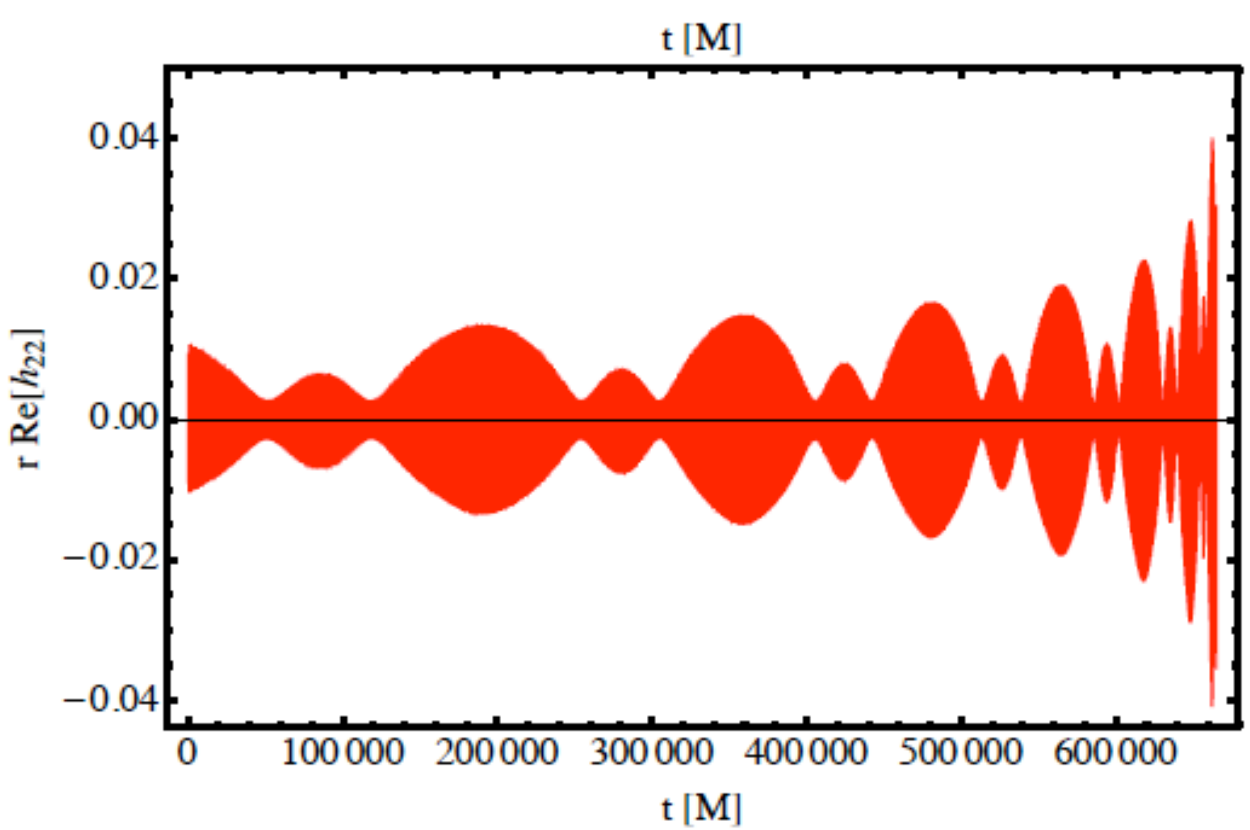


$\vec{J}$ $\vec{L}$

A path toward handling precession

- In a co-rotating frame phasing is essentially unaffected by precession - “simple standard form” of a precessing WF: align z-axis with principal axis of the radiation quadrupole moment [Schmidt+ PRD 2011, Boyle+ PRD 2011]
- Spherical harmonic mode structure in standard frame corresponds to non-precessing case -> “twisting up” accurate aligned spin model with “post-Newtonian” Euler angles works well [Schmidt+ PRD 2012, Hannam+ PRL 2014, Pan+ PRD 89, 2014]



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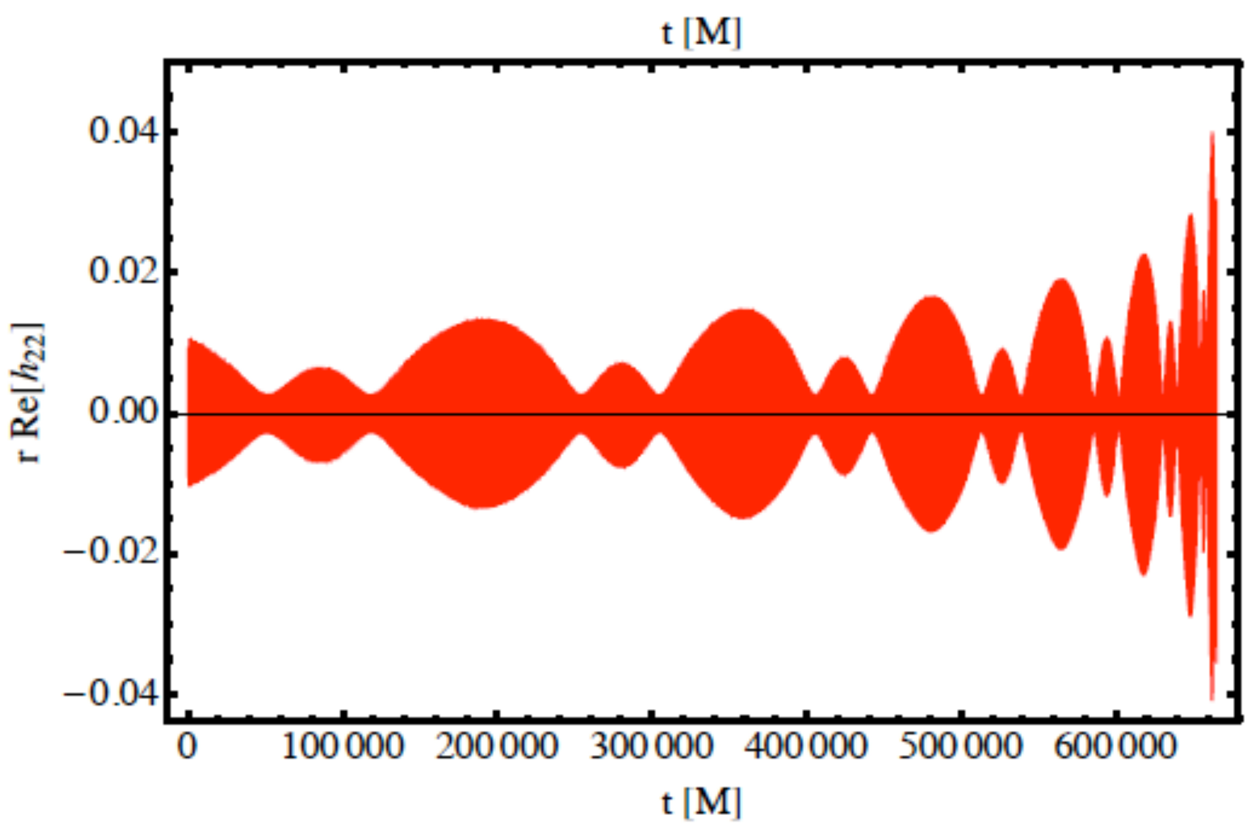

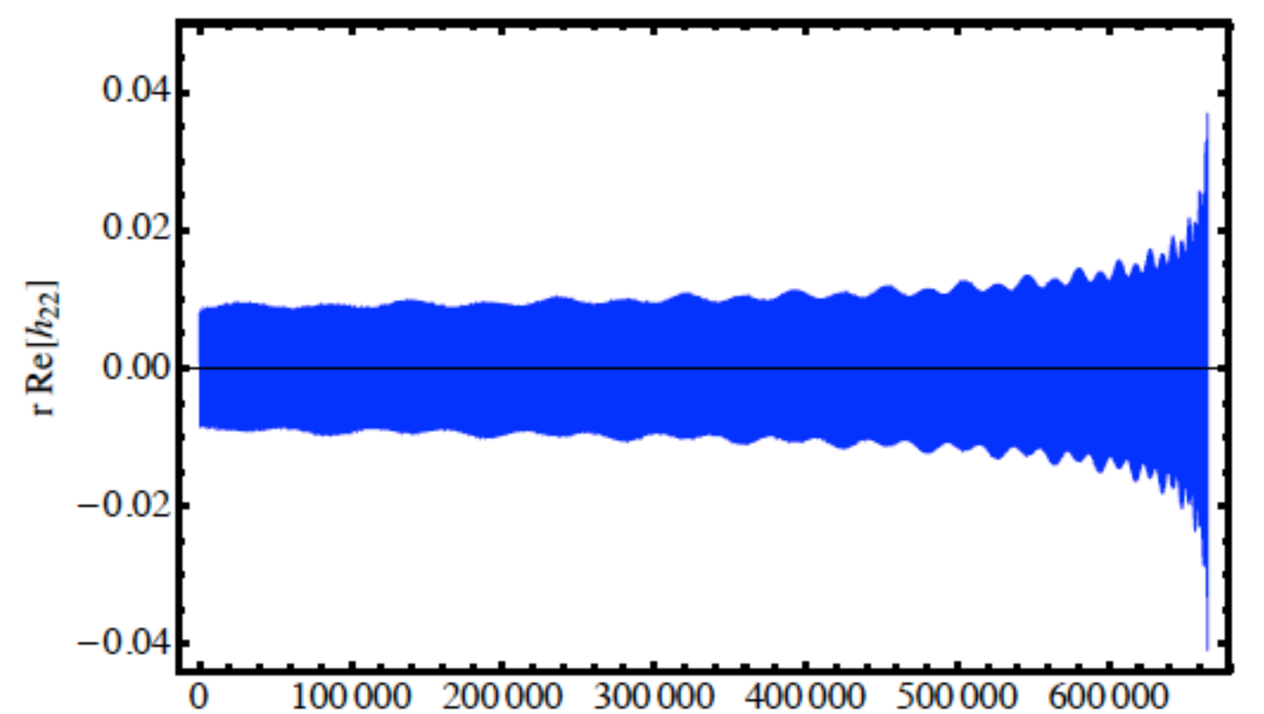

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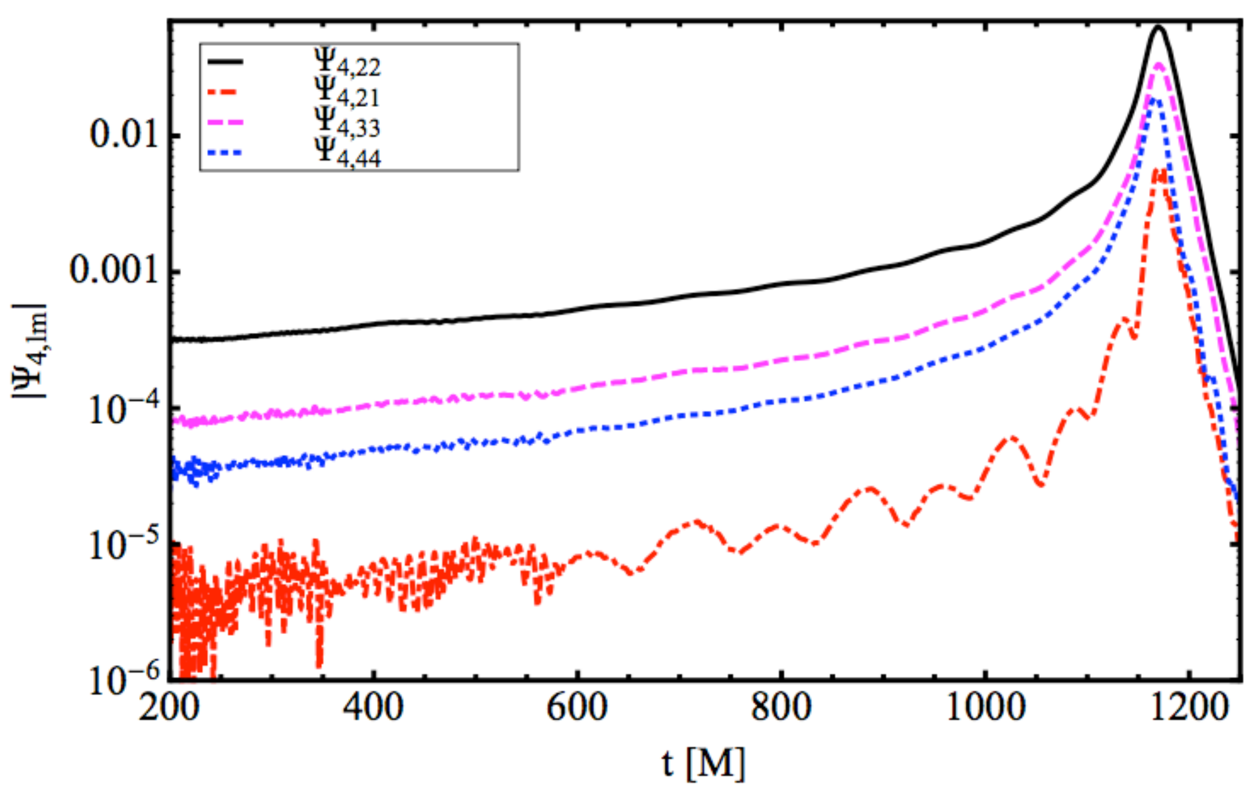

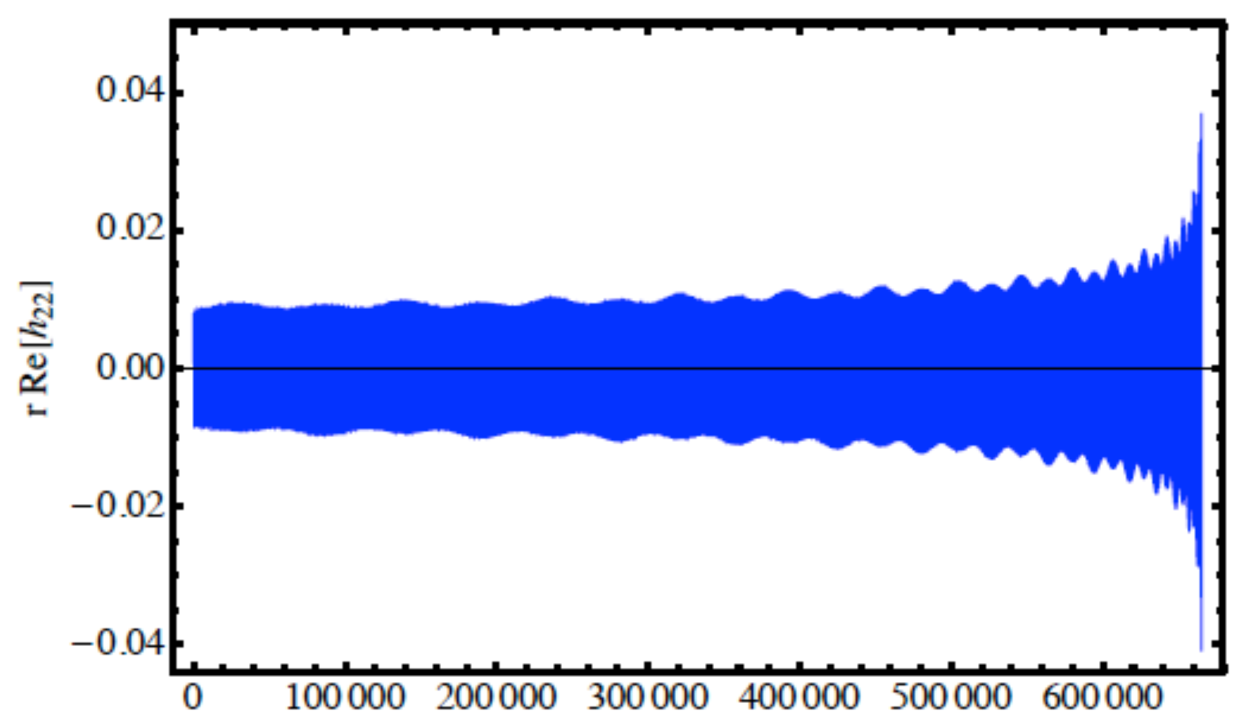

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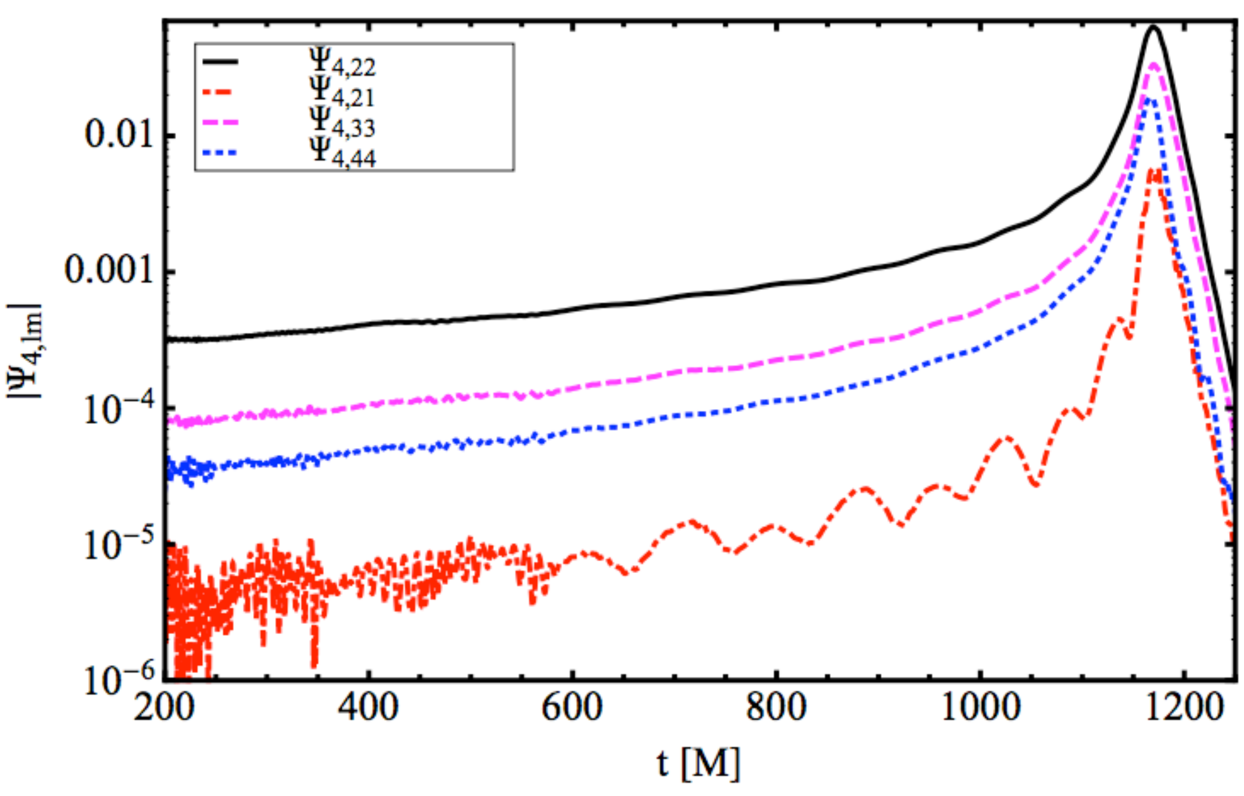
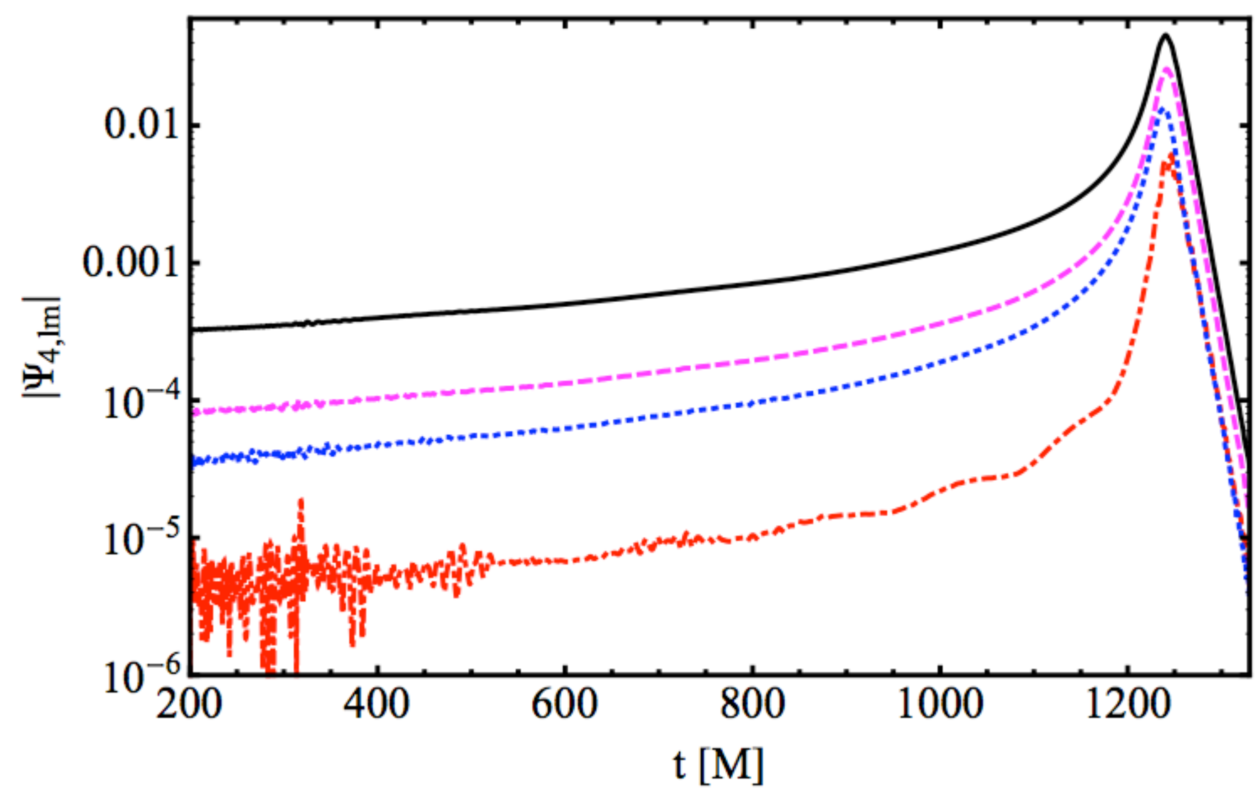


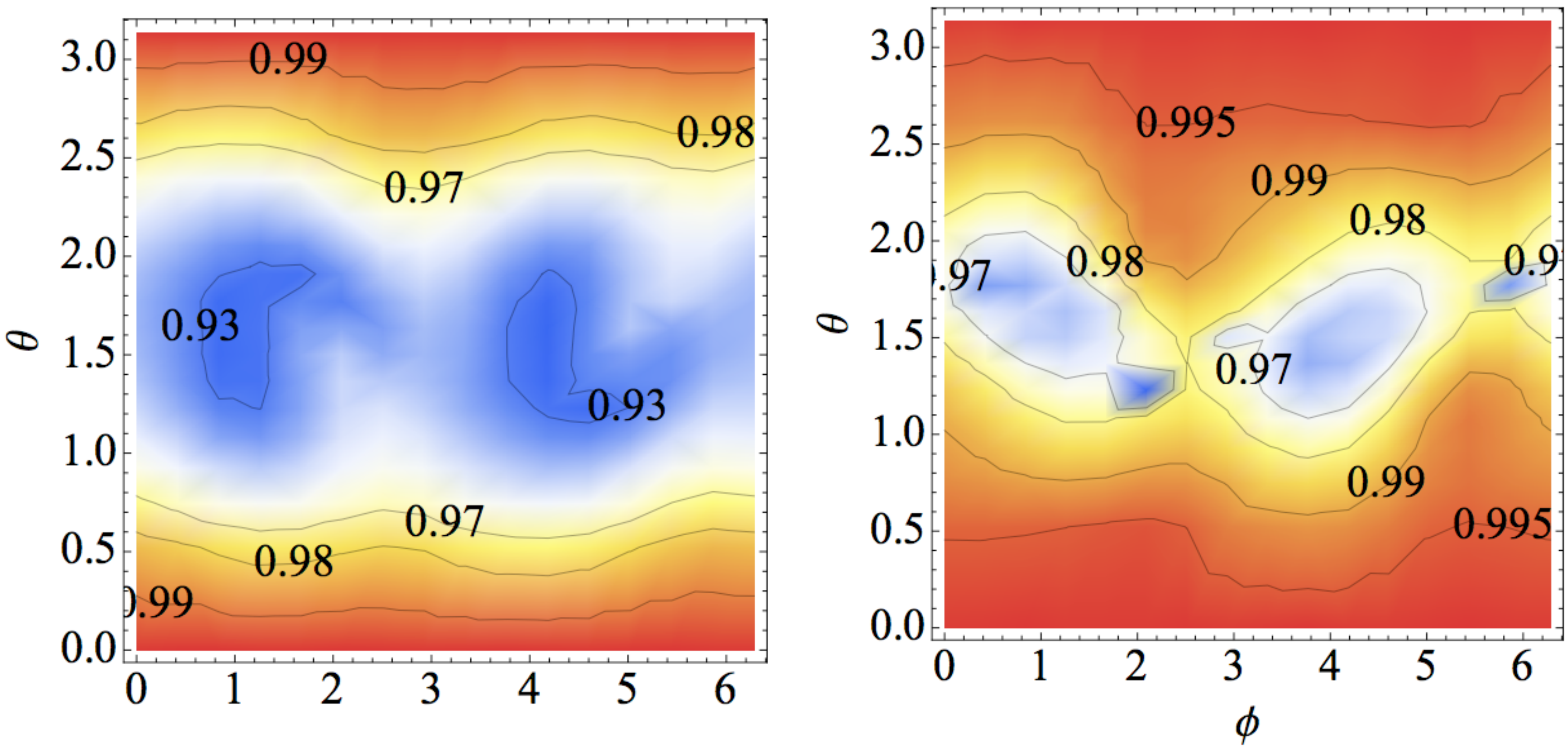


PhenomP model [Cardiff+UIB, arXiv:1308.3271]

- Start with PhenomC/D model for l=|m|=2 aligned spin WFs:
  (TaylorF2 at low frequencies)
  $$h(f, M, \eta, \chi_{\rm eff}) = A(f)e^{i\Phi(f)}$$
- Modify ringdown frequency with estimate for final spin that takes into account precession:
  - final spin depends on individual spins and radiated L
    (negligible dependence on precession).
- Twist up l=|m|=2 modes in time domain with Euler angles α, ι, ε, obtained from Newtonian angular momentum, calculated in PN:
  $$h^P_{2m}(t) = e^{-im\alpha} \sum_{|m'|=2} e^{im'\epsilon} d^2_{m',m}(-\iota) h_{2,m'}(t)$$
- Parameterize precession by a single parameter χ_P - magnitude of spin in orbital plane - neglects double spin oscillations and “superkick” symmetry breaking.
- Angles vary slowly during inspiral -> perform SPA calculation to obtain explicit expressions in Fourier domain.
  $$h^P_{+,\times}(Mf; \eta, \chi_{\rm eff}, \chi_p, \theta, \phi)$$

Fitting factors: PhenomC & PhenomP vs. hybrids

PhenomP: PhenomC twisted up with 2PN PN+SPA Euler angles



PhenomP shortcomings:

- Not tuned to actual precessing NR waveforms.
- Does not include symmetry-breaking “superkick recoil” effect
- SPA for Euler angles @ merger-ringdown
- PhenomC not very accurate -> PhenomPv2 (based on PhenomD) paper in preparation

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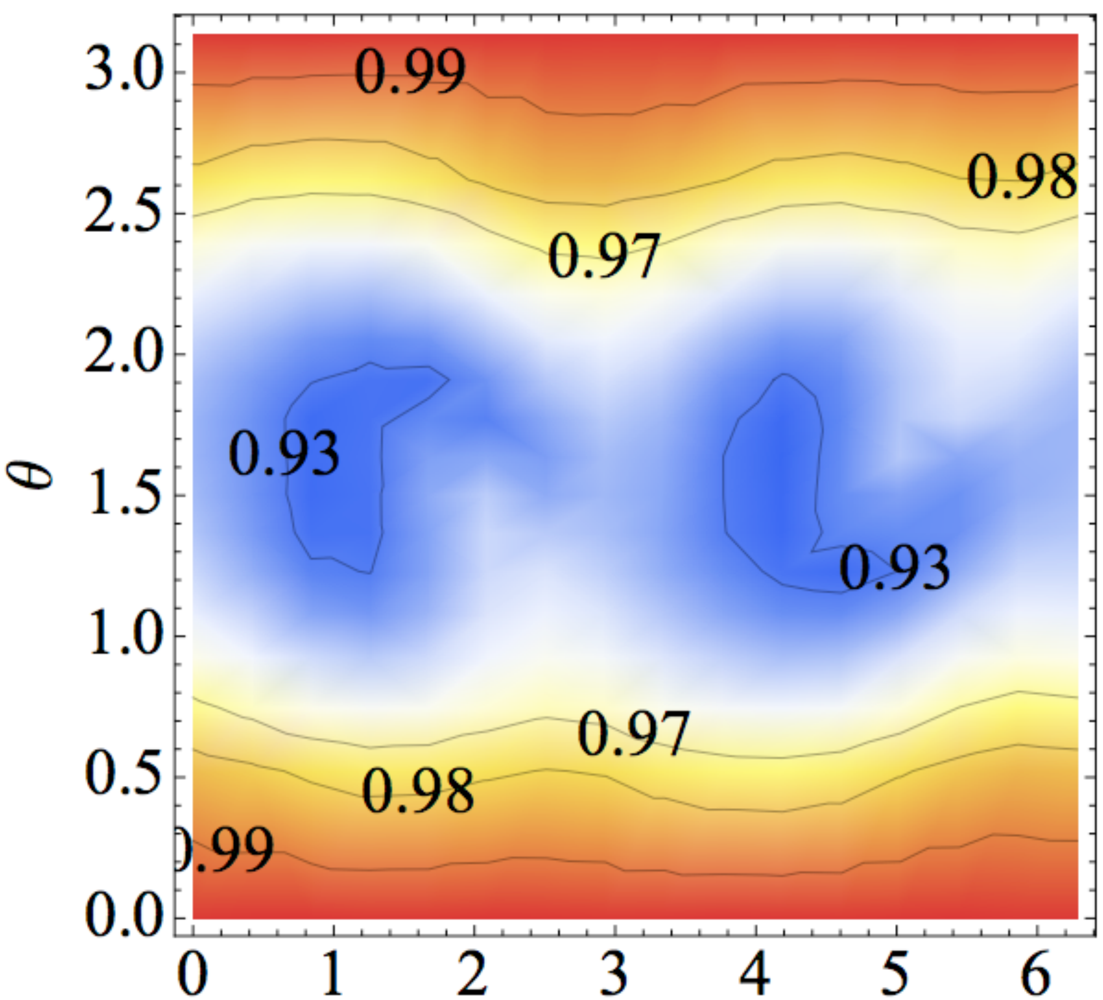

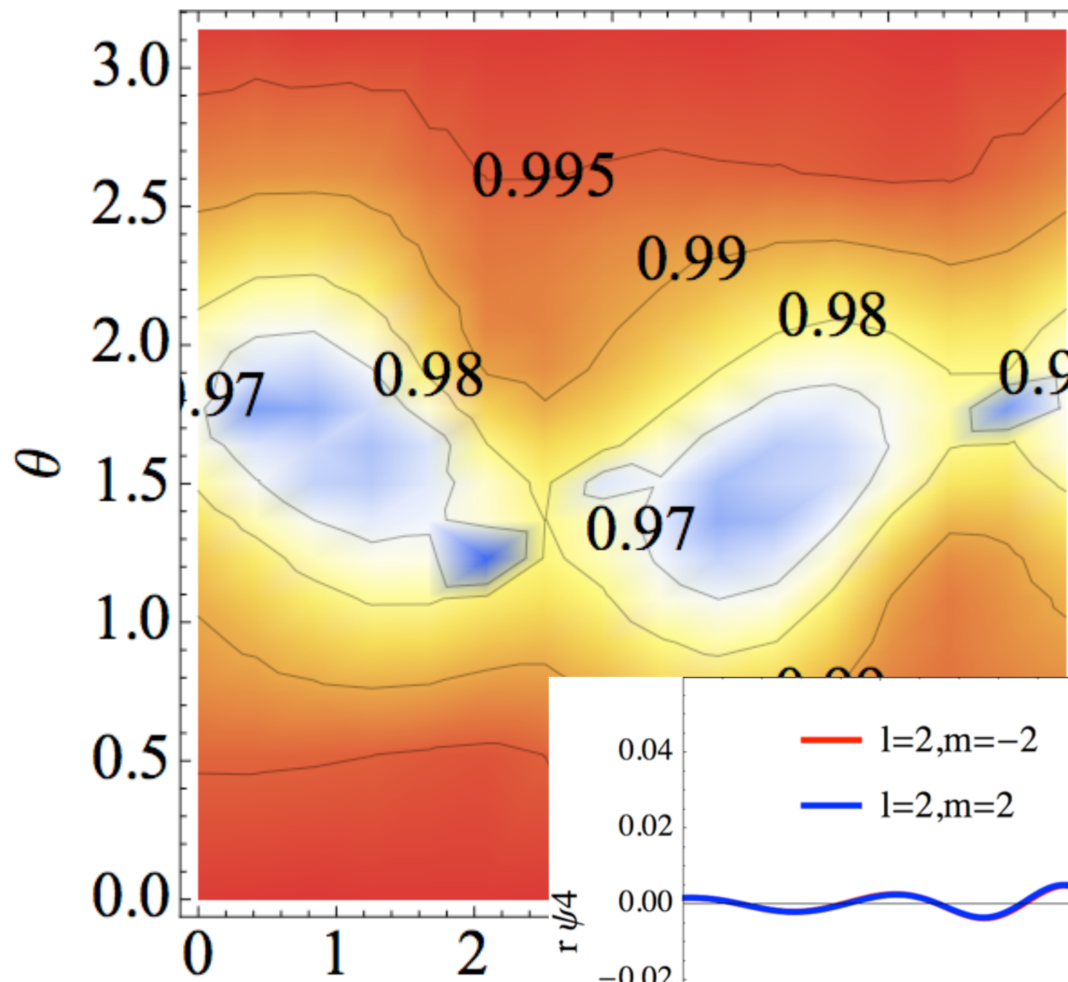

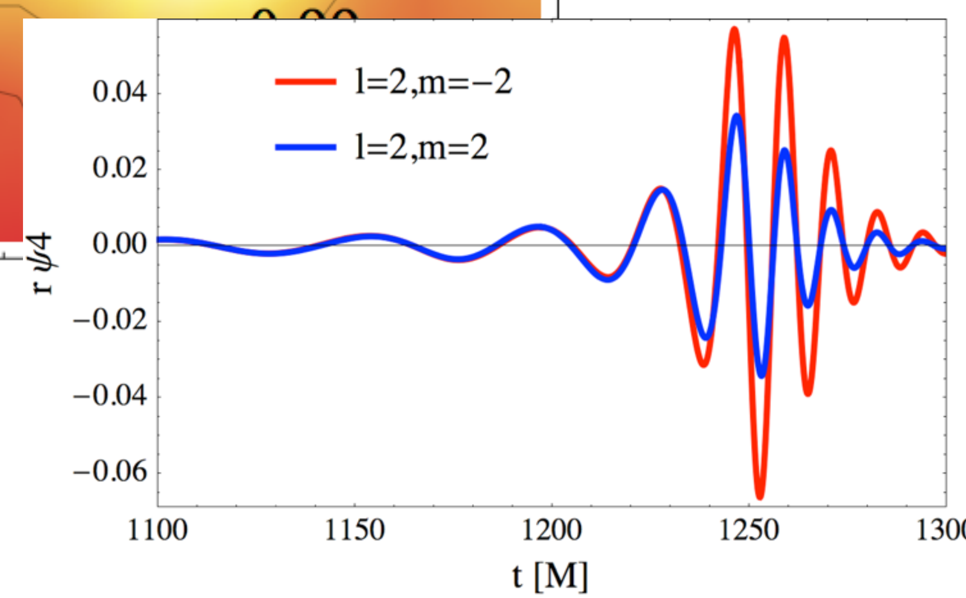

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gIMR [PRL116 (2016)]:

let parameters of Phenom model vary freely, around value of zero = GR.

Problem: For sufficiently large variations the resulting WF may be pathological - need a controlled perturbations and better error-bars for the GR model.

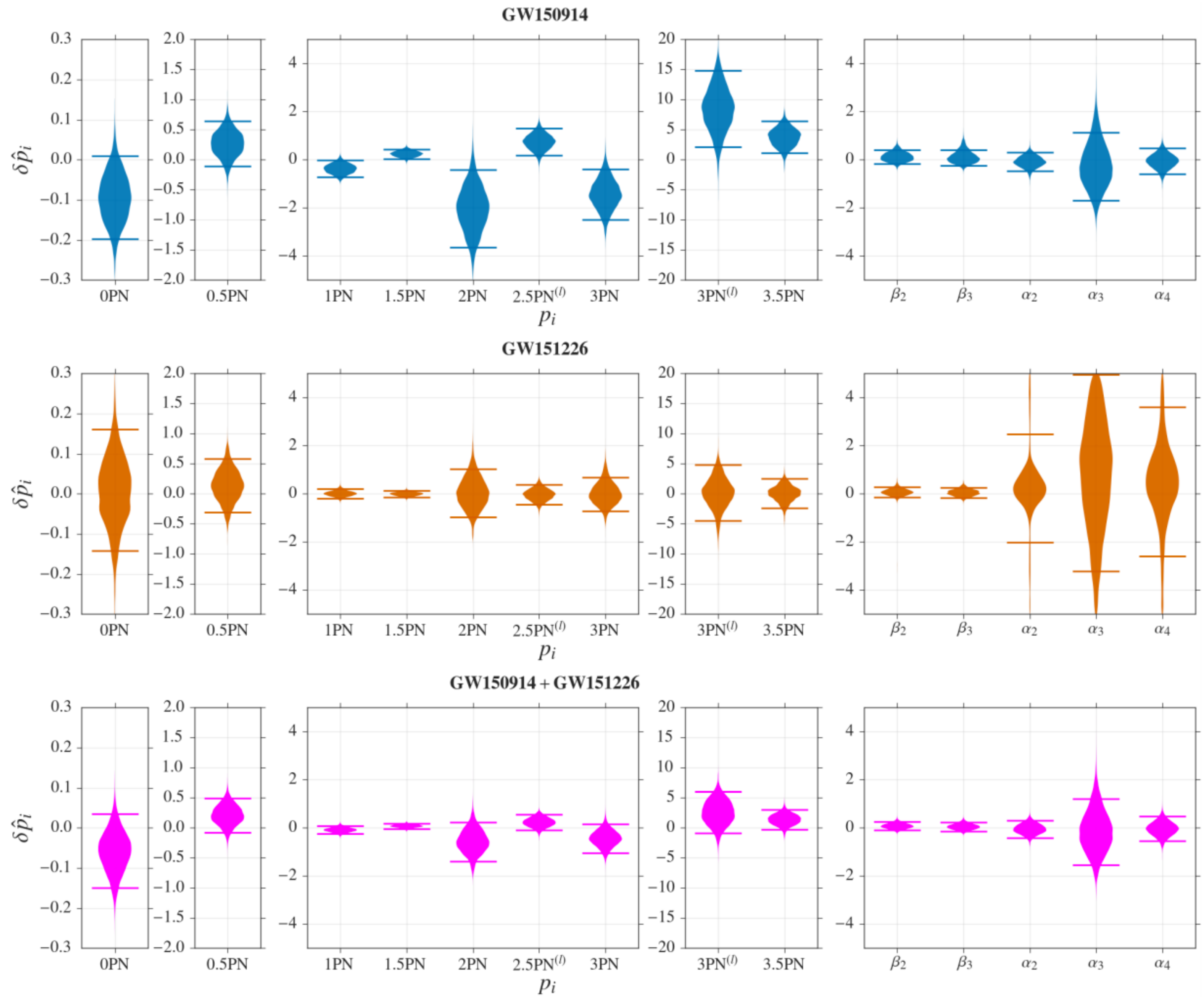

FIG. 6. Posterior density distributions and 90% credible intervals for relative deviations $\delta\hat{p}_i$ in the PN parameters p_i, as well as intermediate parameters β_i and merger-ringdown parameters α_i. The top panel is for GW150914 by itself and the middle one for GW151226 by itself, while the bottom panel shows *combined* posteriors from GW150914 and GW151226. While the posteriors for deviations in PN coefficients from GW150914 show large offsets, the ones from GW151226 are well-centered on zero as well as being more tight, causing the combined posteriors to similarly improve over those of GW150914 alone. For deviations in the β_i, the combined posteriors improve over those of either event individually. For the α_i, the joint posteriors are mostly set by the posteriors from GW150914, whose merger-ringdown occurred at frequencies where the detectors are the most sensitive.

Conclusions

Success of waveform models relies on combined efforts from different communities:

- perturbative methods: post-Newtonian, self-force, …
- numerical relativity
- GW data analysis

We live in the best possible world:

- within a decade, full inspiral-merger-ringdown waveform models were developed, which are good enough for the first detections.
- Open questions remain, more ideas needed, papers to be written ….

Recent progress with the phenomenological waveforms approach: getting ready for LIGO/Virgo upgrades and theoretical/MDC studies for next generation ground based detectors & LISA.