Gravitational waves from neutron stars: theoretical challenges

Ian Jones
University of Southampton

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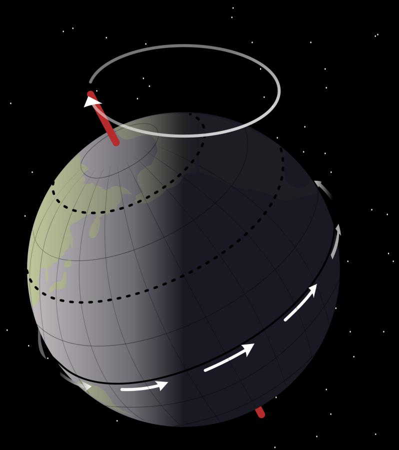
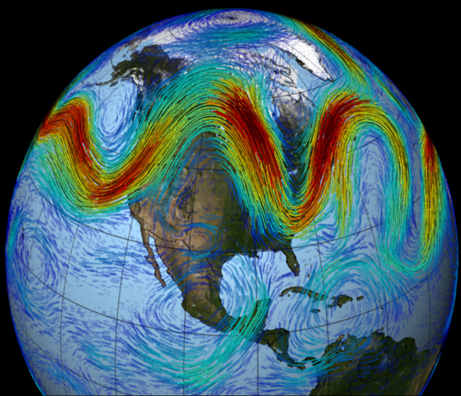
Three gravitational wave emission mechanisms

"Mountains" – non-axisymmetric deformation



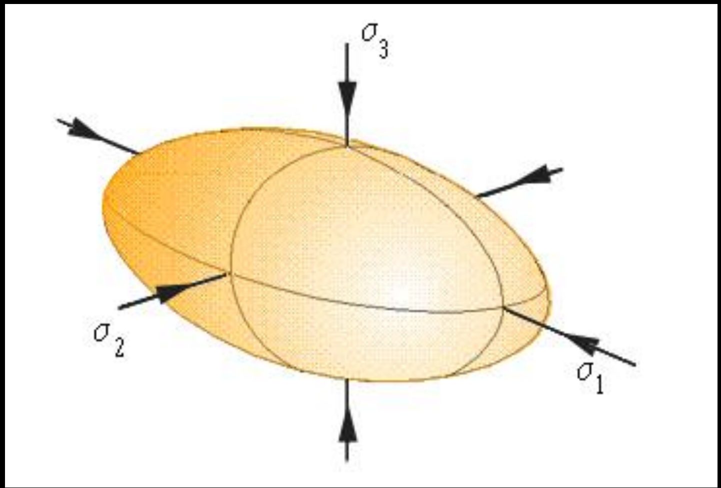
"Wobble" - free precession
Most general motion of rigid body



Fluid oscillations – many possible sorts.
Won't address this issue here.

Gravitational waves from mountains

A *triaxial* neutron star, rotating steadily, emits gravitational waves:



$$h = 3 \times 10^{-28} \left(\frac{\epsilon}{10^{-6}}\right) \left(\frac{f_{\rm spin}}{10\,{\rm Hz}}\right)^2 \left(\frac{1\,{\rm kpc}}{r}\right)$$

$$\epsilon = \frac{I_{yy} - I_{xx}}{I_{zz}}$$

Dimensionless asymmetry in moment of inertia tensor

Spin frequency

Distance to source

Will discuss two issues:

1. Ellipticities (strength of the signal)
2. Frequencies (stability of the signal)

Spin-down upper limits

For pulsars with measured period P and spin-down rate P-dot can obtain upper limit on ellipticity by assuming 100% conversion of kinetic energy into GW energy:

$$\epsilon_{\rm spindown} = \left[\frac{5\dot{P}P^3}{32(2\pi)^4 I_{zz}}\right]^{1/2}$$

For instance, for Crab pulsar, $\varepsilon_{spindown} \approx 7.6 \times 10^{-4}$.

If distance known, can convert into an upper limit on GW amplitude.

For instance, for Crab pulsar, $h_0 \approx 1.4 \times 10^{-24}$

GW upper limits: spin-down and direct

Spindown upper limit beaten for 8 pulsars.

Example: for Crab, $\varepsilon < 3 \times 10^{-5}$.

Implies no more than 0.2% of spin-down energy is going into the GW channel.

Is this interesting?

What does theory have to say about ellipticity?

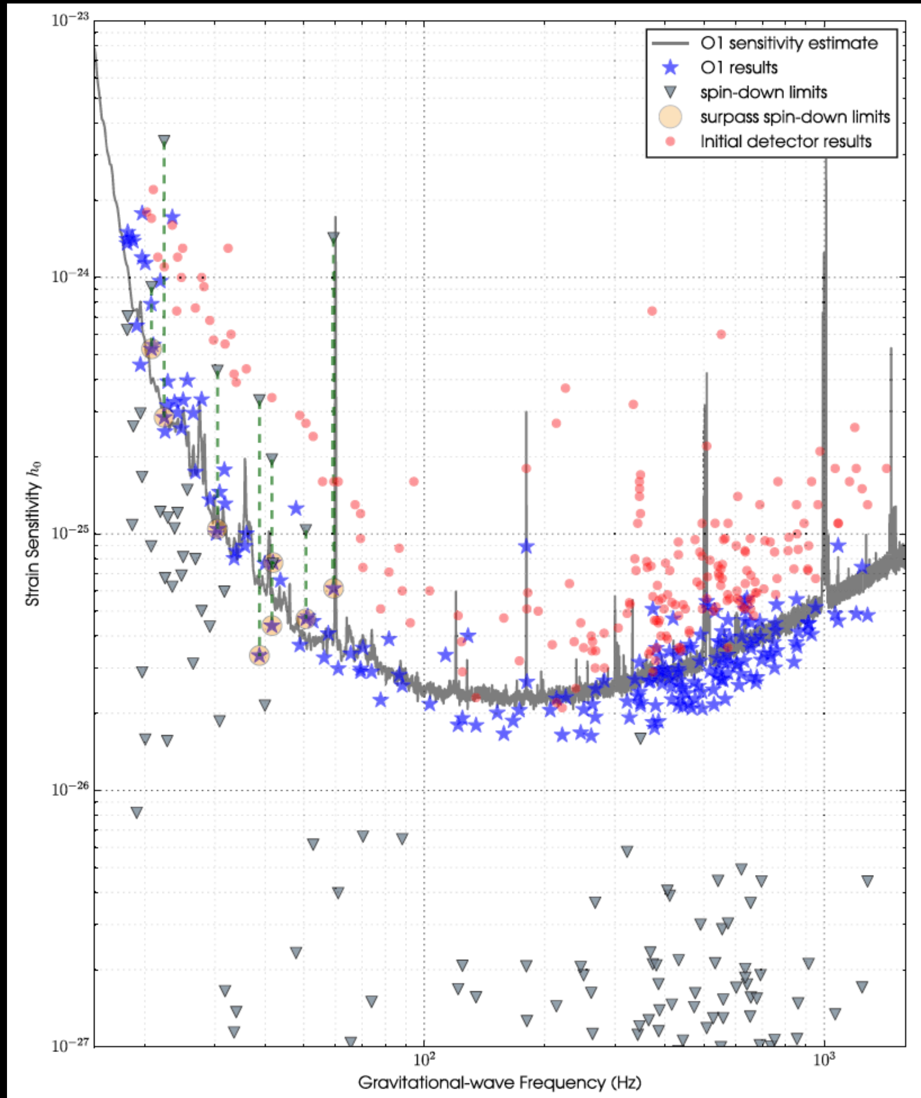


Figure: Aasi+ 2017; O1 data

Elastic mountain: Back-of-the-envelope

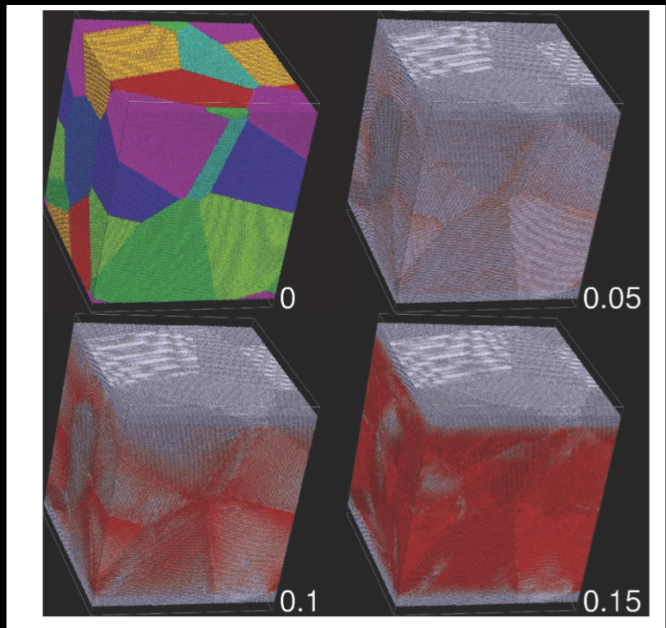
Maximum elastic mountain size determined by balance between gravitational and elastic forces:

$$\epsilon \approx \frac{\mu V_{\rm crust}}{GM^2/R} \times u_{\rm break} \approx 10^{-6} \left(\frac{u_{\rm break}}{10^{-1}} \right)$$

Shear modulus μ, ~ 10^{29} erg cm^{-3} for *crust*

Breaking strain u_{break} less well understood

Molecular dynamics of Horowitz & Kadau (2009) indicate high breaking strain, ~ 0.1 (see Figure).



Plastic flow may relax crust on longer timescales (Chugunov & Horowitz 2010)

“Exotic” elastic mountains

Crust may not be the only solid phase

$\varepsilon_{max} \sim 10^{-1}$ possible for solid quark stars, 10^{-3} for hybrid stars (Johnson-McDaniel & Owen 2013).

Crystalline colour superconducting quark matter also relevant (Mannarelli et al 2007) leading to similarly large maximum ellipticities (Haskell et al 2007 and Lin 2007)

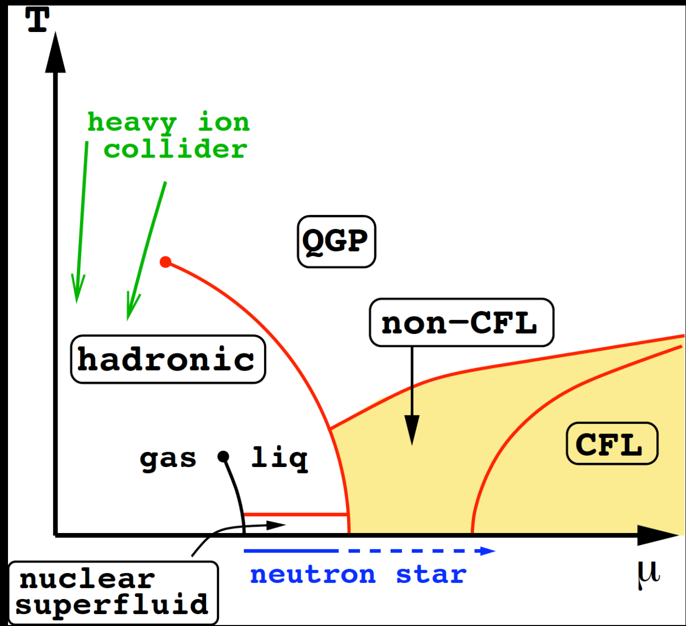


Figure: Alford et al

Lack of detection of such a large mountain *does not* rule out such exotic states of matter…

. . . need estimates of *likely* ellipticities, not just upper bounds!

Mountain building

Bildsten (1998) proposed building mountain via temperature/composition asymmetry in accreting stars.

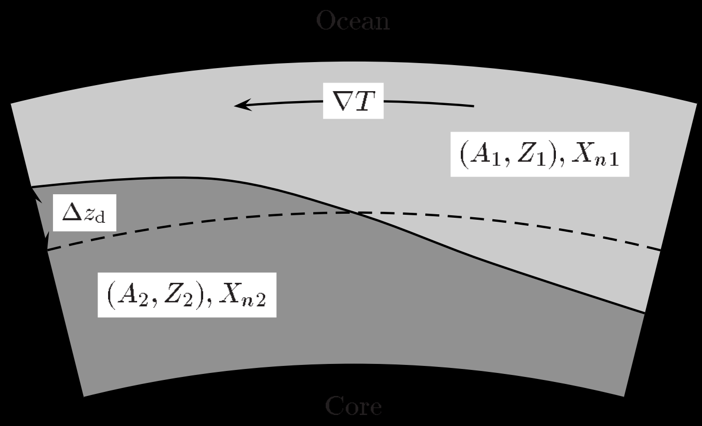


Figure credit: Ushomirsky, Cutler & Bildsten (2000)

Viability of mechanism confirmed by Ushomirsky, Cutler & Bildsten (2000).

Possible evidence for mechanism at work proposed recently (Haskell & Patruno 2017).

But key unknown is likely level of temperature/composition asymmetry.

Magnetic mountains: back-of-the-envelope

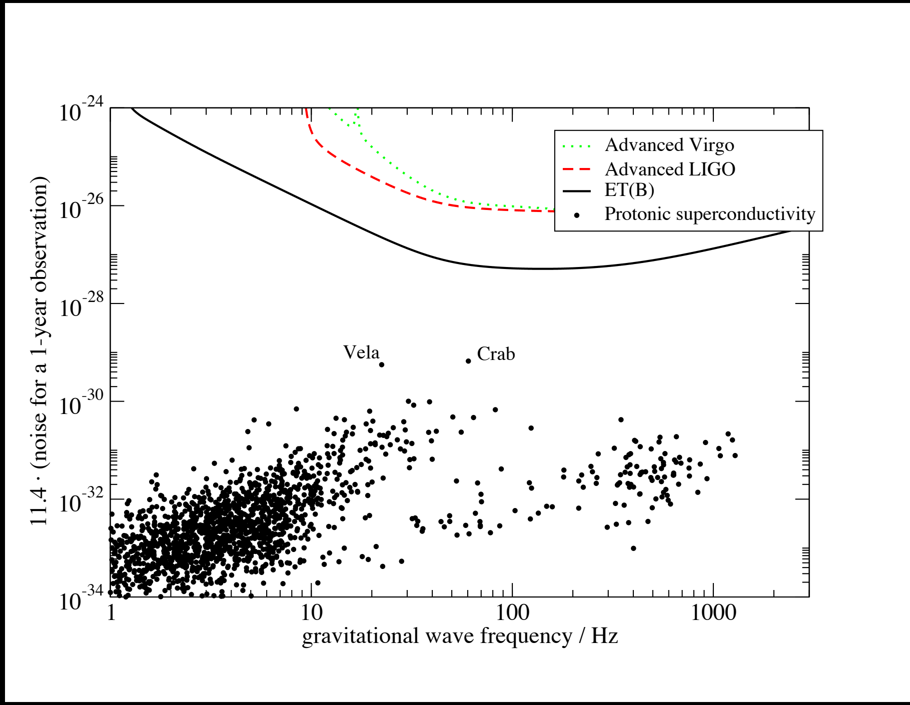
Magnetic field lines have an effective tension, and deform star; roughly:

$$\epsilon \sim \frac{\int B^2 \, dV}{GM^2/R} \sim 10^{-12} \left(\frac{B}{10^{12}\,\mathrm{G}} \right)^2$$

If protons form type II superconductor, magnetic field confirmed to fluxtubes. Effect of this is to increase tension by a factor of H_c/B, where $H_c \sim 10^{15}$ G, increasing ellipticity:

$$\epsilon \sim 10^{-9} \frac{B}{10^{12}\,\mathrm{G}}$$

But even then, the GW emission from known pulsars is not detectable.



Can get stronger emission from local field burial in accreting systems (Haskell et al 2015).

“Exotic” magnetic mountains

If CFL or 2SC phases occur in neutron star cores, can get *colour-magnetic flux tubes* (Iida & Baym 2002, Iida 2005, Alford & Sedrakian 2010).

This leads to flux tube tension ~ 10^3 larger than in protonic superconductivity case. Glampedakis, DIJ & Samuelsson (2012) estimate ellipticity:

$$\epsilon_{\rm CFL} \sim 10^{-7} \left(\frac{f_{\rm vol}}{1/2}\right) \left(\frac{B_{\rm int}}{10^{12}\,{\rm G}}\right) \left(\frac{\mu_{\rm q}}{400\,{\rm MeV}}\right)^2$$

Fraction of stellar volume in CFL/2SC state

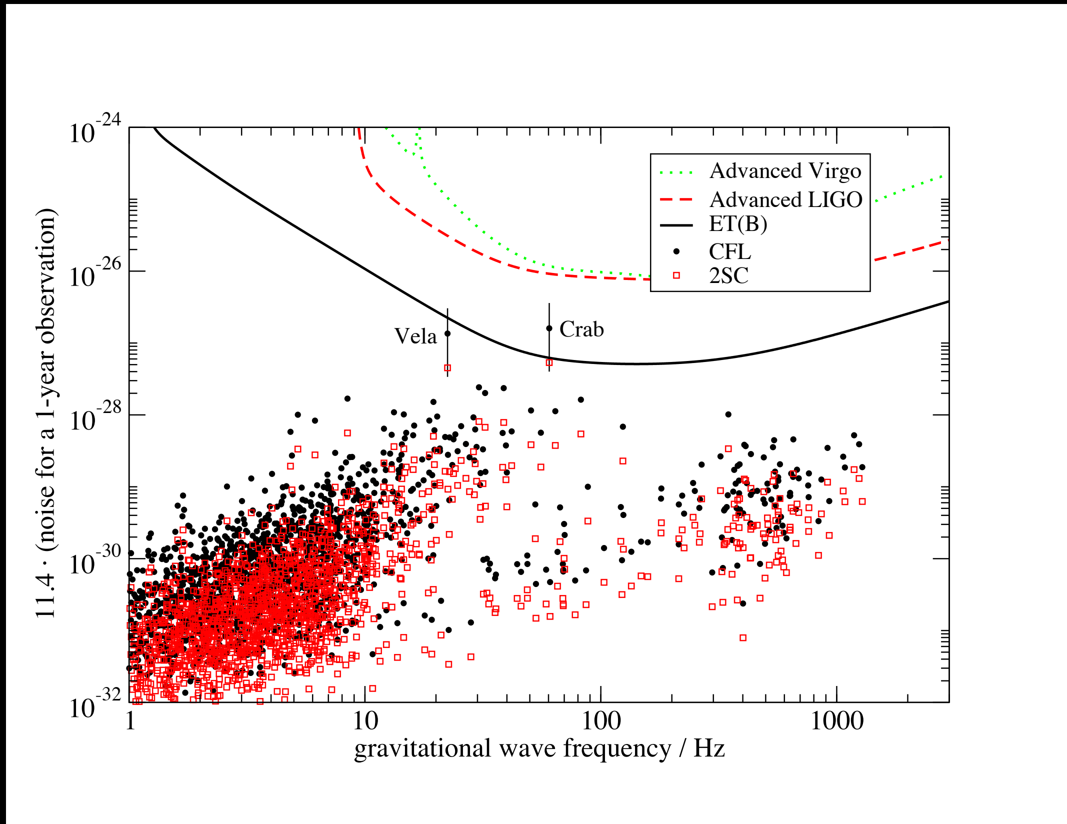
Internal field strength; set $B_{int} = \alpha B_{ext}$

Quark chemical potential

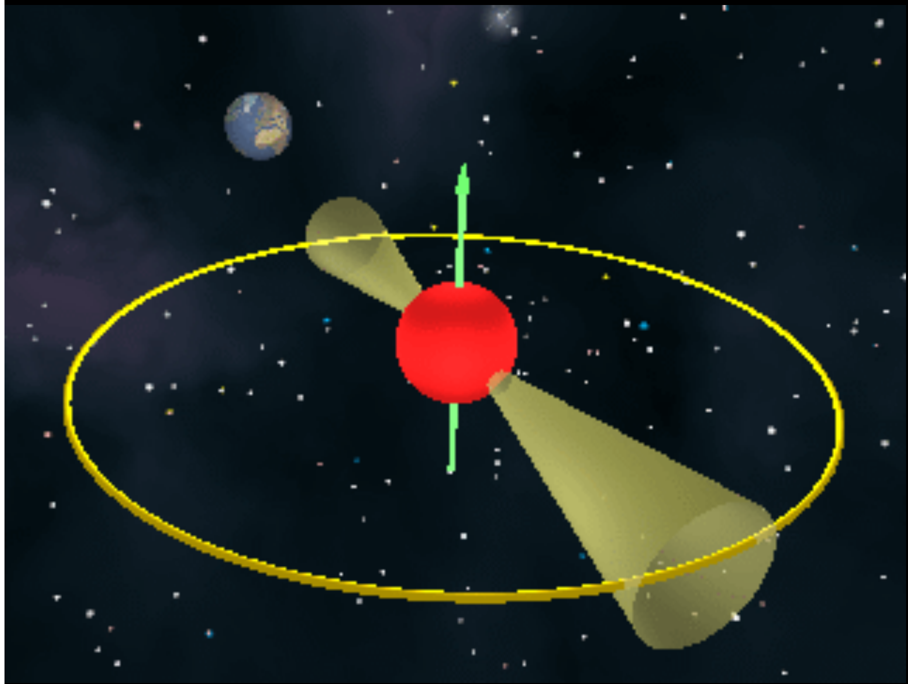
“Exotic” magnetic mountains: detectability

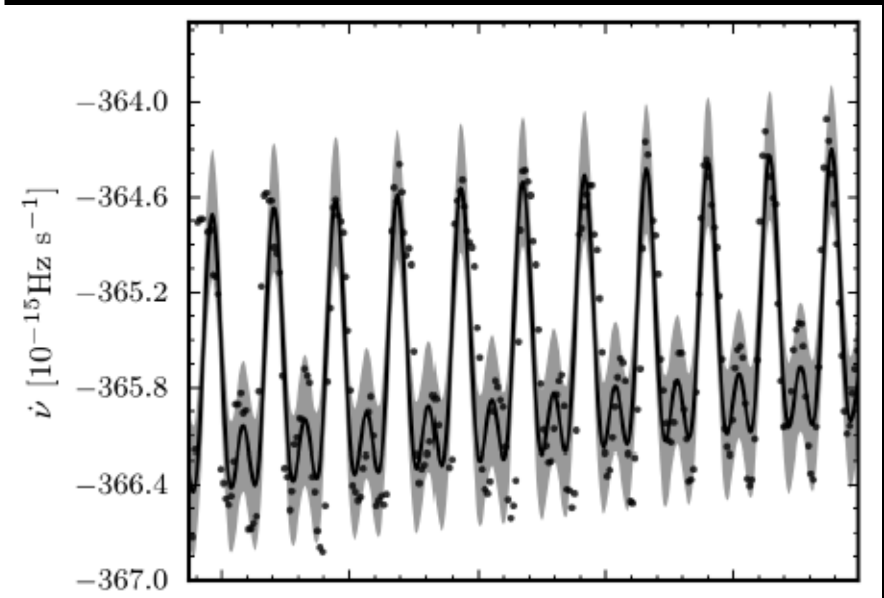
For given stellar parameters f_{vol}, α and μ_q can then balance observed spin-down of pulsars against combined GW & EM torque to estimate B_{int} and hence h.

GW amplitudes scale as $h \sim f_{vol}\, \alpha\, \mu_q^2$; for sensible values ($f_{vol} = 0.5$, $\alpha = 2$, $\mu_q = 400$ MeV) obtain:



Free precession





Precessing biaxial body will emit GWs at both f and 2f.

Precession will also leave imprint on radio pulsar timing (Jodrell animation).

Period of free precession depends upon interaction between superfluid and rest of star.

Doesn't seem to be common in observed radio pulsar population.

Most targeted gravitational wave searches have looked only at 2f, but…

Data courtesy of Andrew Lyne

GW emission: effect of a pinned superfluid

Part of interior superfluid can "pin" to rest of star:

$$J_a = I^{\rm C}_{ab}\Omega^{\rm C}_b + I^{\rm SF}\Omega^{\rm SF}_a$$

Usual contribution from rotating crust

Extra piece due to pinned superfluid

Pinned superfluid acts as a gyroscope, sewn into the star!

A steadily rotating star can then rotate about an arbitrary axis, giving GW emission at both f and 2f (DIJ 2010):

Observation of GW at both f and 2f from a steadily spinning star will provide evidence for pinned superfluidity within the star.

"Dual harmonic" search carried out on S5 data; no detection (Pitkin+ 2015).

Gravitational wave searches

Initial LIGO ran from 2002 to 2010.

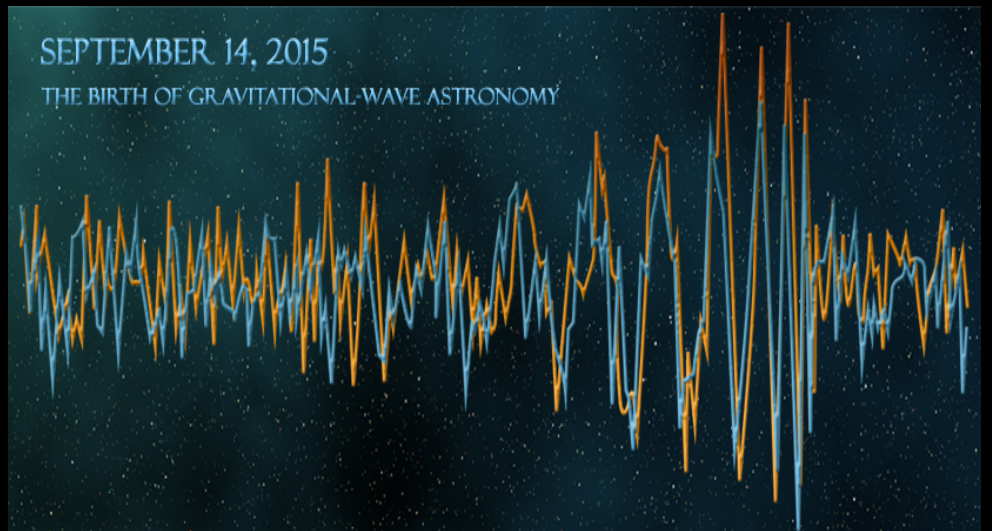
~100 papers, ~25 on GWs from spinning neutron stars.

All upper limits.

Advanced LIGO had first observing run September 2015 – January 2016.

Detection of binary black holse made.

Data analysis for spinning neutron stars underway.; one paper published so far.

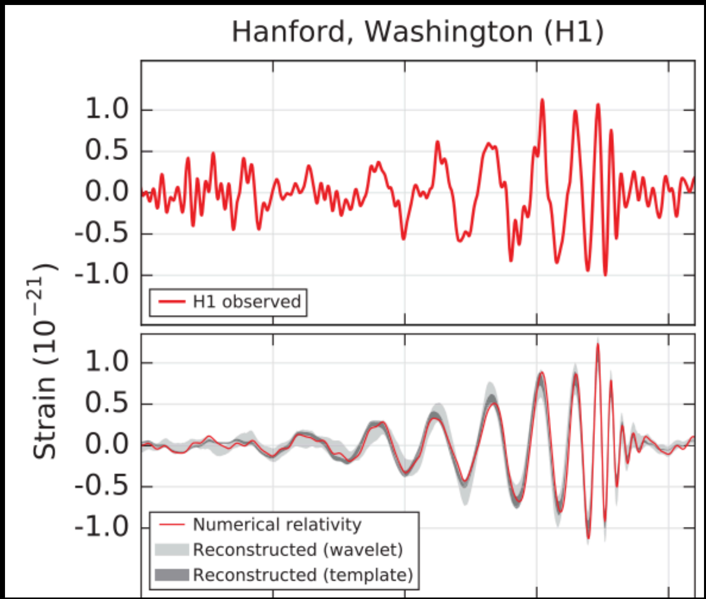


Gravitational wave searches

GW signal is weak, so need to carry out *matched filtering* to make detection.

Need to match phase of signal accurately over lifetime of signal.

GW150914 lasted about 0.3 seconds, ~ 10 cycles:



Abbott et al (2015)

Signals from spinning neutron stars could last months/years, $\sim 10^{10}$ cycles

Need *accurate* search templates.

Targeted searches

Look for GWs from known pulsars with *known timing solutions*, e.g. Crab pulsar

Frequency evolution well-fit by a simple Taylor series:

$$f(t) = f_0 + \dot{f}_0(t - t_0) + \frac{1}{2}\ddot{f}_0(t - t_0)^2 + \ldots$$

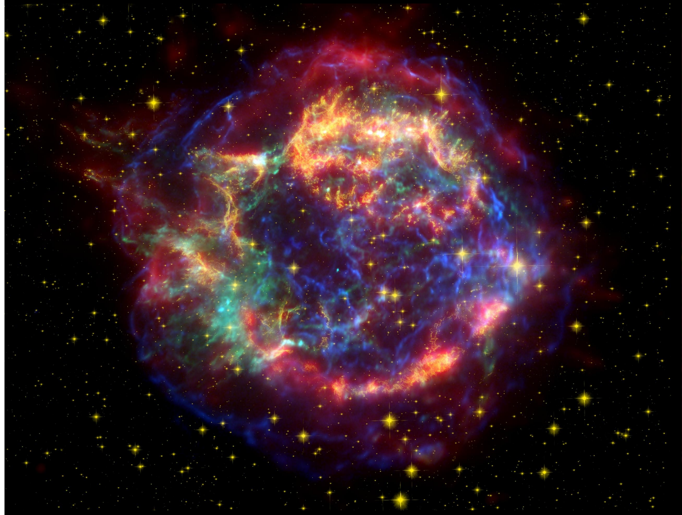
Directed searches

Searches over small sky regions, e.g.

- Supernova remnants
- Globular clusters
- Galactic centre

Timing solution not available: assume a Taylor series.

Of moderate computational cost.



Cas A

Globular cluster 47 Tuc

Galactic centre

All-sky searches

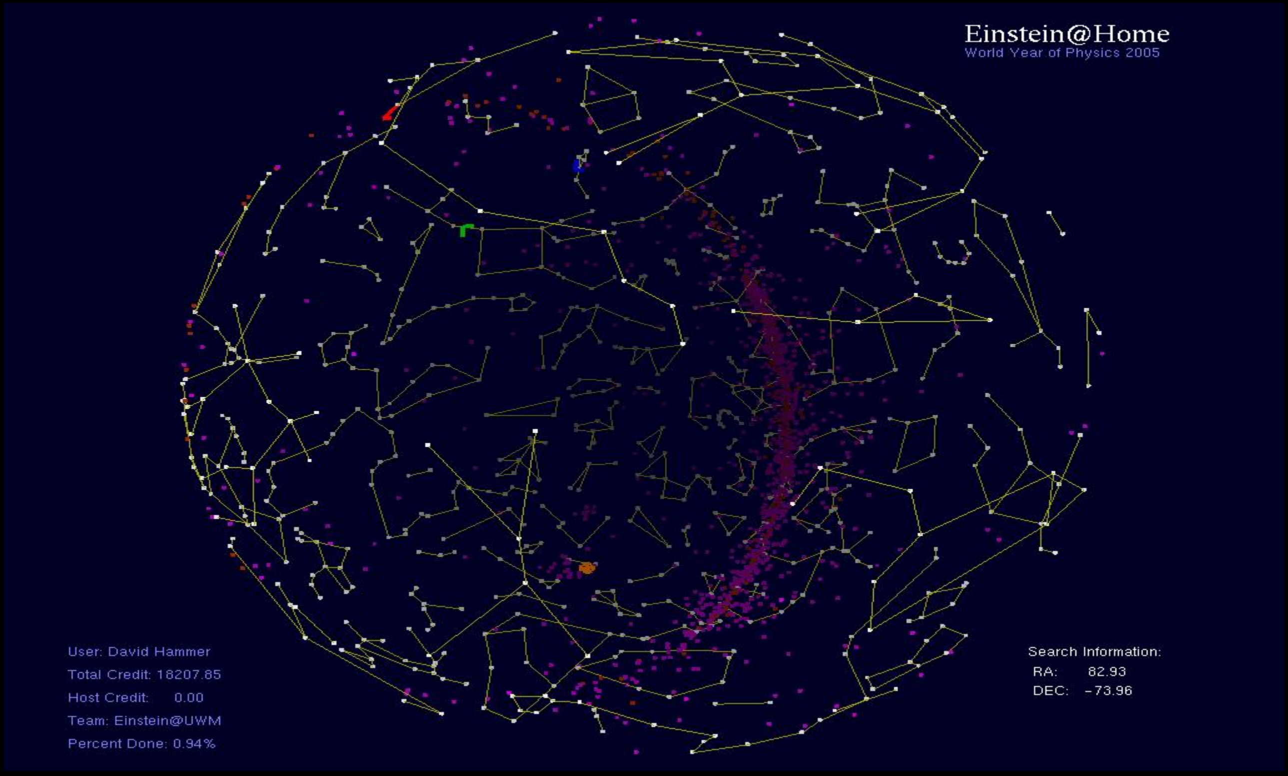
Timing solution not available: assume a Taylor series.

Need to search over:

- All sky directions
- Ranges in spin parameters

Extremely computationally expensive; Einstein@Home used by some searches.

Computational expense reduced by dividing data into N_{seg} segments, and combining the results Incoherently.



Effect of glitches on CW searches

Continuous Wave GW searches typically assume smooth spin evolution.

But glitches are seen in many pulsars: sudden step in frequency:

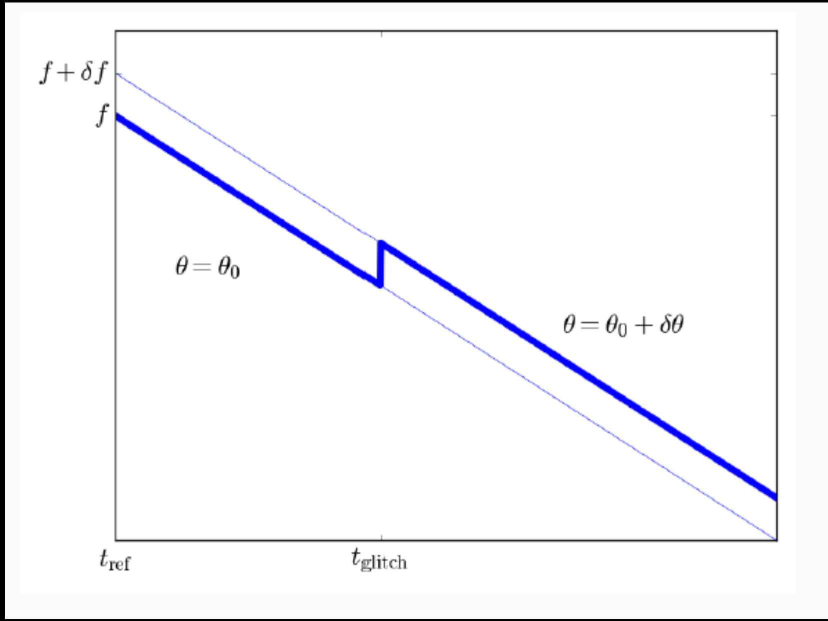


Image credit: Greg Ashton

Potential impact of this on directed and all-sky CW searches assessed:
Ashton, Prix & DIJ 2017 (arXiv:1704.00742).

Quantifying the problem

If one fails to allow for glitch in signal, detection statistic ρ will be degraded.

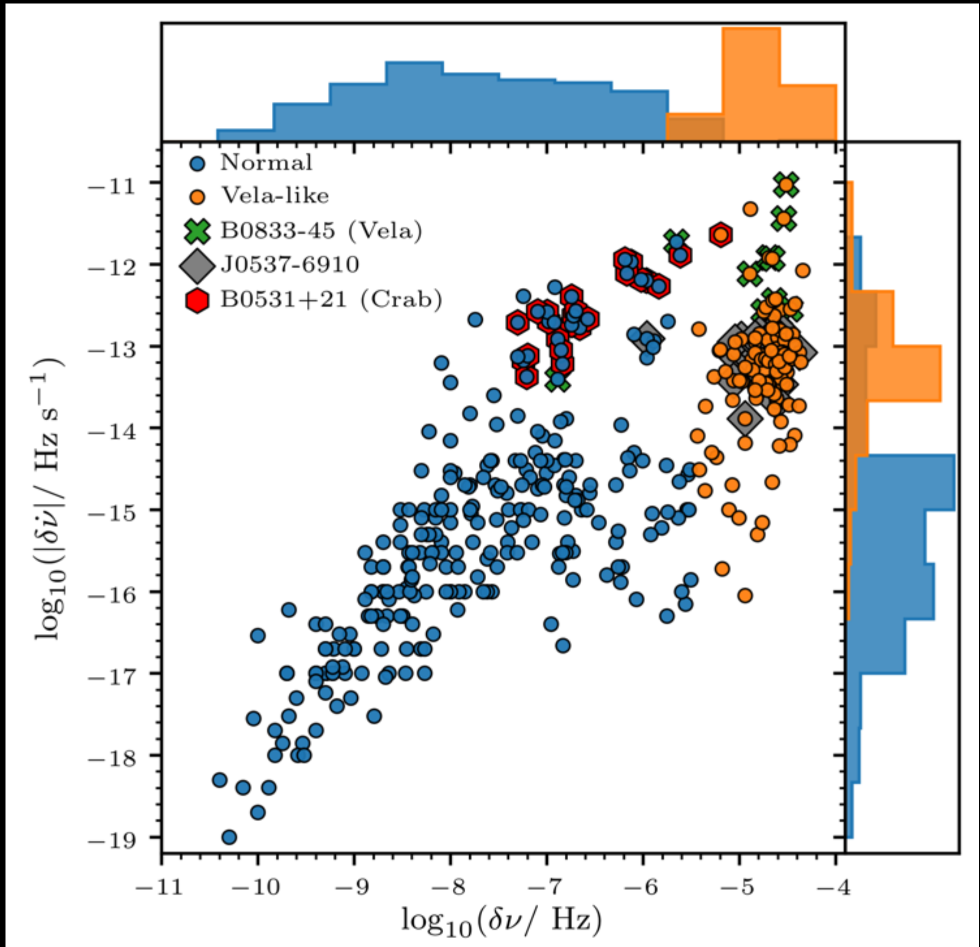
Quantify by mismatch:

$$\mu = \frac{\rho^2_{\mathrm{perfect}} - \rho^2_{\mathrm{template}}}{\rho^2_{\mathrm{perfect}}}$$

For given CW search, can used observed pulsar population to estimate number and magnitude of glitches.

Can then estimate mismatch for each point in search parameter space.

Sizes of glitches



Glitches typically produce changes in both spin frequency and spindown rate.

Glitches often partitioned into two subclasses, “normal” and “Vela-like”.

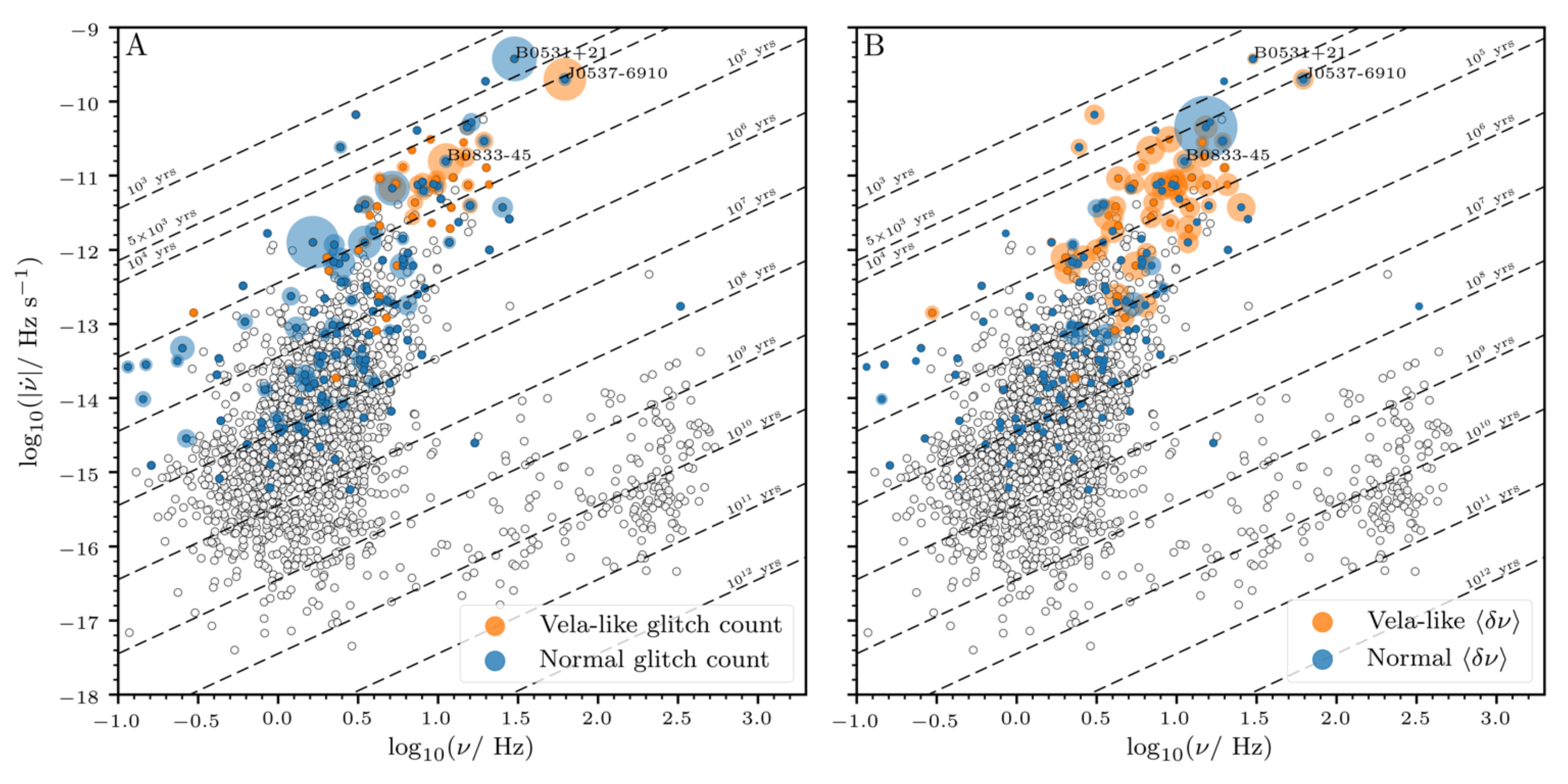
Figure Ashton, Prix, DIJ 2017.

Data taken from glitch catalogue; see Espinoza+ 2011.

Which pulsars glitch?

Number of glitches

Size of glitches

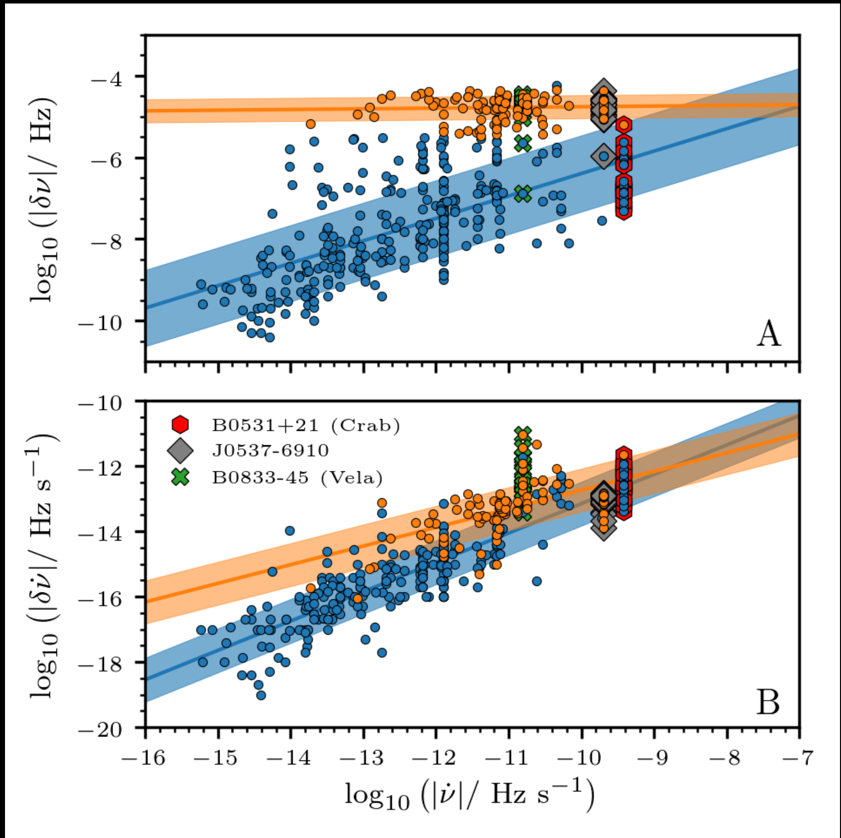


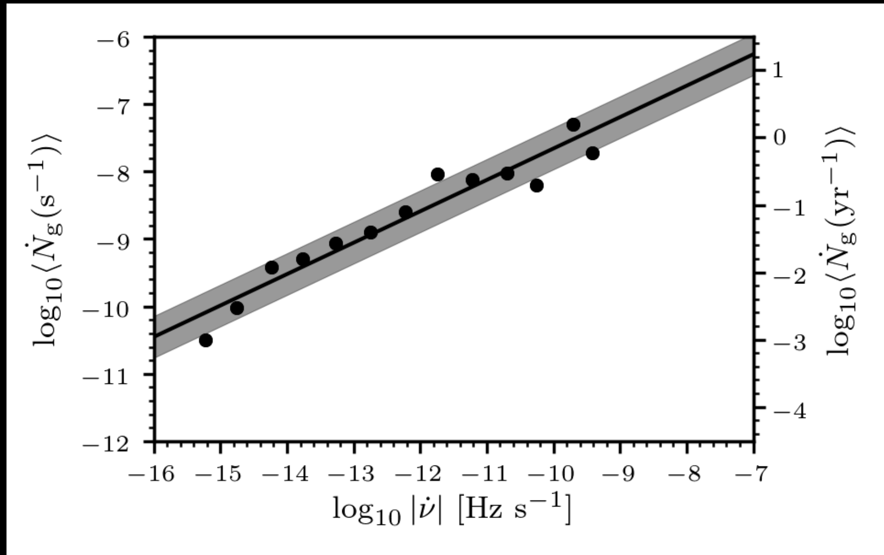
Ashton, Prix & DIJ 2017

Younger pulsars tend to have larger and more frequent glitches.

Estimating size and frequency of glitches:

Both size and frequency reasonably well fit in terms of spin-down rate.

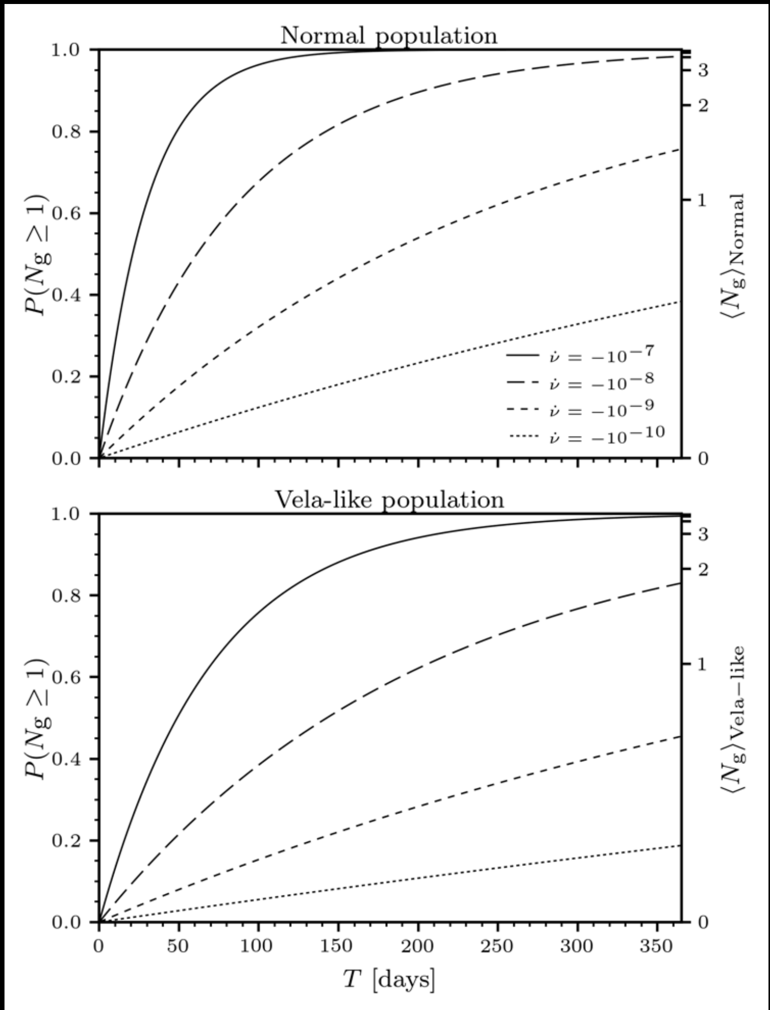




Plot reconstructed from Espinoza+ 2011,
Fit from Ashton, Prix & DIJ 2017

Ashton, Prix & DIJ 2017

Probability of a glitch occurring



Estimate of number of glitches occurring is then a function of assumed spindown rate and duration of CW observation.

Assuming Poisson distribution, can convert to probability of one or more glitches occurring.

Ashton, Prix & DIJ 2017

Impact on past/future CW searches

	$\min(\dot{f}_s)$ [nHz/s]	T_{seg} [hrs]	N_{seg}	T [days]	Ref.	Normal population				Vela-like population			
						$\langle N_g \rangle$	$P_{N_g \geq 1}$	$\langle \hat{\mu}^{(0)} \rangle_{N_g \geq 1}$	$\langle \bar{\mu}^{(0)} \rangle_{N_g \geq 1}$	$\langle N_g \rangle$	$P_{N_g \geq 1}$	$\langle \hat{\mu}^{(0)} \rangle_{N_g \geq 1}$	$\langle \bar{\mu}^{(0)} \rangle_{N_g \geq 1}$
S6 E@H all-sky	−2.7	60	90	255	[43]	1.1	68%	0.16 (0.35)	0.40 (0.54)	0.5	39%	0.18 (0.29)	0.34 (0.47)
S6 E@H all-sky HFU (stage 4)	−2.7	280	22	257	[19]	1.1	68%	0.27 (0.36)	0.44 (0.53)	0.5	39%	0.26 (0.31)	0.39 (0.47)
S6 E@H Cas. A	−106.0	140	44	257	[44]	6.3	100%	0.64 (0.63)	0.93 (0.84)	2.7	93%	0.44 (0.45)	0.76 (0.66)
O1 E@H all-sky	−2.6	210	12	105	[45]	0.5	37%	0.17 (0.33)	0.25 (0.46)	0.2	18%	0.17 (0.30)	0.25 (0.43)
O1 E@H Mult. Dir. Cas. A	−144.0	245	12	122.5	[45]	3.5	97%	0.54 (0.54)	0.77 (0.71)	1.5	78%	0.38 (0.40)	0.59 (0.56)
O1 E@H Mult. Dir. Vela Jr.	−67.9	369	8	123.0	[45]	2.5	91%	0.47 (0.49)	0.66 (0.64)	1.1	65%	0.35 (0.39)	0.51 (0.52)
O1 E@H Mult. Dir. G357.3	−29.7	489	6	122.25	[45]	1.7	81%	0.41 (0.46)	0.59 (0.58)	0.7	51%	0.34 (0.39)	0.50 (0.49)

Ashton, Prix & DIJ 2017

Bottom line: we probably should be worrying about glitches!

Summary

- Maximum/likely GW amplitudes depend sensitively on the state of matter at high density.
- Initial LIGO observations began to probe regimes of astrophysics interest, but detection possible only if
  - Elastic mountains in exotic phases, and they are close to maximally strained, or…
  - … star has non-standard magnetic field configuration.
- Detection of signal simultaneously at f, 2f would give (independent) evidence of superfluid pinning.
- Prospects getting better (Advanced LIGO 10 times more sensitive), but detection far from guaranteed.
- Need to think how to deal with glitches in directed/all-sky searches.