The Giant Graviton on $AdS_4 \times \mathbb{CP}^3$

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DG, JM & AP: 1108.3084 [hep-th]

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Outline

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Gauge theory / gravity dualities

Quantum field theories in flat space with very large local symmetry groups (gauge groups) at strong coupling λ are dual to weakly coupled theories of gravity.

This leads naturally to the following question:

How are geometry and topology
(both of spacetime and membranes embedded in spacetime)
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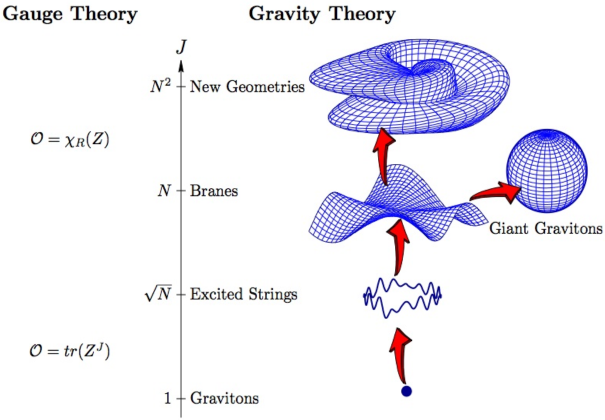
Gauge Theory
Gravity Theory
J
N^2 New Geometries
$\mathcal{O} = \chi_R(Z)$
N Branes
Giant Gravitons
$\sqrt{N}$ Excited Strings
$\mathcal{O} = tr(Z^J)$
1 Gravitons

The four-brane giant graviton on $AdS_4 \times \mathbb{CP}^3$ is a *non-spherical membrane*, embedded and moving in the complex projective space, which *changes shape as it grows*.

We asked the question:

How is the changing shape of this membrane visible in the dual ABJM model?

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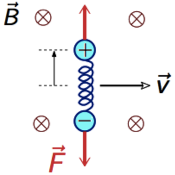
Giant gravitons

A lower dimensional analogy: An electric dipole

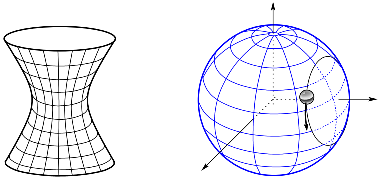
An electric dipole moving perpendicular to a magnetic field $\vec{B}$ (coupling to the EM one-form potential $A_1 = A_\mu dx^\mu$) experiences a force which keeps the charges separated.



The faster it moves, the bigger the dipole! (The greater the equilibrium separation distance between the $+$ve and $-$ve charges.)

Sphere giant gravitons on AdS$_5 \times S^5$

The sphere giant is a D3-brane embedded on an $S^3 \subset S^5$. It is both embedded and moving on the five-sphere space in the background spacetime. The extension of this $\frac{1}{2}$-BPS object is supported by a coupling to the 4-form potential C_4.



[McGreevy, Susskind & Toumbas: hep-th/0003075]

[Grisaru, Myers & Tafjord: hep-th/0008015]

The dual operator in $\mathcal{N} = 4$ SYM is Schur polynomial, constructed from $n \sim O(N)$ single complex scalar field Z and labeled by the totally antisymmetric representation of S_n:

$$\chi_{\tiny\begin{array}{|c|}\hline \\ \hline \\ \hline \vdots \\ \hline \\ \hline \end{array}}(Z) \propto \mathcal{O}_n^{\text{subdet}}(Z) = \epsilon_{a_1 \ldots a_n a_{n+1} \ldots a_N} \, \epsilon^{b_1 \ldots b_n a_{n+1} \ldots a_N} \, Z_{b_1}^{a_1} \cdots Z_{b_n}^{a_n}$$

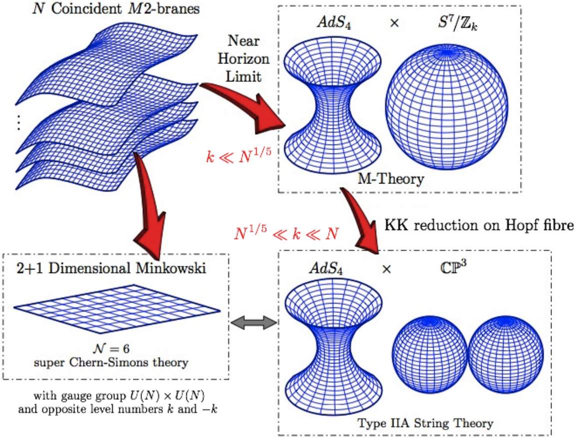
proportional to a subdeterminant with maximum size $n = N$.

[Balasubramanian et. al.: hep-th/0107119]

[Corley, Jevicki & Ramgoolam: hep-th/0111222]

A natural interpretation of this maximum length from the string theory point of view is that the sphere giant cannot grow to be bigger than the compact S^5 space.

Outline



[Aharony, Bergman, Jafferis & Maldacena (ABJM): 0806.1218 [hep-th]]

ABJM Model

t'Hooft coupling: $\lambda = \frac{N}{k}$

two sets of two complex scalars: $(A_1)^a_\alpha$, $(A_2)^a_\alpha$, $(B_1)^a_\alpha$, $(B_2)^a_\alpha$

in the bifundamental representation of the $U(N) \times U(N)$ gauge group. (Here a and α are indices in different $U(N)$'s.)

four composite scalars:

We can build composite scalars

$$(\phi_{11})^a_b = (A_1)^a_\alpha \, (B_1^\dagger)^\alpha_b, \qquad (\phi_{12})^a_b = (A_1)^a_\alpha \, (B_2^\dagger)^\alpha_b,$$
$$(\phi_{21})^a_b = (A_2)^a_\alpha \, (B_1^\dagger)^\alpha_b, \qquad (\phi_{22})^a_b = (A_2)^a_\alpha \, (B_2^\dagger)^\alpha_b$$

transforming in the first $U(N)$ of the product gauge group, out of which we can build long, gauge invariant operators.

Type IIA string theory on $\text{AdS}_4 \times \mathbb{CP}^3$

The metric of the $\text{AdS}_4 \times \mathbb{CP}^3$ background is

$$ds^2 = R^2 \left(ds^2_{\text{AdS}_4} + 4\, ds^2_{\mathbb{CP}^3} \right)$$

There is a constant non-zero dilaton Φ satisfying $e^{2\Phi} = \frac{4R^2}{k^2}$. The field strength forms are given by

$$F_2 \equiv dC_1 = 2k\, dJ \qquad \text{with} \quad C_1 = 2kJ$$
$$F_8 = *F_2$$

$$F_4 \equiv dC_3 = -\tfrac{3}{2} kR^2 \ \text{vol}\left(\text{AdS}_4\right)$$
$$F_6 = *F_4 \equiv dC_5 = \tfrac{3}{2} \left(2^6\right) R^4 \ \text{vol}\left(\mathbb{CP}^3\right)$$

The $\mathbb{CP}^3$ giant graviton on $\mathrm{AdS}_4 \times \mathbb{CP}^3$

The $\mathbb{CP}^3$ giant graviton is a D4-brane extended and moving in the complex projective space. Its extension is supported by a coupling to the 5-form potential C_5.

If we turn on a worldvolume gauge field, then this D4-brane will also couple to the C_1 potential through $F \wedge F \wedge C_1$.

The dual operator of length $n \sim O(N)$ is a Schur polynomial constructed from the single composite field $A_1 B_1^\dagger$ and labeled by the totally antisymmetric representation of S_n:

$$\chi_{\begin{smallmatrix}\square\\ \vdots \\ \square\end{smallmatrix}}(A_1 B_1^\dagger) \propto \mathcal{O}_n^{\text{subdet}}(A_1 B_1^\dagger) = \epsilon_{a_1 \ldots a_n a_{n+1} \ldots a_N}\, \epsilon^{b_1 \ldots b_n a_{n+1} \ldots a_N}\, (A_1 B_1^\dagger)_{b_1}^{a_1} \cdots (A_1 B_1^\dagger)_{b_n}^{a_n}$$

which factorizes at maximum size into the product of two full determinants

$$\mathcal{O}_N^{\text{subdet}}(A_1 B_1^\dagger) = (\det A_1)\,(\det B_1^\dagger)$$

These are ABJM dibaryons, which are dual to four-branes wrapped on different non-trivial $\mathbb{CP}^2 \subset \mathbb{CP}^3$ subspaces.

[Gutíerrez, Lozano & Rodríguez-Gómez: 1004.2826 [hep-th]]

[JM & AP: 1103.1163 [hep-th]]

Outline

Parameterization of the complex projective space

Let us now parameterize the homogenous coordinates z^a of the complex projective space $\mathbb{CP}^3$ as follows:

$$z^1 = \cos\zeta \sin\frac{\theta_1}{2}\; e^{i(\frac{1}{2}\chi - \frac{1}{4}\varphi_1 + \frac{1}{4}\varphi_2)} \qquad z^2 = \cos\zeta \cos\frac{\theta_1}{2}\; e^{i(\frac{1}{2}\chi + \frac{3}{4}\varphi_1 + \frac{1}{4}\varphi_2)}$$

$$z^3 = \sin\zeta \sin\frac{\theta_2}{2}\; e^{i(-\frac{1}{2}\chi - \frac{1}{4}\varphi_1 + \frac{1}{4}\varphi_2)} \qquad z^4 = \sin\zeta \cos\frac{\theta_2}{2}\; e^{i(-\frac{1}{2}\chi - \frac{1}{4}\varphi_1 - \frac{3}{4}\varphi_2)}$$

so that the $\mathbb{CP}^3$ metric becomes

$$ds^2_{\mathbb{CP}^3} = d\zeta^2 + \cos^2\zeta \sin^2\zeta \left[d\chi + \cos^2\frac{\theta_1}{2} d\varphi_1 + \cos^2\frac{\theta_2}{2} d\varphi_2\right]^2 + \frac{1}{4}\cos^2\zeta\left(d\theta_1^2 + \sin^2\theta_1 d\varphi_1^2\right) + \frac{1}{4}\sin^2\zeta\left(d\theta_2^2 + \sin^2\theta_2 d\varphi_2^2\right)$$

Note that $\theta_1 = \pi$ and $\theta_2 = \pi$ define two $\mathbb{CP}^2$ subspaces.

We can split the metric of the complex projective space into radial and angular parts:

$$ds^2_{\mathbb{CP}^3} = \frac{1}{4}\left\{ds^2_{\rm rad} + ds^2_{\rm ang}\right\},$$

where

$$ds^2_{\rm rad} = 4\,d\zeta^2 + \cos^2\zeta\; d\theta_1^2 + \sin^2\zeta\; d\theta_2^2$$

$$\begin{aligned} ds^2_{\rm ang} = {} & 4\cos^2\zeta\sin^2\zeta\left[d\chi + \cos^2\tfrac{\theta_1}{2}\,d\varphi_1 + \cos^2\tfrac{\theta_2}{2}\,d\varphi_2\right]^2 \\ & + \cos^2\zeta\sin^2\theta_1\; d\varphi_1^2 + \sin^2\zeta\sin^2\theta_2\; d\varphi_2^2. \end{aligned}$$

The homogeneous coordinates of $\mathbb{CP}^3$ can be associated with the scalars in ABJM theory

$$z^1 \longrightarrow A_1, \quad z^2 \longrightarrow A_2, \quad z^3 \longrightarrow B_1, \quad z^4 \longrightarrow B_2$$

in that the momenta in these directions can be associated with the $\mathcal{R}$-charges of the scalar fields. Hence we deduce

radial direction $\sqrt{1-\alpha^2}$ direction of motion

$$\begin{aligned}
z^1\,\bar{z}_3 &= \tfrac{1}{2}\,\boxed{\sin(2\zeta)\sin\tfrac{\theta_1}{2}\sin\tfrac{\theta_2}{2}}\,e^{i\chi} & \longrightarrow\quad & A_1 B_1^\dagger \\
z^2\,\bar{z}_4 &= \tfrac{1}{2}\sin(2\zeta)\cos\tfrac{\theta_1}{2}\cos\tfrac{\theta_2}{2}\,e^{i(\chi+\varphi_1+\varphi_2)} & \longrightarrow\quad & A_2 B_2^\dagger \\
z^2\,\bar{z}_3 &= \tfrac{1}{2}\sin(2\zeta)\cos\tfrac{\theta_1}{2}\sin\tfrac{\theta_2}{2}\,e^{\frac{1}{2}i(\chi+\varphi_1)} & \longrightarrow\quad & A_2 B_1^\dagger \\
z^1\,\bar{z}_4 &= \tfrac{1}{2}\sin(2\zeta)\sin\tfrac{\theta_1}{2}\cos\tfrac{\theta_2}{2}\,e^{\frac{1}{2}i(\chi+\varphi_2)} & \longrightarrow\quad & A_1 B_2^\dagger
\end{aligned}$$

Giant Graviton Ansatz

* Point-like in the AdS_4 (with $r = 0$) and moving only in time t.
* Radial ansatz in the $\mathbb{CP}^3$

$$\sin(2\zeta)\sin\frac{\theta_1}{2}\sin\frac{\theta_2}{2} = \sqrt{1-\alpha^2},$$

* Motion in $\mathbb{CP}^3$ along the angular direction $\chi = \chi(t)$.
* Turn off the worldvolume field strength $F = dA = 0$.
* We shall make use of the worldvolume coordinates

$$\sigma^a = (t, y, z_1, \varphi_1, \varphi_2)$$

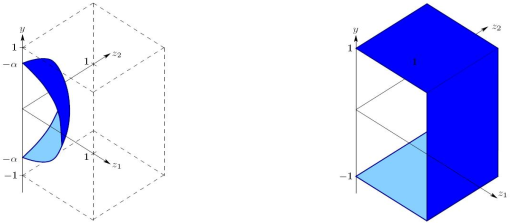
Here we define the radial coordinates

$$y \equiv \cos(2\zeta) \qquad z_1 \equiv \cos^2\frac{\theta_1}{2} \qquad z_2 \equiv \cos^2\frac{\theta_2}{2}$$

and the ansatz becomes

$$\left(1-y^2\right)\left(1-z_1\right)\left(1-z_2\right) = 1-\alpha^2$$

A sketch of the submaximal and maximal $\mathbb{CP}^3$ giants in radial (y, z_1, z_2) space.



(a) Submaximal giant graviton $0 < \alpha < 1$

(b) Maximal giant graviton $\alpha = 1$

The shape of this four-brane changes as the size α increases:

The small giant graviton: $\alpha \ll 1$

The giant graviton ansatz becomes

$$y^2 + z_1 + z_2 \approx \alpha^2$$

which describes a two-sphere in radial $(y, \sqrt{z_1}, \sqrt{z_2})$ space.

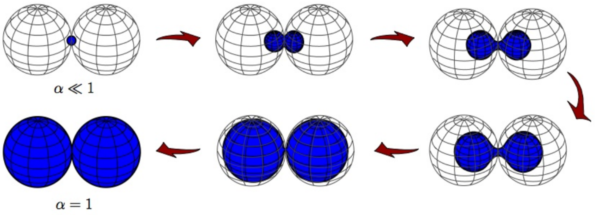
The maximal giant graviton: $\alpha = 1$

The giant graviton ansatz becomes

$$z_1 = 1 \qquad \text{or} \qquad z_2 = 1$$

which describes two separate $\mathbb{CP}^2$ cycles.

Cartoon representation of the growth of the four-brane giant graviton:



The small giant graviton with $\alpha \ll 1$ is nearly spherical, but pinches off as it grows, until it factorizes at maximum size $\alpha = 1$ into two four-branes, each wrapped on a $\mathbb{CP}^2 \subset \mathbb{CP}^3$ cycle.

D4-brane Action

The D4-brane action $S_{\mathrm{D4}} = S_{\mathrm{DBI}} + S_{\mathrm{WZ}}$, which describes the dynamics of the four-brane giant graviton. Here

$$S_{\mathrm{DBI}} = -T_4 \int_{\Sigma} d^5\sigma \; e^{-\Phi} \sqrt{-\det\left(\mathcal{P}\left[g\right] + 2\pi F\right)},$$

and

$$S_{\mathrm{WZ}} = T_4 \int_{\Sigma} \left\{ \mathcal{P}\left[C_5\right] + \mathcal{P}\left[C_3\right] \wedge (2\pi F) + \frac{1}{2} \mathcal{P}\left[C_1\right] \wedge (2\pi F) \wedge (2\pi F) \right\},$$

with $T_4 \equiv \frac{1}{(2\pi)^4}$ the tension and Σ the worldvolume of the giant.

Substituting this ansatz into the D4-brane action

$$S_{\rm D4} = \int dt\ \ L_{\rm D4} \qquad \text{with} \qquad L_{\rm D4} = \int_{-\alpha}^{\alpha} dy \int_0^{\frac{\alpha^2 - y^2}{1-y^2}} dz_1\ \ \mathcal{L}_{\rm D4}(y, z_1)$$

associated with the radial Lagrangian density

$$\mathcal{L}_{\rm D4}(y, z_1) = -\frac{N}{2}\frac{1}{(1-z_1)}\left[\tfrac{1}{2}(1+y)(1-z_1) + \tfrac{1}{2}(1-y)(1-z_2) - \left(1-\alpha^2\right)\right] \times \left\{\sqrt{1 + \frac{\left(1-\dot{\chi}^2\right)\left(1-\alpha^2\right)}{\left[\tfrac{1}{2}(1+y)(1-z_1) + \tfrac{1}{2}(1-y)(1-z_2) - \left(1-\alpha^2\right)\right]}} - \dot{\chi}\right\},$$

where $z_2(z_1) = 1 - \frac{(1-\alpha^2)}{(1-y^2)(1-z_1)}$ and $N \equiv \frac{kR^4}{2\pi^2}$ denotes the flux of the 6-form field strength through the complex projective space.

The conserved momentum conjugate to the χ takes the form

$$P_\chi = \int_{-\alpha}^{\alpha} dy \int_0^{\frac{\alpha^2-y^2}{1-y^2}} dz_1 \;\; \mathcal{P}_\chi(y, z_1),$$

written in terms of the radial momentum density

$$\mathcal{P}_\chi(y,z_1) = \frac{N}{2}\frac{1}{(1-z_1)}\left[\tfrac{1}{2}(1+y)(1-z_1)+\tfrac{1}{2}(1-y)(1-z_2)-\left(1-\alpha^2\right)\right] \times \left\{\frac{\frac{\left(1-\alpha^2\right)\dot{\chi}}{\left[\frac{1}{2}(1+y)(1-z_1)+\frac{1}{2}(1-y)(1-z_2)-\left(1-\alpha^2\right)\right]}}{\sqrt{1+\frac{\left(1-\dot{\chi}^2\right)\left(1-\alpha^2\right)}{\left[\frac{1}{2}(1+y)(1-z_1)+\frac{1}{2}(1-y)(1-z_2)-\left(1-\alpha^2\right)\right]}}}+1\right\}.$$

The energy $H = P_\chi \dot{\chi} - L$ of this D4-brane configuration can hence be determined as a function of its size α and angular velocity $\dot{\chi}$:

$$H = \int_{-\alpha}^{\alpha} dy \int_0^{\frac{\alpha^2 - y^2}{1 - y^2}} dz_1 \;\; \mathcal{H}(y, z_1)$$

with radial Hamiltonian density

$$\mathcal{H}(y, z_1) = \frac{N}{2} \frac{1}{(1 - z_1)} \frac{\left[\frac{1}{2}(1 + y)(1 - z_1) + \frac{1}{2}(1 - y)(1 - z_2)\right]}{\sqrt{1 + \frac{\left(1 - \dot{\chi}^2\right)\left(1 - \alpha^2\right)}{\left[\frac{1}{2}(1+y)(1-z_1) + \frac{1}{2}(1-y)(1-z_2) - \left(1 - \alpha^2\right)\right]}}}$$

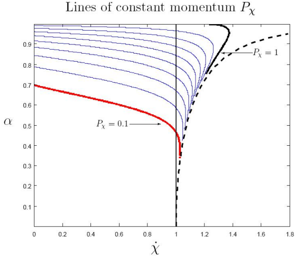
where $z_2(z_1) = 1 - \frac{(1-\alpha^2)}{(1-y^2)(1-z_1)}$.

Note on the Numerics:

The D4-brane energy $H(\alpha, \dot{\chi})$ and momentum $P_\chi(\alpha, \dot{\chi})$ become singular along the curve

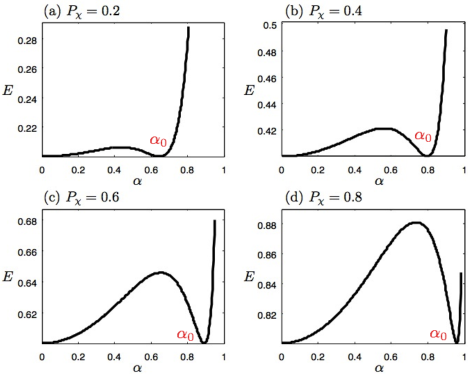
$$\dot{\chi}^4 = \frac{1}{(1-\alpha^2)}$$

Decreasing α from the maximal size $\alpha = 1$, the lines of constant momentum P_χ approach this curve in $(\dot{\chi}, \alpha)$-space. At small α, the numerics therefore become problematic.



Energy Plots:

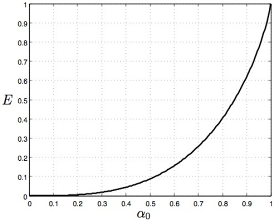
The energy of the four-brane, plotted as a function of the size α_0 at fixed momentum P_χ, in units of the flux N:



The finite $\alpha = \alpha_0$ degenerate minimum in the energy occurs when $\dot{\chi} = 1$ and the four-brane energy is

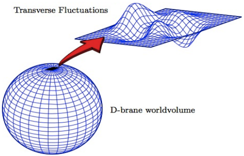
$$H = P_\chi = N \left\{ \alpha_0 + \frac{1}{2} \left(1 - \alpha_0^2\right) \ln \left(\frac{1 - \alpha_0}{1 + \alpha_0} \right) \right\}$$

indicating a BPS configuration - this is the $\mathbb{CP}^3$ giant graviton.



Outline

Let us consider small fluctuations about the worldvolume of the four-brane giant graviton:



transverse or scalar fluctuations:

$$v_k(\sigma^a) = \varepsilon\, \delta v_k(\sigma^a), \qquad \alpha(\sigma^a) = \alpha_0 + \varepsilon\, \delta\alpha(\sigma^a), \qquad \chi(\sigma^a) = t + \varepsilon\, \delta\chi(\sigma^a)$$

longitudinal or worldvolume fluctuations:

$$F(\sigma^a) = \varepsilon\, \frac{R^2}{2\pi}\, \delta F(\sigma^a),$$

A suitable choice of worldvolume coordinates was a problem!

In the fluctuation analysis, we made use of

$$\sigma^a = (t, x_1, x_2, \varphi_1, \varphi_2)$$

with $x_i(\alpha, z_i)$ any generic radial worldvolume coordinates, with ranges independent of α_0.

The equations of motion for the small fluctuations are

$$(\Box\ \delta v_k) + h^{tt}\ \delta v_k = 0$$

$$(\Box\ \delta\alpha) + g^{\alpha\alpha}_{\rm rad}\ \ \partial_a \left(\frac{1}{g^{\alpha\alpha}_{\rm rad}}\right) h^{ab}\,(\partial_b\delta\alpha) - \frac{g^{\alpha\alpha}_{\rm rad}}{\sqrt{-h}}\ \ \partial_i\left(\sqrt{-h}\ \ \frac{g^{\alpha i}_{\rm rad}}{g^{\alpha\alpha}_{\rm rad}}\ \ h^{tb}\right)(\partial_b\delta\chi) = 0$$

$$(\Box\ \delta\chi) + \left(g^{\chi\chi}_{\rm ang} - 1\right)\ \partial_a\left(\frac{1}{g^{\chi\chi}_{\rm ang} - 1}\right) h^{ab}\,(\partial_b\delta\chi) + \frac{\left(g^{\chi\chi}_{\rm ang} - 1\right)}{\sqrt{-h}}\ \ \partial_i\left(\sqrt{-h}\ \ \frac{g^{\alpha i}_{\rm rad}}{g^{\alpha\alpha}_{\rm rad}}\ \ h^{tb}\right)(\partial_b\delta\alpha) = 0.$$

with h_{ab} the worldvolume metric.

The $\mathbb{CP}^3$ fluctuations $\delta\alpha$ and $\delta\chi$ are clearly coupled. It is not immediately obvious, without making a specific choice for the radial worldvolume coordinates x_1 and x_2, how to define new $\mathbb{CP}^3$ fluctuations $\delta\beta_\pm$, in terms of a linear combination of $\delta\alpha$ and $\delta\chi$, such that the equations of motion for $\delta\beta_+$ and $\delta\beta_-$ decouple.

However, once these equations of motion have been decoupled, the obvious ansätze

$$\delta v_k(t, x_1, x_2, \varphi_1, \varphi_2) = e^{i\omega_k t}\, e^{im_k \varphi_1}\, e^{in_k \varphi_2}\, f_k(x_1, x_2)$$
$$\delta \beta_\pm(t, x_1, x_2, \varphi_1, \varphi_2) = e^{i\omega_\pm t}\, e^{im_\pm \varphi_1}\, e^{in_\pm \varphi_2}\, f_\pm(x_1, x_2)$$

should reduce these problems to second order decoupled partial differential equations for $f_k(x_1, x_2)$ and $f_\pm(x_1, x_2)$. We are interested in solving for the spectrum of eigenfrequencies ω_k and $\omega_\pm$ in terms of the two pairs of integers m_k and n_k, and $m_\pm$ and $n_\pm$ respectively.

Radial worldvolume coordinates:

The radial worldvolume shall now be described using two sets of nested polar coordinates (r_1, θ) and (r_2, ϕ):

$$y = r_1(\alpha, \theta) \cos\theta, \qquad z_1 = r_2^2(\alpha, \theta, \phi) \cos^2\phi, \qquad z_2 = r_2^2(\alpha, \theta, \phi) \sin^2\phi,$$

with the polar radii r_1 and r_2 the positive roots of

$$r_1^2(\alpha, \theta) = \frac{2}{\sin^2(2\theta)} \left\{ 1 - \sqrt{1 - \alpha^2 \sin^2(2\theta)} \right\}$$

$$r_2^2(\alpha, \theta, \phi) = \frac{2}{\sin^2(2\phi)} \left\{ 1 - \sqrt{1 - r_1^2(\alpha, \theta) \sin^2\theta \sin^2(2\phi)} \right\},$$

where we observe that $\alpha = \alpha_0$ describes the radial worldvolume of the submaximal giant graviton. Here the radial worldvolume coordinates $x_1 \equiv \theta \in [0, \pi]$ and $x_2 \equiv \phi \in [0, \frac{\pi}{2}]$ have fixed ranges.

Small giant graviton $\alpha_0 \ll 1$

We can expand the square roots in r_1 and r_2 in orders of α. The first term in the expansion gives $r_1(\theta) \approx \alpha$ and $r_2(\theta, \phi) \approx \alpha \sin\theta$.

Our radial coordinates then become

$$\begin{aligned} y &\approx \alpha \cos\theta \\ z_1 &\approx \alpha^2 \sin^2\theta \cos^2\phi \\ z_2 &\approx \alpha^2 \sin^2\theta \sin^2\phi \end{aligned}$$

in the vicinity of the $\alpha = \alpha_0$ surface. This approximate radial projection of the giant is a 2-sphere in $(y, \sqrt{z_1}, \sqrt{z_2})$-space.

Leading order analysis

* The leading-order equations of motion can easily be decoupled.
* Analytic solutions can be obtained in terms of hypergeometric and Heun functions.
* The spectrum is independence of the size α_0 to leading order.
* However, this leading order spectrum is different from that of the maximal giant graviton (dual to two dibaryons).

[JM & AP: 1103.1163 [hep-th]]

Next-to-leading order analysis

* The next-to-leading-order equations of motion also decouple.
* The next-to-leading order equations of motion are dependent on the size α_0 and hence the shape of the giant graviton.
* These equations of motion did *not*, however, admit any obvious analytic solution. We were not able to verify this dependence at the level of the spectrum.

Outline

Take home message

Membranes in the ABJM duality have non-trivial geometries!

It appears that traces of this non-trivial geometry are visible in the fluctuation spectrum.

Future Research

We would now like to understand how this non-trivial geometry is encoded in the dual ABJM model. In particular, we would like to

* Find a complete, orthogonal basis for operators in the holomorphic sector of the ABJM model and study the action of the dilatation operator to leading order in $\frac{1}{N}$.
  [de Mello Koch, JM & AP: work in progress]
* Note that this semiclassical $\frac{1}{N}$ limit includes contributions from non-planar diagrams (and must therefore be seen as distinct from the usual t'Hooft limit) and describes membrane interactions from the point of view of the dual ABJM model.