Super amplitudes

A review

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Outline

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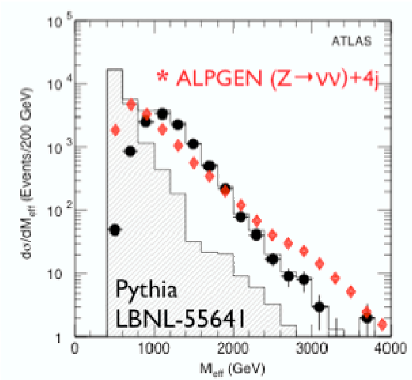
Why compute amplitudes?

- Collider physics
- Structure of Yang-Mills theory; integrability of $\mathcal{N} = 4$ SYM
- Gravity; divergences of supergravity

QCD background at LHC



SUSY missing energy signal, Monte Carlo background, Exact $Z + 4$ jet background [Gianotti, Mangano]

Conjecture of Iterative Structure in Planar $\mathcal{N} = 4$ SYM

[Catani; Sterman, Tejeda-Yeomans; Anastasiou, Bern, Dixon, Kosower; Bern, Dixon, Smirnov]

BDS Ansatz

$$\left(\frac{A_n^{(L)}}{A_n^{\text{tree}}}\right) = \sum_{l=0}^{L-1} \text{Div}_n^{(L-l)} \left(\frac{A_n^{(l)}}{A_n^{\text{tree}}}\right) + F_n^{(l)}$$

Formula based on soft & collinear limits.

Finite discrepancies begin at 6 gluons, 2-loop order.

What is the correct version?

Is $\mathcal{N} = 8$ supergravity finite?
Is gravity a double copy of gauge theory?

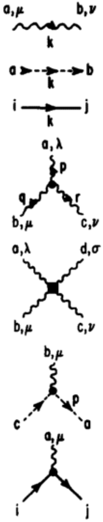
Possible: UV behavior is the same for $\mathcal{N} = 8$ supergravity and $\mathcal{N} = 4$ SYM, so finite in 4 dimensions.
e.g. power counting of the 2-loop 4-point amplitude is the same.

$N = 4$ SYM and $N = 8$ supergravity:

$$D_c = 4 + \frac{6}{L} \qquad L = 2, 3, 4$$

True through 4 loops. 5 loops underway.

Feynman Rules



$$-i\delta^{ab}\left[\left(g_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)/k^2 + a k_\mu k_\nu/k^4\right]$$

$$-i\delta^{ab}/k^2$$

$$i\delta^{ij}\not{k}/k^2$$

$$-gf^{abc}\left[(p-q)_\nu g_{\lambda\mu} + (q-r)_\lambda g_{\mu\nu} + (r-p)_\mu g_{\nu\lambda}\right]$$

$$-ig^2 f^{abe} f^{cde}(g_{\lambda\nu} g_{\mu\sigma} - g_{\lambda\sigma} g_{\mu\nu})$$
$$-ig^2 f^{ace} f^{bde}(g_{\lambda\mu} g_{\nu\sigma} - g_{\lambda\sigma} g_{\mu\nu})$$
$$-ig^2 f^{ade} f^{cbe}(g_{\lambda\nu} g_{\mu\sigma} - g_{\lambda\mu} g_{\sigma\nu})$$

$$gf^{abc}p^\mu$$

$$-ig\gamma^\mu T^a_{ij}$$

Feynman diagrams don't respect symmetries

Feynman diagrams are both numerous and tedious.

In the end, one finds many cancellations ending with a gauge-invariant result.

Symmetry offers clues for improvement. Amplitudes have remarkable structure!

Understanding the simplicity of gauge theory amplitudes leads to new, efficient techniques for calculation.

Conversely: the challenge of difficult calculations triggers discovery of shortcuts.

Why compute amplitudes?

- Collider physics
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- Gravity; divergences of supergravity

Why compute amplitudes?

- Collider physics
- Structure of Yang-Mills theory; integrability of $\mathcal{N} = 4$ SYM
- Gravity; divergences of supergravity
- String theory & gauge/gravity duality
- Why are amplitudes so simple?
- New perspectives on quantum field theory!

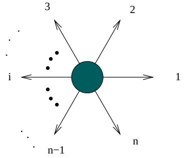
Outline

Color decomposition

[Berends, Giele; Mangano, Parke, Xu]

$$\mathcal{A}_n^{\rm tree} = g^{n-2} \sum_{\sigma \in S_n/Z_n} {\rm Tr}(T^{a_{\sigma(1)}} \dots T^{a_{\sigma(n)}}) A(\sigma(p_1^\mu, \epsilon_1^\mu), \dots, \sigma(p_n^\mu, \epsilon_n^\mu))$$

"color ordered partial amplitudes"



Color decomposition

[Berends, Giele; Mangano, Parke, Xu]

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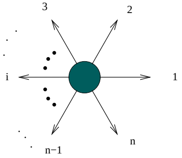
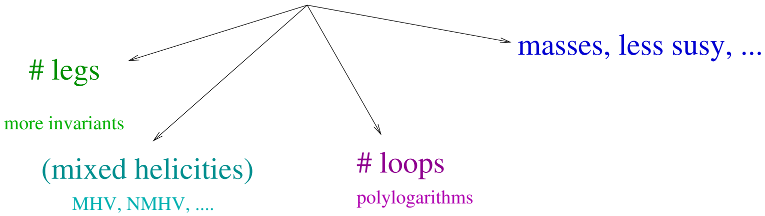
"color ordered partial amplitudes"



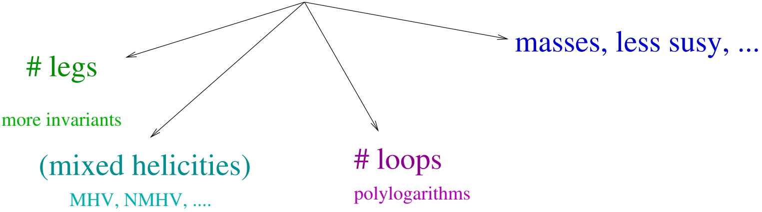
Cyclic ordering; color-ordered (planar) Feynman rules.
Convention: momenta are outgoing; $0 \to n$

$$\delta^{(4)}(\sum p_i)$$

Complexity of an amplitude



Complexity of an amplitude

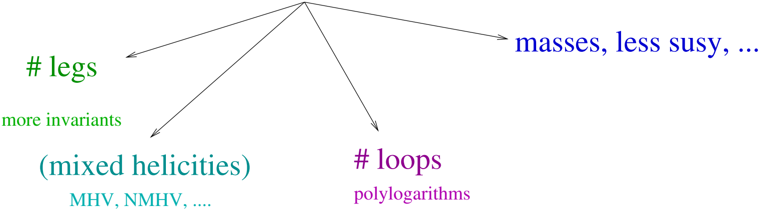


Maximally Helicity Violating (MHV) Amplitudes:

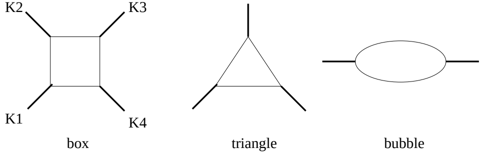
$$A(1^+,\ldots,i^-,\ldots,j^-,\ldots,n^+) = \frac{\langle i\ j\rangle^4 \quad \delta^{(4)}(\sum p_i)}{\langle 1\ 2\rangle\langle 2\ 3\rangle\ldots\langle n-1\ n\rangle\langle n\ 1\rangle}$$

[Parke, Taylor; Berends, Giele; Nair]

Complexity of an amplitude



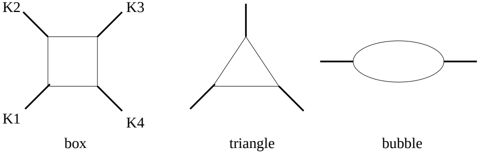
Master integrals



$A^{1-\text{loop}} = \sum_i c_i I_i,$ c_i are rational functions.

Analytically known at 1-loop, some special cases beyond.

Master integrals

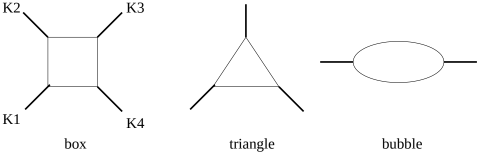


$A^{1-\mathrm{loop}} = \sum_i c_i I_i,$ $\quad$ c_i are rational functions.

Analytically known at 1-loop, some special cases beyond.
e.g.: If $K_3^2 = K_4^2 = 0$,

$$
\begin{aligned}
I_4^{2m\ h} \quad = \quad & \frac{2r_\Gamma}{st}\,\frac{1}{\epsilon^2}\left[\frac{1}{2}(-s)^{-\epsilon} + (-t)^{-\epsilon} - \frac{1}{2}(-K_1^2)^{-\epsilon} - \frac{1}{2}(-K_2^2)^{-\epsilon}\right] \\
& -\frac{2r_\Gamma}{st}\left[-\frac{1}{2}\ln\left(\frac{s}{K_1^2}\right)\ln\left(\frac{s}{K_2^2}\right) + \frac{1}{2}\ln^2\left(\frac{s}{t}\right)\right. \\
& \left. + \mathrm{Li}_2\left(1 - \frac{K_1^2}{t}\right) + \mathrm{Li}_2\left(1 - \frac{K_2^2}{t}\right)\right] + \mathcal{O}(\epsilon).
\end{aligned}
$$

Master integrals



$A^{1-\mathrm{loop}} = \sum_i c_i I_i,$ c_i are rational functions.

Cancellations for SYM multiplets in the loop:

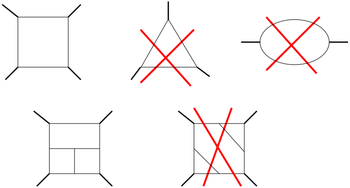
$\mathcal{N}$	Box	Triangle	Bubble	Rational
4	$\surd$			
1	$\surd$	$\surd$	$\surd$	
0	$\surd$	$\surd$	$\surd$	$\surd$

The no-triangle property for maximal supergravity

[Bern, Dixon, Perelstein, Rozowsky; Bern, Bjerrum-Bohr, Dunbar, Ita, Perkins, Risager; Bjerrum-Bohr, Vanhove]

Cancellations within the supermultiplet loop for $\mathcal{N}=4$ SYM and $\mathcal{N}=8$ supergravity eliminate triangle and bubble subgraphs.

Thus: softer divergences.



Supergravity cancellations are even stronger than those implied by no-triangle.

Gravity amplitudes: KLT relations

- Simple, despite infinity of vertices and no color ordering
- Kawai, Lewellen, Tye (KLT) expressed closed string amplitudes as sums of products of open string amplitudes (left and right moving modes).
- The field theory limit relates graviton and gauge field amplitudes.

  [Berends, Giele, Kuijf; Bern, Dixon, Perelstein, Rozowsky]
- Nicer versions found via recursion relations

  [Bern, Carrasco, Forde, Ita, Johansson; Elvang, Freedman]

$$A^{\rm grav}(1^-,2^-,3^+,\ldots,n^+) = \sum_{\mathcal{P}(i_3,\cdots,i_n)} s_{1i_n}\left(\prod_{s=4}^{n-1}\beta_s\right) A^{\rm YM}(1^-,2^-,i_3^+,\ldots,i_n^+)^2$$

$$\beta_s \equiv -\frac{\langle i_s, i_{s+1}\rangle}{\langle 2, i_{s+1}\rangle}\langle 2|i_3+\cdots+i_{s-1}|i_s].$$

Outline

Clever coordinates

- Spinors and superfields $p = \lambda\tilde{\lambda};\ \eta$

$$p_{a\dot{a}} = \sigma^{\mu}_{a\dot{a}}\ p_{\mu} \qquad\qquad a, \dot{a} = 1, 2$$

Massless case:

$$0 = p^2 = \det(p_{a\dot{a}}) \implies p_{a\dot{a}} = \lambda_a \tilde{\lambda}_{\dot{a}}.$$

Lorentz-invariant spinor products:

$$\langle \lambda\ \lambda' \rangle \equiv \epsilon_{ab}\ \lambda^a\ \lambda'^b$$

$$[\tilde{\lambda}\ \tilde{\lambda}'] \equiv \epsilon_{\dot{a}\dot{b}}\ \tilde{\lambda}^{\dot{a}}\ \tilde{\lambda}'^{\dot{b}}$$

$$A(1^+, \ldots, i^-, \ldots, j^-, \ldots, n^+) = \frac{\langle i\ j \rangle^4 \quad \delta^{(4)}(\sum p_i)}{\langle 1\ 2 \rangle \langle 2\ 3 \rangle \ldots \langle n-1\ n \rangle \langle n\ 1 \rangle}$$

Clever coordinates

- Spinors and superfields $p = \lambda\tilde{\lambda};\ \eta$

$$A_n^{\mathrm{MHV}} = \frac{\delta^{(4)}\left(\sum_i \lambda_i^a \tilde{\lambda}_i^{\dot{a}}\right) \delta^{(8)}\left(\sum_i \lambda_i^a \eta_i^A\right)}{\langle 1\ 2\rangle\langle 2\ 3\rangle \ldots \langle n-1\ n\rangle\langle n\ 1\rangle}$$

Superamplitude has components corresponding to the on-shell superfield

$$\Phi = G^+ + \eta^A \Gamma_A + \frac{1}{2!}\eta^A\eta^B S_{AB} + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\overline{\Gamma}^D + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^- .$$

Clever coordinates

- Spinors and superfields $p = \lambda\tilde{\lambda};\ \eta$

  Spinors allow us to write polarization vectors too:

$$\epsilon^{-}_{a\dot{a}} = \frac{\lambda_a \tilde{\mu}_{\dot{a}}}{[\tilde{\lambda}, \tilde{\mu}]}, \qquad \epsilon^{+}_{a\dot{a}} = \frac{\tilde{\lambda}_a \mu_{\dot{a}}}{[\lambda, \mu]}.$$

  Gravity:

$$\epsilon^{-}_{a\dot{a},b\dot{b}} = \epsilon^{-}_{a\dot{a}}\epsilon^{-}_{b\dot{b}}, \qquad \epsilon^{+}_{a\dot{a},b\dot{b}} = \epsilon^{+}_{a\dot{a}}\epsilon^{+}_{b\dot{b}}$$

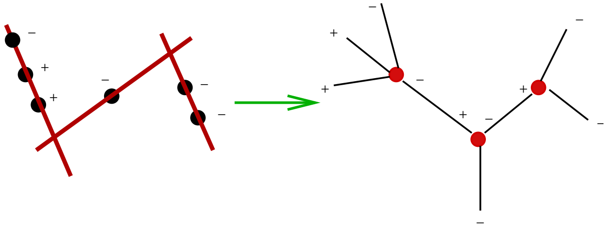
  Therefore we specify helicity states.

  [Berends, Chang, Kleiss, De Causmaecker, Gastmans, Gunion, Kunszt, Stirling, Troost, Wu, Xu, Zhang]

Clever coordinates

- Spinors and superfields $p = \lambda\tilde{\lambda};\ \eta$
- Twistors $\lambda, \mu;$ $\mu + y \cdot \lambda = 0$

$$\mu_{\dot{a}} \equiv \partial/\partial\tilde{\lambda}^{\dot{a}}$$



Points and (null) lines are exchanged.

[Penrose; Nair; Witten; Hodges]

Clever coordinates

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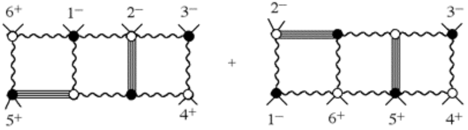
Hodges diagrams



Conformal generators are 1st order

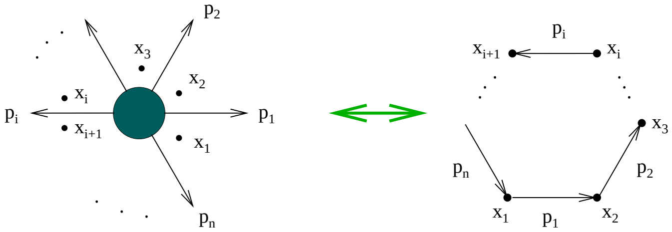
$$p_{a\dot{a}} = i\lambda_a \frac{\partial}{\partial\tilde{\mu}^{\dot{a}}} \qquad j_{ab} = \frac{i}{2}\left(\lambda_a \frac{\partial}{\partial\lambda^b} + \lambda_b \frac{\partial}{\partial\lambda^a}\right)$$

$$k_{a\dot{a}} = i\mu_{\dot{a}} \frac{\partial}{\partial\lambda^a} \qquad \tilde{j}_{ab} = \frac{i}{2}\left(\mu_{\dot{a}} \frac{\partial}{\partial\mu^{\dot{b}}} + \mu_{\dot{b}} \frac{\partial}{\partial\mu^{\dot{a}}}\right)$$

$$d = \frac{i}{2}\left(\lambda_a \frac{\partial}{\partial\lambda^a} - \mu_{\dot{a}} \frac{\partial}{\partial\mu^{\dot{a}}}\right)$$

Clever coordinates

- Spinors and superfields $p = \lambda\tilde{\lambda};\ \eta$
- Twistors $\lambda, \mu; \qquad \mu + y \cdot \lambda = 0$
- Dual coordinates / region momenta $p_i = x_i - x_{i+1}$



Momentum conservation $\Leftrightarrow$ closed null-edged polygon

Clever coordinates

- Spinors and superfields $p = \lambda\tilde{\lambda};\ \eta$
- Twistors $\lambda, \mu; \qquad \mu + y \cdot \lambda = 0$
- Dual coordinates / region momenta $p_i = x_i - x_{i+1}$
- Dual chiral superspace $\{\lambda_i, x_i, \theta_i\}$ [Drummond, Henn, Korchemsky, Sokatchev]

$$A_n^{\rm NMHV} = A_n^{\rm MHV} \mathcal{P}_n^{\rm NMHV}, \qquad \mathcal{P}_n^{\rm NMHV} = \sum_{r,s} R_{n,rs},$$

$$R_{t,rs} = \frac{\langle r, r-1\rangle\langle s, s-1\rangle \delta^{(4)}(\langle t|x_{tr}x_{rs}|\theta_{st}\rangle + \langle t|x_{ts}x_{sr}|\theta_{rt}\rangle)}{x_{rs}^2 \langle t|x_{tr}x_{rs}|s\rangle\langle t|x_{tr}x_{rs}|s-1\rangle\langle t|x_{ts}x_{sr}|r\rangle\langle t|x_{ts}x_{sr}|r-1\rangle}.$$

$$x_{ij} \equiv x_i - x_j, \qquad \theta_{ij} \equiv \theta_i - \theta_j$$

Closed form given for all tree-level superamplitudes in $\mathcal{N} = 4$ SYM. [Drummond, Henn]

Clever coordinates

- Spinors and superfields $p = \lambda\tilde{\lambda};\ \eta$
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- Momentum twistors $\mu + x \cdot \lambda = 0$

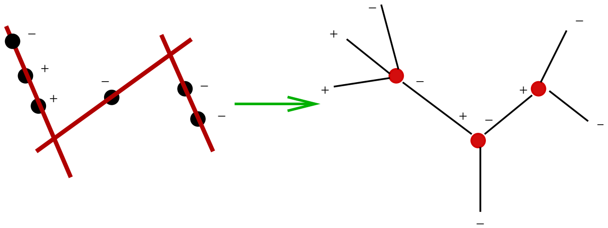
$$R_{t,rs} = \frac{\delta^{0|4}\left(\chi_t \epsilon(r-1, r, s-1, s) + 4 \text{ cyclic permutations}\right)}{\epsilon(t, r-1, r, s-1)\epsilon(r-1, r, s-1, s)\epsilon(r, s-1, s, t)\epsilon(s-1, s, t, r-1)\epsilon(s, t, r-1, r)}.$$

Implement momentum conservation and on-shell conditions simultaneously. [Hodges]

On-shell techniques

- CSW/MHV rules

[Cachazo, Svrček, Witten]



MHV amplitudes are vertices of new diagrams.

Motivated by localization of MHV amplitudes on lines in twistor space = points in Minkowski space.

On-shell techniques

- CSW/MHV rules
- BCFW recursion relations [RB, Cachazo, Feng, Witten]

Residue theorem on a momentum-shifted, on-shell amplitude:

$$p_i(z) = p_i + zq; \qquad p_j(z) = p_j - zq.$$

If $\lim_{z\to\infty} A(z) = 0$, then

$$0 = A(0) + \sum_{\text{poles } z_\pi \text{ of } A(z)} \mathrm{Res}\left(\frac{A(z)}{z}\right)_{z=z_\pi}.$$

$$A(0) = \sum_\pi A_L(z_\pi)\frac{i}{P_R^2 - M^2} A_R(z_\pi).$$

Compact expressions.

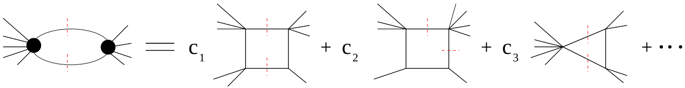
Stronger boundary behavior → "bonus" relations.

[Arkani-Hamed, Kaplan; Spradlin, Volovich; Elvang, Freedman; Badger, Henn; Feng, Huang, Jia]

On-shell techniques

- CSW/MHV rules
- BCFW recursion relations
- Unitarity cuts for loop amplitudes

[Bern, Dixon, Dunbar, Kosower]



$$\Delta A^{1-\mathrm{loop}} = \sum_i c_i \Delta I_i$$

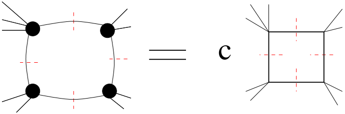
1-loop amplitudes are cut-constructible.

Supersymmetric amplitudes are 4d cut-constructible.

On-shell techniques

- CSW/MHV rules
- BCFW recursion relations
- Unitarity cuts for loop amplitudes
- Generalized unitarity cuts with complex momenta

  [RB, Cachazo, Feng]



Solves 4d $\mathcal{N} = 4$ SYM and $\mathcal{N} = 8$ supergravity at 1-loop, given tree amplitudes and master integrals.

Similar "maximal" cuts for more loops, but less clean.

On-shell techniques

- CSW/MHV rules
- BCFW recursion relations
- Unitarity cuts for loop amplitudes
- Generalized unitarity cuts with complex momenta
- OPP algorithm

  [Ossola, Papadopoulos, Pittau]

Numerical 1-loop amplitudes

Classification of integrands; coefficients from cuts.

Complex Analysis

- Residue theorem [BCFW recursion relations]
- Branch cuts [Unitarity method]

Complex Analysis

- Residue theorem [BCFW recursion relations]
- Branch cuts [Unitarity method]
- Multi-dimensional counterpart:

Global Residue Theorem

[known in math literature]

Calculates leading singularities (maximal cuts) of multiloop integrals.

[Arkani-Hamed, Cachazo, Cheung, Kaplan; Kosower, Larsen]

Symmetries

- Supersymmetry as a tool
  - Tree-level QCD is effectively supersymmetric; can use on-shell superfields
  - Supersymmetry decomposition for QCD loop amplitudes

Symmetries

- Supersymmetry as a tool
- Dual (super)conformal symmetry in $\mathcal{N} = 4$ SYM

  [Drummond, Henn, Korchemsky, Sokatchev; Berkovits, Maldacena; Brandhuber, Heslop, Travaglini]

  - Motivated by duality: conformal invariance of Wilson loops.
  - Combine superconformal and dual superconformal symmetries into Yangian $Y(PSU(2,2|4))$
  - Regulator of IR divergences breaks both symmetries. But the anomaly may be understood.
  - Dual-conformal-invariant cross ratios = building blocks for amplitudes & integrands $\frac{x_{ij}^2 x_{kl}^2}{x_{ik}^2 x_{jl}^2}$

Integrands before integrals

Focus on integrands and integration separately.

- Integrands are uniquely defined in planar theories (region momenta)
- Constructible from recursion relations
- Yangian invariant
- Derive amplitude/Wilson loop duality, extended beyond MHV

[Arkani-Hamed, Bourjaily, Cachazo, Caron-Huot, Trnka; Boels; Adamo, Bullimore, Mason, Skinner]

Hope to improve integration independently, perhaps by motivic algebra (symbols).

Grassmannians as contours

[Arkani-Hamed, Cachazo, Cheung, Kaplan]

Geometry of momentum conservation → Grassmannian manifolds: space of k-planes through the origin of $\mathbb{C}^n$.

$$A_n^{\mathrm{N^kMHV}} = A_n^{\mathrm{MHV}} \mathcal{P}_n^{\mathrm{N^kMHV}}$$

$$\mathcal{P}_n^{\mathrm{N^kMHV}} = \frac{1}{(2\pi i)^{k(n-k)}} \oint_{\Gamma \subset G(k,n)} \prod_{r=1}^{k} \delta^{4|4}(T^r \cdot Z) \times \frac{D^{k(n-k)}T}{(12\cdots k)(23\cdots k+1)\cdots(n1\cdots k-1)}.$$

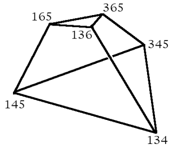
Tree amplitudes and leading singularities of all multi-loop amplitudes. Manifestly and generally Yangian.

Related: a local form, from momentum-twistor space

[Arkani-Hamed, Bourjaily, Cachazo, Trnka]

Amplitudes as polytopes

Compact formulas are elements of a triangulation. [Hodges]



$$\frac{[4|5+6|1\rangle^3}{[34][23]\langle 56\rangle\langle 61\rangle[2|3+4|5\rangle s_{234}} + \frac{[6|1+2|3\rangle^3}{[61][12]\langle 34\rangle\langle 45\rangle[2|3+4|5\rangle s_{612}} =$$

$$\frac{s_{123}^3}{[12][23]\langle 45\rangle\langle 56\rangle[1|2+3|4\rangle[3|4+5|6\rangle} + \frac{\langle 12\rangle^3[45]^3}{\langle 16\rangle[34][3|4+5|6\rangle[5|6+1|2\rangle S_{612}}$$

$$+\frac{\langle 23\rangle^3[56]^3}{\langle 34\rangle[16][1|2+3|4\rangle[5|6+1|2\rangle S_{234}}$$

Higher-loop integrands are polytopes in projective spaces; dual to Wilson loops

[Arkani-Hamed, Bourjaily, Cachazo, Hodges, Trnka]

To the integral: symbols for polylogarithms

- Discrepancy from BDS Ansatz discovered in general from Wilson loops

  [Alday, Maldacena; Brandhuber, Heslop, Travaglini]
- First discrepancy at $R_6^{(2)}$; expression written down

  [Bern, Dixon, Kosower, Roiban, Spradlin, Vergu, Volovich; Del Duca, Duhr, Smirnov]
- $R_6^{(2)}$ simplified dramatically with an ansatz suggested by "symbols" [Goncharov, Spradlin, Vergu, Volovich]

$$\log R \to R$$
$$\log R_1 \log R_2 \to R_1 \otimes R_2 + R_2 \otimes R_1$$
$$\mathrm{Li}_2 R \to -(1-R) \otimes R$$

$$\cdots \otimes (R_a R_b) \otimes \cdots = \cdots \otimes R_a \otimes \cdots + \cdots \otimes R_b \otimes \cdots$$
$$\cdots \otimes (cR) \otimes \cdots = \cdots \otimes R \otimes \cdots$$

To the integral: symbols for polylogarithms

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  [Bern, Dixon, Kosower, Roiban, Spradlin, Vergu, Volovich; Del Duca, Duhr, Smirnov]

- $R_6^{(2)}$ simplified dramatically with an ansatz suggested by "symbols" [Goncharov, Spradlin, Vergu, Volovich]
- Find a direct route! Ideally a shortcut to integration.
- Recent application: 2 loop MHV, n gluons, in multi-Regge kinematics [Prygarin, Spradlin, Vergu, Volovich]

Relations between color and kinematics

- BCJ relations found in SYM while computing gravity

  [Bern, Carrasco, Johansson]

$$A_m = g^{m-2} \sum_j \frac{c_j n_j}{\prod_{\alpha_j} p^2_{\alpha_j}}$$

Kinematic numerators n_j satisfy same Jacobi identities as color factors c_j
but only in a particular gauge!

Relations between color and kinematics

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  [Bern, Carrasco, Johansson]

$$A_m = g^{m-2} \sum_j \frac{c_j n_j}{\prod_{\alpha_j} p_{\alpha_j}^2}$$

- Amplitude relations proved from open string worldsheet, and BCFW construction

  [Bjerrum-Bohr, Damgaard, Vanhove; Stieberger; Feng, Huang, Jia; Chen, Du, Feng]

- Leads to new YM and gravity Lagrangians

  [Bern, Dennen, Huang, Kiermaier]

- Duality might be yet deeper and more explicit

  [Bern, Dennen; Bern, Carrasco, Dixon, Johansson, Roiban]

- Hence there is a Lie algebra for the kinematics. MHV, self-dual sector studied [Monteiro, O'Connell]

Outline

General theories

- Validity of on-shell constructions in general theories

  [Cohen, Elvang, Kiermaier; Boucher-Veronneau, Larkoski; Benincasa, Conde]

- Progress towards 2-loop integral and cut classification

  [Gluza, Kazda, Kosower; Mastrolia, Ossola; Kosower, Larsen]

- Unitarity extended to special massive 1-loop integrals

  [RB, Mirabella]

- Spinors in 6 or 10 dimensions; superamplitudes in even dimensions

  [Cheung, O'Connell; Boels; Caron-Huot, O'Connell; Boels, O'Connell]

Maximally supersymmetric SYM

- Integrands inherit simplifications of tree-level permutation sums [Boels, Isermann; Du, Feng, Fu]
- 4-point integrands through 7 loops, constrained by divergences and symmetry [Bourjaily, DiRe, Shaikh, Spradlin, Volovich]
- Complete 5-point integrands at 1 and 2 loops, with color-kinematics duality manifest [Carrasco, Johansson]
- 3-loop, 6-point remainder function in the Regge limit; all-loop dispersion relations [Bartels, Lipatov, Prygarin]
- Dual conformal symmetry of 6- and 10-d tree amplitudes (dual only) [Bern, Carrasco, Dennen, Huang, Ita; Caron-Huot, O'Connell]
- Form factors

  [Brandhuber, Gurdogan, Mooney, Spence, Travaglini; Bork, Kazakov, Vartanov; Gehrmann, Henn, Huber]

$\mathcal{N} < 4$ or massive SYM

On-shell superfield formalism is extended and allows for efficient recursion

- Compact tree amplitudes in $\mathcal{N} < 4$, and the origin of bubble integrals
- Massive amplitudes on the Coulomb branch of $\mathcal{N} = 4$ SYM
- Dual conformal generators constructed
- Also examples in (S)QCD, Abelian Higgs model, massive vector boson currents

[Elvang, Huang, Peng; Craig, Elvang, Kiermaier, Slatyer; Boels, Schwinn; Elvang, Freedman, Kiermaier; Boels, Isermann]

Supergravity

- SUSY and $E_{7(7)}$ constraints on counterterms for $\mathcal{N} = 8$ supergravity rule out divergences below 7 loops

  [Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger]

- $\mathrm{Grav} = \mathrm{YM}^2$ in 1-loop, 4- or 5-pt amplitudes in $\mathcal{N} \geq 4$ supergravity

  [Bern, Boucher-Veronneau, Johansson]

- $\mathcal{N} = 7$ recursion relation simplifies tree amplitudes; momentum twistors play a role

  [Hodges]

Triality in $\mathcal{N} = 4$ SYM

- Planar MHV amplitudes dual to Wilson loops on contour of null momenta. Wilson loops are easier to calculate.

  [Alday, Maldacena; Brandhuber, Heslop, Travaglini]

- Correlation function of 1/2 BPS operators in the limit of lightlike separation, with pole singularities removed, and regularized, is the Wilson loop squared.

  [Alday, Eden, Korchemsky, Maldacena, Sokatchev; Adamo, Bullimore, Mason, Skinner]

- Last link: lightlike separation limit of correlation functions gives amplitudes.
  Loop corrections by Lagrangian insertions, then "dual infrared" dimensional regularization.

  [Eden, Korchemsky, Sokatchev]

Triality in $\mathcal{N} = 4$ SYM

- amplitudes $\leftrightarrow$ Wilson loops fails beyond MHV;
  amplitudes $\leftrightarrow$ correlators is ok, through 6 points.

  [Belitsky, Korchemsky, Sokatchev; Eden, Heslop, Korchemsky, Sokatchev]

- Detailed correspondence of integrands, through 4 loops for 4 points

- Twistor space is the natural setting for the Wilson loops

- Duality from Wilson loops $\rightarrow$ descent equations relating amplitudes of different orders

  [Caron-Huot, He; Bullimore, Skinner]

ABJM amplitudes

Amplitudes of $\mathcal{N} = 6$ Chern-Simons matter theory [Aharony, Bergman, Jafferis, Maldacena]

exhibit an $OSp(6|4)$ Yangian symmetry, [Bargheer, Loebbert, Meneghelli; Gang, Huang, Koh, Lee, Lipstein]

and a corresponding Grassmannian integral description. [Lee]

Two-loop confirmation of amplitude/Wilson loop duality; one-loop evidence for correlator/Wilson loop duality.

[Bianchi, Leoni, Mauri, Penati, Ratti, Santambrogio]

Two-loop correction = one-loop correction to $\mathcal{N} = 4$ SYM; conjectured to all orders in dim. reg.

[Chen, Huang; Bianchi, Leoni, Mauri, Penati]

Witten diagrams

- BCFW recursion relations in flat space have the physics of a hard particle blasting through a soft background at a *point*

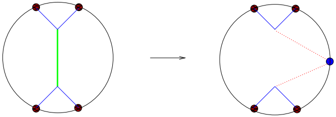
  [Arkani-Hamed, Kaplan]

- Generalize to AdS: integrate over interaction points

  [Raju]

- Derive recursion relations in terms of 3-point "transition amplitudes"

  [Raju]



- Diagrammatic recursion for CFT correlators

  [Fitzpatrick, Kaplan, Penedones, Raju, Van Rees]

String amplitudes

- Momentum kernel shown to factorize closed into open string amplitudes, give minimal basis in YM

  [Bjerrum-Bohr, Damgaard, Sondergaard, Vanhove]

- String scattering in flat space: in high-energy limit, similar behavior as in AdS: minimal surfaces, gluing analog of KLT relations

  [Caputa, Hirano]

- Pure spinor technology $\rightarrow$ N-point string disk amplitudes in compact forms

  [Mafra, Schlotterer, Stieberger]

- Three-point amplitudes with massive legs

  [Boels]

Outline

Outlook

- Precision calculations for LHC, at NLO and looking towards NNLO
- $\mathcal{N} = 4$ SYM: all-order results, direct path to remainder function; Regge limit and form factors
- $\mathcal{N} = 8$ supergravity: divergent behavior does/does not match SYM at 5 loops; cleaner formulations
- CFT correlation functions from AdS amplitudes
- Understanding color/kinematics interplay
- Results in theories with mass, less supersymmetry

Themes

- Tough, interesting computations $\longrightarrow$ new insights
- String theory and twistor space keep coming back
- New insights $\longrightarrow$ new results are possible
- Glimpses of a new understanding of quantum field theory