Beyond the Unitarity Bound in AdS / $CFT_{(A)dS}$

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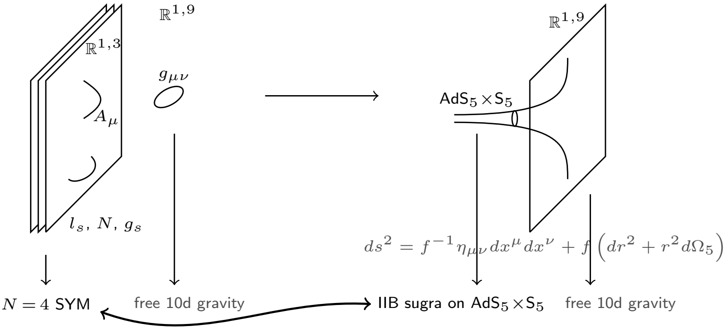
Iberian Strings, Bilbao 2012

[JHEP01(2012)123 Tomás Andrade, CFU]

Introduction

AdS/CFT prototype:



▷ many more brane configurations, gauge/gravity etc.

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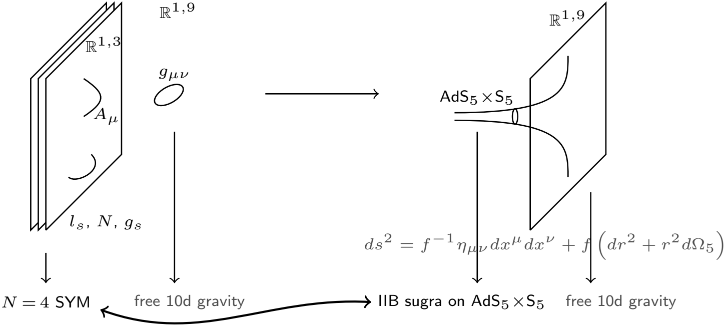

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Boundary metric (A)dS:

[Marolf, Rangamani, van Raamsdonk '11]
[Aharony, Marolf, Rangamani '11]

▷ curved-space QFT on boundary

▷ branes ending on branes [Aharony, Berdichevsky, Berkooz, Shamir '11]

▷ multi-layered AdS/CFT

$$\int \mathcal{D}\hat{\phi}\Big|_{\hat{\phi}|_{\partial\mathcal{M}}=\delta\phi} e^{iS_{\mathsf{sugra}}[\hat{\phi}]} \quad = \quad \left\langle e^{i\int_{\partial\mathcal{M}}\delta\phi\,\mathcal{O}}\right\rangle_{\mathsf{SCFT}}$$

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unitarity bound $\Delta_{\mathcal{O}} \geq d/2 - 1 \longleftrightarrow$ Neumann b.c. only for $\nu^2 \leq 1$ ✓

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- ▷ 2-point function sick on Poincaré AdS

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Here: set up holographic renormalization for CFTs on $\mathrm{(A)dS}$
study unitarity holographically

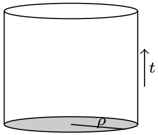
Outline:

- ▷ CFT on AdS from the holographic perspective:
  - the geometry
  - regularization and renormalization
  - beyond the unitarity bound
  - multi-layered AdS/CFT
- ▷ CFT on dS
  - geometry
  - beyond the unitarity bound

AdS/CFT$_{\text{AdS}}$ – the geometry

AdS$_{d+1}$ in global coordinates:

$$ds^2 = -\left(1+\rho^2\right)dt^2 + \frac{d\rho^2}{1+\rho^2} + \rho^2 d\Omega_{d-1}^2$$



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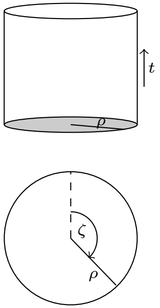

$(\rho,\zeta) \rightarrow (R,z)$:

$$\rho^2 = \csc^2 z \cosh^2 R - 1$$

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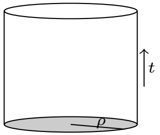

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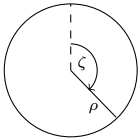
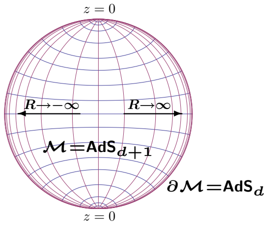


$$\to ds^2 = dR^2 + \cosh^2 R\, ds^2_{\mathsf{AdS}_d}$$

$$ds^2_{\mathsf{AdS}_d} = \frac{1}{\sin^2 z}\left(dz^2 - dt^2 + \cos^2 z\, d\Omega_{d-2}^2\right)$$

conformal boundary at $|R| = \infty$ is $2\times$AdS$_d$



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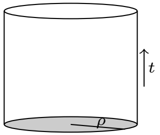

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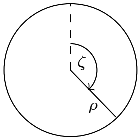

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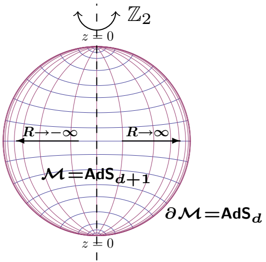
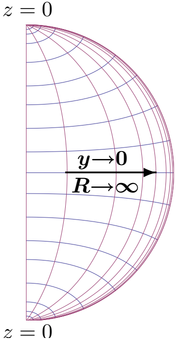
$\to \mathbb{Z}_2$ orbifold for single copy



Divergences and Renormalization

Klein-Gordon field ϕ on $\mathsf{AdS}_{d+1}/\mathbb{Z}_2$, $y := e^{-R}$, $m_\phi^2 = -d^2/4 + \nu^2$:



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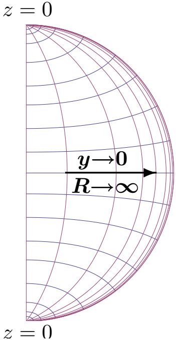

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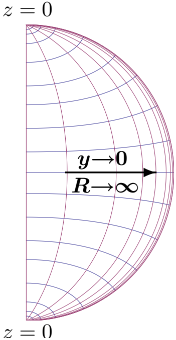
$$\delta S = \mathsf{EOM} + \int\limits_{y=1} \delta\phi\partial_y\phi + \int\limits_{y=0} \delta\phi\partial_y\phi + \int\limits_{z=0} \delta\phi\partial_z\phi$$



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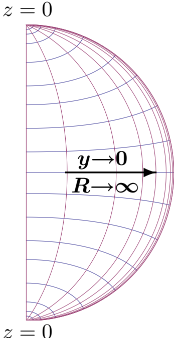

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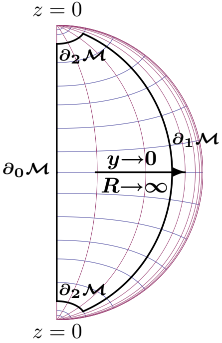

$\delta S \supset$ divergences from $y/z \to 0$: regularize $y \geq \epsilon_1$, $z \geq \epsilon_2$

$\rightarrow \partial\mathcal{M}$: $\mathbb{Z}_2$-fixed surface $\partial_0\mathcal{M}$, $y = \epsilon_1/z = \epsilon_2$ surfaces $\partial_{1/2}\mathcal{M}$

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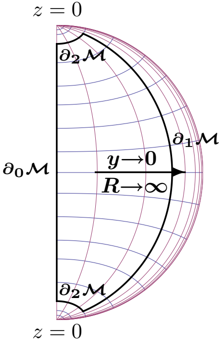

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Simplest case: Dirichlet_d $\chi_{\mathrm{D}} = \delta\chi_{\mathrm{D}} = 0$ s.t. no $\epsilon_2 \to 0$ divergences

$\to$ correlation functions for $\mathrm{CFT}_{\mathrm{AdS}}$ w/ Dirichlet b.c.

Dirichlet$_d$ and the unitarity bound

Choose $\nu \in (1,2)$ and fixed Dirichlet$_d$. For finite δS:

$$S_{\text{ct}} = -\frac{1}{2}\int_{\partial_1\mathcal{M}}\left[(d/2-\nu)\phi^2 + \frac{1}{2(\nu-1)}\phi\,\Box^W\phi\right] + \frac{1}{4(\nu-1)}\int_{\partial\partial\mathcal{M}}\phi\mathcal{L}_n\phi$$

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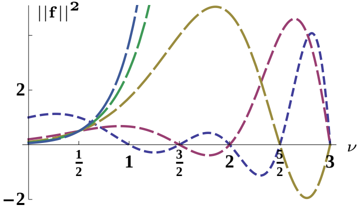
Spectrum: $\mu = 2n + \frac{1}{2} - a_{\text{D/N}}\nu + b_{\text{even/odd}}$, $||f||^2 = \frac{(n+b/2)!}{\mu\,\Gamma(2\mu-2n-b)}$

- ✓ $||f||^2 > 0$ for Dirichlet$_{d+1}$
- ✓ ghosts for $\nu > 1$ Neumann$_{d+1}$
- ▷ results extend to generic ν
- ▷ log-modes for $2\nu \in \mathbb{Z}$



Saturating the unitarity bound: $\nu = 1$

$\rightarrow$ $\log$ terms in asymptotic expansion

$\rightarrow$ new finite combination of boundary terms $\propto \kappa$

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in summary: satisfactory renormalization procedure for Dirichlet$_d$

CFT unitarity violation reflected in bulk by ghost states

Neumann$_d$: dimensional reduction

$\phi = \sum_n \alpha_n \phi_n$, $\quad \phi_n = \chi_{\mu_n} \cdot f_n,\ n \in \mathbb{N}$ $\quad \sim$ 'tower' of AdS$_d$ modes

$$\langle \phi_n, \phi_m \rangle = \underbrace{\langle \chi_{\mu_n}, \chi_{\mu_m} \rangle}_{\substack{\text{finite for} \\ \text{Dirichlet}_d}} \cdot \underbrace{\langle f_n, f_m \rangle}_{\to\ \text{finite } \langle f_n, f_m \rangle_{\text{ren}}\ \text{w/ c.t. on } \partial_1 \mathcal{M}}$$

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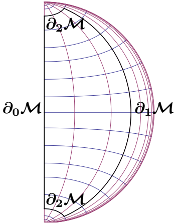
Neumann$_d$: χ_{μ_n} normalizable only for $\mu_n^2 < 1$ $(z \to 0$ divergences)

add c.t. on $\partial_2 \mathcal{M}$ s.t. all $\langle \chi_{\mu_n}, \chi_{\mu_m} \rangle$ finite?



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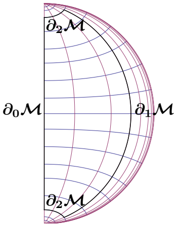
$$S_{\text{ct}} = \int_{\partial_2 \mathcal{M}} \alpha_1 \phi^2 + \alpha_2 \phi \Box \phi + \alpha_3 \mathcal{R} \phi^2 + \ldots$$

need $\alpha_i = \alpha_i(\mu)$, e.g. $\alpha_2 = 1/(1 - \mu_n)$

$\to$ w/ fixed α_i only one $\mu_n^2 > 1$ normalizable



$\phi = \sum_n \alpha_n \phi_n$, $\phi_n = \chi_{\mu_n} \cdot f_n,\ n \in \mathbb{N}$ ~ 'tower' of AdS$_d$ modes

$$\langle \phi_n, \phi_m \rangle = \underbrace{\langle \chi_{\mu_n}, \chi_{\mu_m} \rangle}_{\text{finite for Dirichlet}_d} \cdot \underbrace{\langle f_n, f_m \rangle}_{} \rightarrow \text{finite } \langle f_n, f_m \rangle_{\text{ren}} \text{ w/ c.t. on } \partial_1 \mathcal{M}$$

Neumann$_d$: χ_{μ_n} normalizable only for $\mu_n^2 < 1$ $(z \to 0$ divergences)

add c.t. on $\partial_2 \mathcal{M}$ s.t. all $\langle \chi_{\mu_n}, \chi_{\mu_m} \rangle$ finite?

$$S_{\text{ct}} = \int_{\partial_2 \mathcal{M}} \alpha_1 \phi^2 + \alpha_2 \phi \Box \phi + \alpha_3 \mathcal{R} \phi^2 + \ldots$$

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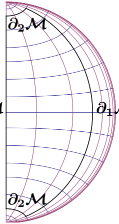

reduction of (infinite) tower of AdS$_d$ modes to finitely many

~ dimensional reduction, no localization in radial direction

Including $\mu^2 \geq 1$ modes: $\langle \chi_\mu, \chi_\mu \rangle$ indefinite $\rightarrow$ ghosts [Andrade, Marolf '11]

$\rightarrow$ add no c.t. on $\partial_2 \mathcal{M}$ s.t. only $\mu^2 < 1$ normalizable

$$\mu^2 < 1 \iff -\tfrac{3}{2} < 2n + b_{\text{even/odd}} - a_{\mathrm{D/N}} \nu < \tfrac{1}{2}$$

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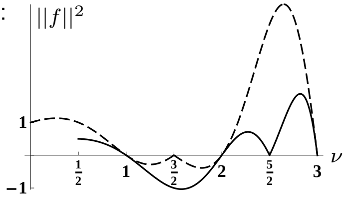
norm positive ✓

Neumann$_{d+1}$:



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$||f||^2 > 0$ for $\nu < 1$ ✓

and $\nu \in (2, 3)$?

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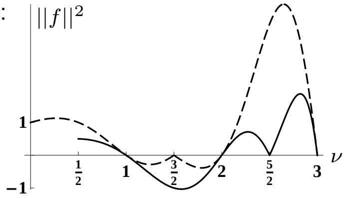

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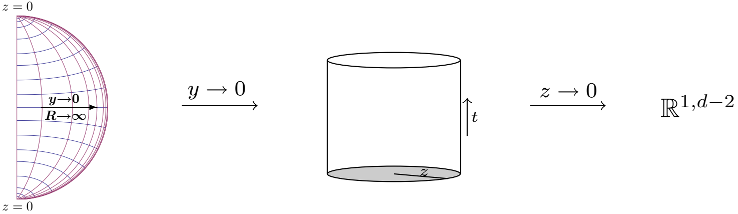
and $\nu \in (2, 3)$?

- ▷ ghost-free Neumann theory for $\nu > 1$
- ▷ truncation breaks conformal invariance $\rightarrow$ no unitarity bound ✓

Prospects for multi-layered AdS/CFT?

(Parental advisory: speculative!)



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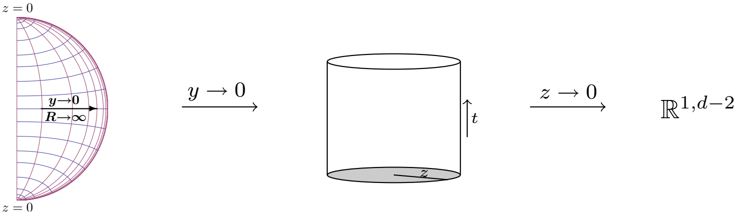

gravity$_{d+1}$

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[Compère, Marolf '08]

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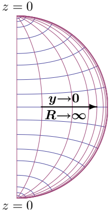
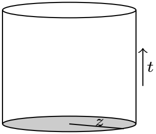


$\xrightarrow{y \to 0}$



$\xrightarrow{z \to 0}$ $\mathbb{R}^{1,d-2}$

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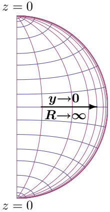

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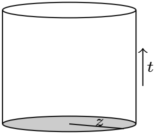

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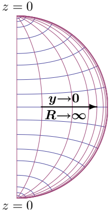

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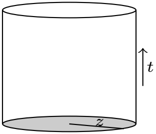

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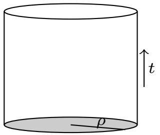
from scalar: expect dim. reduction of gravity$_{d+1}$ w/ Neumann$_d$
$\rightarrow$ multiple iterations trivialized

unitarity in $\mathrm{CFT}_{\mathrm{dS}}$ holographically



AdS_{d+1} in global coordinates:

$$ds^2 = -\left(1+\rho^2\right)dt^2 + \frac{d\rho^2}{1+\rho^2} + \rho^2 d\Omega_{d-1}^2$$

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CFT_{dS} and the unitarity bound

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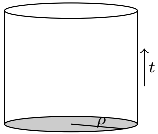

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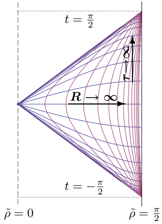

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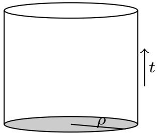

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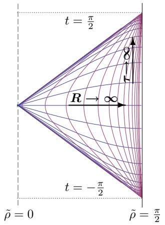

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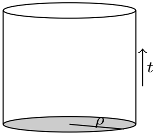

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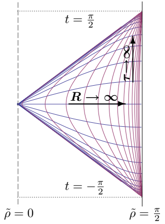

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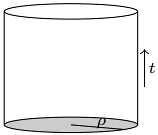

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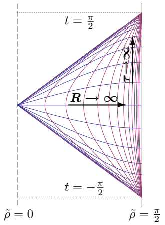

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- ▷ CFT unitarity violation reflected by ghosts immediately
  $\to$ Poincaré subtleties can not be blamed on horizon

Conclusion

Unitarity CFT_{AdS}

- Dirichlet$_d$: boundary non-unitarity reflected in bulk by ghost states
- Neumann$_d$:
  - reduced spectrum of normalizable solutions, ghost-free for certain $\nu > 1$
  - conformal invariance broken $\rightarrow$ unitarity bound does not apply
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Thanks for your attention!