SupergravitiES in $d = 11$

Nucl. Phys. B **855** (2012) 308-319

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Iberian Strings 2012
Euskal Herriko Unibertsitatea
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The Uniqueness of Supergravity in $d = 11$

- SUGRA in $d = 11$ is unique!!
  S. Deser, *The Uniqueness of $D = 11$ Supergravity*, `hep-th/9712064`
- But there are exceptions!
  It is necessary "to read the fine print"!!

The Uniqueness of Supergravity in $d = 11$

- Underlying the uniqueness, there are some hypotheses which are very natural but not unbreakable.
- A simple way of breaking these hypotheses is through a **Chern–Simons Theory**.

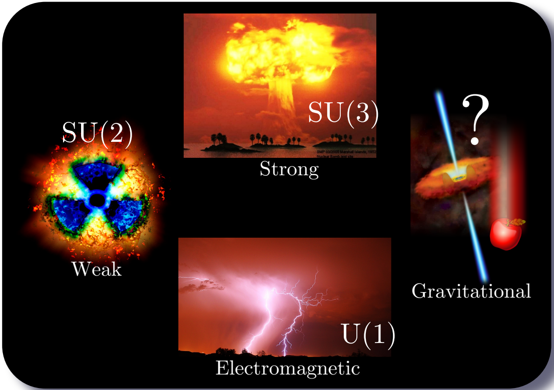
Contents

Chern–Simons SUGRA

Chern–Simons theories in different odd dimensions
started in the 80's-90's with the work of

A. Achúcarro,
M. Bañados,
A. Borowiec,
A. H. Chamseddine,
M. Ferraris,
M. Francaviglia
M. Henneaux
P. K. Townsend,
R. Troncoso,
J. Zanelli,
etc. . .

Gauge Theories X-treme



Gauge theories **do** work.

What about a Supergravity theory where *every* field, *all* of them, belong only to a 1-form *connection*?

Gauge Theories X-treme

$$\boldsymbol{A} = \frac{1}{2}\omega^{ab}{}_{\mu}\mathrm{d}x^{\mu}\boldsymbol{J}_{ab} + \frac{1}{l}e^{a}{}_{\mu}\mathrm{d}x^{\mu}\boldsymbol{P}_{a} + \frac{1}{\sqrt{l}}\bar{\psi}_{\mu}\mathrm{d}x^{\mu}\boldsymbol{Q} + \text{extra bosonic terms.}$$

A really daring idea!

Gauge Theories X-treme

And the Lagrangian??

Gauge Theories X-treme

$$\langle \boldsymbol{F} \wedge *\boldsymbol{F} \rangle$$

Gauge Theories X-treme

$$\langle \boldsymbol{F} \wedge * \boldsymbol{F} \rangle \oslash$$

Hodge dual $* \Rightarrow$ a metric field $g_{\mu\nu}(x) \notin \boldsymbol{A}$!!

Gauge Theories X-treme

:-(

And then?

Gauge Theories X-treme

Chern–Simons Lagrangian Form

$$\mathcal{L}_{\text{CS}}^{(2n+1)}(\boldsymbol{A}) = \kappa(n+1)\int_{\tau=0}^{\tau=1} \mathrm{d}\tau \left\langle \boldsymbol{A} \wedge \left(\tau \mathrm{d}\boldsymbol{A} + \tau^2 \boldsymbol{A} \wedge \boldsymbol{A}\right)^{\wedge n} \right\rangle$$

$\left\langle \boldsymbol{T}_{A_1} \cdots \boldsymbol{T}_{A_{n+1}} \right\rangle =$ (Super)Algebra Invariant Tensor
(Close cousin of the superalgebra killing metric)

Gauge Theories X-treme

Advantages of CS Theories

1. The Lagrangian has a deep mathematical meaning (Chern–Weil)!
2. Genuine **gauge** theories, in fiber bundle sense.
3. Off-shell invariant (without auxiliary fields).
4. They *seem* to be renormalizable (only proven in $d = 3$).
5. In general, no boson-fermion matching.
6. It does include **gravity** (Lanczos–Lovelock)!
7. No ghosts.
8. No higher spin fields.
9. Propagating degrees of freedom in $d > 3$.
10. Background-free.

Gauge Theories X-treme

Disadvantages of CS Theories

1. It works only in **odd** dimensions!
2. Each individual term is not explicitly invariant.
3. The number of terms grows exponentially with d.
4. **Highly non-linear** in $d > 3$!
5. Irregular constraint system (complex dynamics).
6. Quantum behavior is **unknown** in $d > 3$.

Gauge Theories X-treme

An odd number. . . ?

Gauge Theories X-treme

$$11$$

Contents

$\mathfrak{osp}\,(32|1)$-Chern–Simons SUGRA in $d = 11$

Chern–Simons Lagrangian in $d = 11$

$$\mathcal{L}_{\mathrm{CS}}^{(11)}(\boldsymbol{A}) = 6\kappa \int_{\tau=0}^{\tau=1} \mathrm{d}\tau \left\langle \boldsymbol{A} \wedge \left(\tau \mathrm{d}\boldsymbol{A} + \tau^2 \boldsymbol{A} \wedge \boldsymbol{A}\right)^{\wedge 5} \right\rangle$$

$$\langle \boldsymbol{T}_{A_1} \cdots \boldsymbol{T}_{A_6} \rangle = \text{Superalgebra Invariant Tensor}$$

$\mathfrak{osp}\,(32|1)$-Chern–Simons SUGRA in $d = 11$

$$A = \frac{1}{2}\omega^{ab}\boldsymbol{J}_{ab} + \frac{1}{l}e^{a}\boldsymbol{P}_{a} + \frac{1}{5!l}b^{a_1\cdots a_5}\boldsymbol{Z}_{a_1\cdots a_5} + \frac{1}{\sqrt{l}}\bar{\psi}\boldsymbol{Q}$$

$\mathfrak{osp}\,(32|1)$-connection 1-form.

R. Troncoso, J. Zanelli, Phys. Rev. D **58** (1998) 101703, `hep-th/9710180`

SUGRAs in $d = 11$

But $d = 11$ is already inhabited!

SUGRAs in $d = 11$

Standard SUGRA, E. Cremmer, B. Julia and J. Scherk (1978),

$$\mathcal{L}^{(11)}_{\mathrm{CJS}} = -\frac{1}{9!}\frac{1}{4\kappa^2}\epsilon_{a_1\cdots a_{11}}R^{a_1a_2}\wedge e^{a_3}\wedge\cdots\wedge e^{a_{11}} + \frac{i}{2}\bar{\psi}\wedge\Gamma_{(8)}\wedge\mathrm{D}\psi +$$
$$+\frac{i}{8}\left(T^a - \frac{i\kappa^2}{4}\bar{\psi}\wedge\Gamma^a\psi\right)\wedge e_a\wedge\bar{\psi}\wedge\Gamma_{(6)}\wedge\psi - \frac{1}{2}F\wedge * F +$$
$$+(*F + b)\wedge(\mathrm{d}A_3 - a) + \frac{1}{2}a\wedge b - \frac{1}{3}A_3\wedge\mathrm{d}A_3\wedge\mathrm{d}A_3,$$

the "only" theory of SUGRA in $d = 11$ (under several hypotheses).

B. Julia, S. Silva, JHEP **0001** (2000) 026, `hep-th/9911035`

SUGRAs in $d = 11$

$$\Gamma_{(n)} = \frac{1}{n!}\Gamma_{a_1 \cdots a_n} e^{a_1} \wedge \cdots \wedge e^{a_n}$$

$$a = \frac{i\kappa}{4}\bar{\psi} \wedge \Gamma_{(2)} \wedge \psi,$$

$$b = \frac{i\kappa}{4}\bar{\psi} \wedge \Gamma_{(5)} \wedge \psi.$$

SUGRAs in $d = 11$

What to do with two SUGRA theories in $d = 11$?

SUGRAs in $d = 11$

Could there be a link between both SUGRA theories?

Surprisingly hard to answer!

SUGRAs in $d = 11$

In particular, could it be possible
to consider standard SUGRA as a sector of a Chern–Simons system?
(A priori, **it seems to be no reason to think so!**)

WARNING!!!

- CS SUGRA is *very* complicated! Lagrangian with more than a 1000, non-explicitly invariant terms!
- Jumping to compare both of them would be hazardous for mental health!

DANGER!!

WARNING!!!

- The way of finding the “needle in the haystack” is (carefully) using
  - Cartan’s Homotopy Formula[1],
  - arguments of dimensional consistency and
  - numerical algoritms[2].

[1] hep-th/0603061

[2] 1106.1648 [math-ph]

The Needle in the Haystack

CS SUGRA and powers of l

$$\mathcal{L}_{\mathrm{CS}}^{(11)} = \sum_{n=0}^{11} l^{-n} \mathcal{L}_n$$

Just for the sake of simplicity, a very restricted sector will be explored:

- Order l^{-9},
- Only $\mathcal{L}_{\mathrm{CS}}^{(11)}\Big|_{b^{a_1 \cdots a_5}=0}$ and $\mathcal{L}_{\mathrm{CJS}}^{(11)}\Big|_{A_3=0}$ will be compared!
- Only the most simple invariant tensor will be used!

SUGRAs in $d = 11$

Restricted sector of *osp* $(32|1)$-CS Lagrangian:

$$\mathcal{L}_{\text{CS}}^{(11)}\Big|_{\substack{l^{-9} \\ b=0}} = \frac{\lambda}{l^9}\Bigg[-\frac{1}{4\times 9!\kappa_2^2}\epsilon_{a_1\cdots a_{11}}R^{a_1a_2}\wedge e^{a_3}\wedge\cdots\wedge e^{a_{11}}+ \\ -\frac{7i}{4}\bar{\psi}\wedge\Gamma_{(8)}\wedge \mathrm{D}_\omega\psi-\frac{1}{2}a\wedge b+ \\ +\frac{i}{8}\left(T^a-\frac{i\kappa_1^2}{4}\bar{\psi}\wedge\Gamma^a\psi\right)\wedge e_a\wedge\bar{\psi}\wedge\Gamma_{(6)}\wedge\psi+ \\ +\frac{7}{2^4\times 5}a\wedge * a-\frac{1}{2^{10}\times 3^2}\frac{(4!)^5}{6!}b\wedge * b\Bigg].$$

SUGRAs in $d = 11$

Standard SUGRA, E. Cremmer, B. Julia and J. Scherk (1978),

$$\mathcal{L}^{(11)}_{\text{CJS}} = -\frac{1}{9!}\frac{1}{4\kappa^2}\epsilon_{a_1\cdots a_{11}}R^{a_1a_2}\wedge e^{a_3}\wedge\cdots\wedge e^{a_{11}} + \frac{i}{2}\bar{\psi}\wedge\Gamma_{(8)}\wedge \mathrm{D}\psi +$$
$$+\frac{i}{8}\left(T^a - \frac{i\kappa^2}{4}\bar{\psi}\wedge\Gamma^a\psi\right)\wedge e_a\wedge\bar{\psi}\wedge\Gamma_{(6)}\wedge\psi - \frac{1}{2}F\wedge * F +$$
$$+(*F + b)\wedge(\mathrm{d}A_3 - a) + \frac{1}{2}a\wedge b - \frac{1}{3}A_3\wedge \mathrm{d}A_3\wedge \mathrm{d}A_3,$$

SUGRAs in $d = 11$

$$A_3 = ?$$

SUGRAs in $d = 11$

$$A_3 \sim \frac{1}{l^3}$$

SUGRAs in $d = 11$

$$
\begin{aligned}
A_3 = {} & C_{a_1 \cdots a_5 bc} b^{a_1 \cdots a_5} \wedge e^b \wedge e^c \\
& + C_{a_1 \cdots a_5 b_1 \cdots b_5 c} b^{a_1 \cdots a_5} \wedge b^{b_1 \cdots b_5} \wedge e^c + \\
& + C_{a_1 \cdots a_5 b_1 \cdots b_5 c_1 \cdots c_5} b^{a_1 \cdots a_5} \wedge b^{b_1 \cdots b_5} \wedge b^{c_1 \cdots c_5}
\end{aligned}
$$

- A_3 seems to be related to the Chern–Simons 3-form for $\mathrm{osp}(32|1)$!!

SUGRAs in $d = 11$

A deformed version of standard CJS SUGRA in $d = 11$
seems to be contained as a sector of the Chern–Simons Theory!

A priori, no reason to expect such “coincidence”!

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Further Work & Questions

- It is very likely that standard CJS SUGRA can be considered as an small sector of a Chern–Simons system, when a more general invariant tensor is considered.

Further Work & Questions

- What could this relationship between standard SUGRA and Chern–Simons theory in $d = 11$ possibly mean. . . ?

Further Work & Questions

- Could a Chern–Simons theory in $d = 11$ play a rôle in the net of dualities surrounding Strings / M Theory? [1, 2]
- If it does, to which kind of String theory it could be related?

[1] P. Horava, Phys. Rev. D **59** (1999) 046004, `hep-th/9712130`

[2] H. Nastase, `hep-th/0306269`

Thank You!

wxMaxima	Cadabra	Kile	Evince	Inkscape	Gimp
Computational Algebra	Computational Algebra for Quantum Field Theory	LaTeX Editor	PDF Viewer	Vector Graphics Editor	Image Editor

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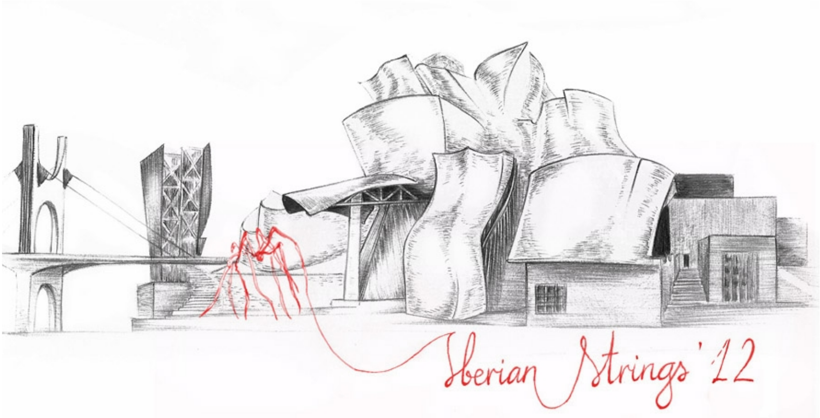
of mathematicians, physicists, programmers and artists from all around the world.

Thank You!

Thank You!



¡Muchas gracias! / Eskerrik asko!