The Kerr/CFT correspondence
A bird-eye view

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Geoffrey Compère
Universiteit van Amsterdam

Aim of the Kerr/CFT correspondence

The **Kerr/CFT correspondence** proposes that *(at least part of)* the dynamics of rotating black holes can be identified with the dynamics of *(a close cousin of)* a conformal field theory (CFT).

Rotating black holes $1+1$ **CFTs**

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• Metric $g_{\mu\nu}$, ...	$\Leftrightarrow$?	..., OPEs, Operators •
• Angular momentum J, ...	$\Leftrightarrow$?	T_L, T_R Temperatures •
• Symmetries $U(1)\times U(1)$	$\Leftrightarrow$?	Virasoro $\times$ Virasoro •
• Entropy	$\Leftrightarrow$?	Entropy •
$S_{BH}(M,J,\dots) = \frac{A}{4G} + \dots$		$S_{CFT} = \frac{\pi^2}{3}(c_R T_R + c_L T_L)$
• Dynamics of probes	$\Leftrightarrow$?	Correlators •
$\Box\phi = 0$, ...		..., $\langle O_1(0) O_2(x)\rangle$

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- $\Leftrightarrow ?$ Correlators

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List of open physics problems in quantum gravity :

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No !

This is an effective (bottom-up) model but one can build UV complete models in string theory.

(Current models use non-realistic string compactifications.)

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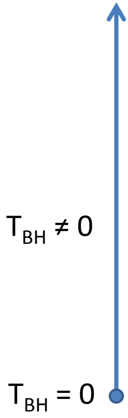
Derivation of Kerr/CFT in three steps



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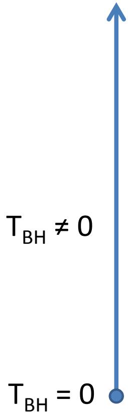

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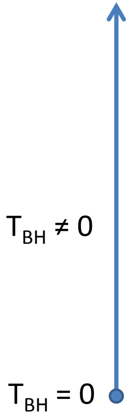

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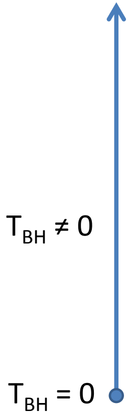

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Entropy still matches !

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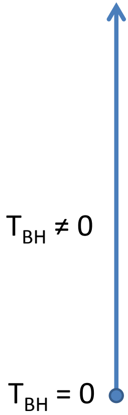

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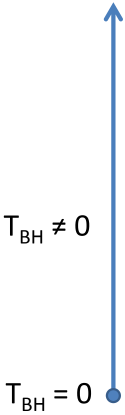

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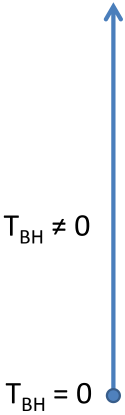
First step of the Kerr/CFT correspondence



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Extremal black holes are nearly physical

Theoretical bounds on the Kerr parameters :

$$\frac{J}{GM^2} \leq 1 \qquad \text{no naked singularity}$$

$$\frac{J}{GM^2} \leq 0.998 \qquad \text{[Thorne] from details of matter}$$

Observed :

$$\frac{J}{GM^2} > 0.98 \qquad \text{[GRS 1915+105 (2006)] } M \sim 10 - 15 M_\odot$$

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Two key statements

① The extremal Kerr black hole can be described by a field theory at the (dimensionless) temperatures

$$T_L = \frac{1}{2\pi}, \qquad T_R = 0$$

② The axial symmetry can be extended to a Virasoro asymptotic symmetry algebra close to the horizon with central charge

$$c = \frac{12J}{\hbar} \in 6\mathbb{N}$$

The black hole entropy can then be reproduced by a CFT counting

$$S_{BH} = \frac{A}{4G\hbar} = \frac{2\pi J}{\hbar} \stackrel{!}{=} \frac{\pi^2}{3}(c\,T_L + c\,T_R) = S_{CFT}$$

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The Kerr metric [Roy Kerr, 1963]

$$\begin{aligned} ds^2 &= -\frac{r^2 - 2GMr + a^2}{r^2 + a^2\cos^2\theta}(dt - a\sin^2\theta d\phi)^2 + (r^2 + a^2\cos^2\theta)d\theta^2 \\ &+\frac{r^2 + a^2\cos^2\theta}{r^2 - 2GMr + a^2}dr^2 + \frac{\sin^2\theta}{r^2 + a^2\cos^2\theta}((r^2 + a^2)d\phi - adt)^2 \end{aligned}$$

where $a = \frac{J}{M}$. Extremality is reached when $J = GM^2$ or $a = GM$. Then

$$T_H = 0, \qquad \Omega_H = \frac{1}{2GM}, \qquad S_{BH} = 2\pi\frac{J}{\hbar}, \qquad J \in \hbar\frac{\mathbb{N}}{2}$$

Take the near-horizon limit $\lambda \to 0$:

$$t^{near} = \lambda t, \qquad r^{near} = \frac{r - GM}{\lambda}, \qquad \phi^{near} = \phi - \Omega_H t$$

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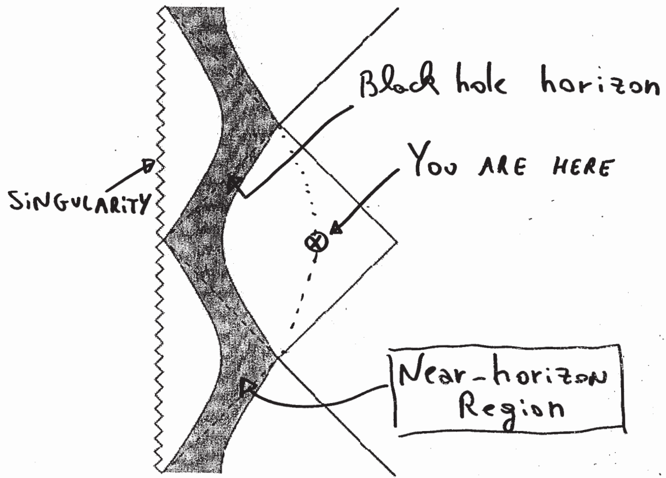
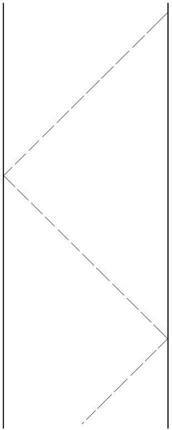
The extremal Kerr causal structure



Near-horizon region of extremal Kerr

[Bardeen-Horowitz, 1999]

$$ds^2 = J(1+\cos^2\theta)\left((-r^2dt^2 + \frac{dr^2}{r^2}) + d\theta^2 + \frac{4\sin^2\theta}{(1+\cos^2\theta)^2}(d\phi + rdt)^2\right)$$



- Decoupling between horizon and flat asymptotics
- This spacetime is geodesically complete.
- Enhancement to $SL(2,\mathbb{R}) \times U(1)$ symmetry [Kunduri, Lucietti, Reall]

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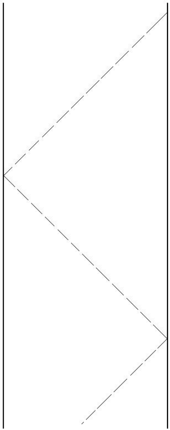

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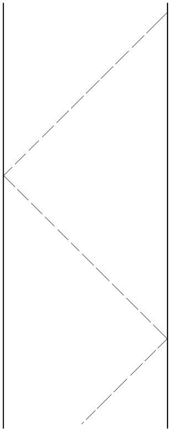

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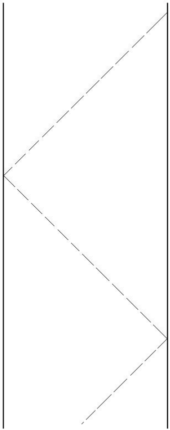

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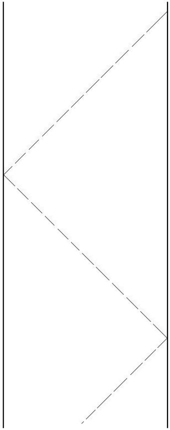

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- [Conjecture] Any solution asymptotic to this geometry is diffeomorphic to it.

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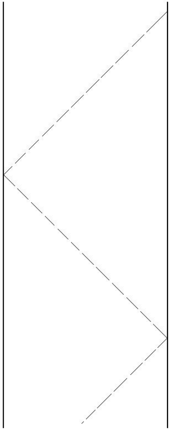

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Temperature of extremal black holes

$$T_H = 0, \qquad \text{but} \ldots$$

First law suggests the existence of a physical temperature

$$\begin{aligned} \delta S &= \frac{1}{T_H}(\delta M - \Omega_H \delta J) \\ &\overset{ext}{=} \frac{1}{T_H}((\frac{\partial M}{\partial J})_{ext} - \Omega_H)\delta J = \frac{1}{T_\phi}\delta J \end{aligned}$$

This suggests that excitations of extreme rotating black holes lie along

$$\frac{\partial}{\partial \phi}.$$

One can then argue that there is a $2d$ QFT description with

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One more statement to make

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Interlude : detour to 3d gravity !

(Main argument to microscopically derive black hole entropy in string theory !)

Remember : Gravity in AdS_3

Near-horizon geometry of many supersymmetric black holes is asymptotic to

$$ds^2 = l_{AdS}^2 \left(-(1+r^2)dt^2 + \frac{dr^2}{1+r^2} + r^2 d\phi^2 \right)$$

Semi-classical analysis valid for $l_{AdS} \gg \hbar G_3$.

- Conformal boundary is a cylinder (t, ϕ) : $x^{\pm} = t \pm \phi$ with conformal symmetries

$$\xi_L = L(x^+)\partial_+ - rL'(x^+)\partial_r \quad \text{and} \quad \xi_R = \bar{L}(x^-)\partial_- - r\bar{L}'(x^-)\partial_r$$

extending

$$SL(2,\mathbb{R})_L \times SL(2,\mathbb{R})_R$$

- Boundary conditions $g_{\mu\nu} = O(\dots)$ exist such that charges

$$L_n \leftrightarrow \xi_L(L(x^+) = e^{inx^+}), \qquad \bar{L}_n \leftrightarrow \xi_R(\bar{L}(x^-) = e^{inx^-}),$$

are finite, well-defined and conserved.

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$$ds^2 = l_{AdS}^2 \left(-(1+r^2)dt^2 + \frac{dr^2}{1+r^2} + r^2 d\phi^2 \right)$$

Semi-classical analysis valid for $l_{AdS} \gg \hbar G_3$.

- Conformal boundary is a cylinder (t, ϕ) : $x^\pm = t \pm \phi$ with conformal symmetries

  $$\xi_L = L(x^+)\partial_+ - rL'(x^+)\partial_r \quad \text{and} \quad \xi_R = \bar{L}(x^-)\partial_- - r\bar{L}'(x^-)\partial_r$$

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  $$SL(2,\mathbb{R})_L \times SL(2,\mathbb{R})_R$$

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⇒ Virasoro algebra with central charges

$$c_L = c_R \overset{Einstein}{=} \frac{3 l_{AdS}}{2G_3}$$

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$$S_{BH} \overset{!}{=} 2\pi\sqrt{\frac{c_L \mathcal{L}_0}{6}} + 2\pi\sqrt{\frac{c_R \bar{\mathcal{L}}_0}{6}} \overset{Cardy}{=} S_{CFT}$$

(One can use this reasoning to derive microscopically the entropy of a large class of supersymmetric black hole in string theory.)

End of Interlude : back to Kerr !

Near horizon geometry of extremal Kerr : Asymptotic symmetries

- Now, we have $AdS_2 \times_{warped} S_2$.
- Idea : conformal symmetries along ∂_ϕ !

$$\boxed{l_n = e^{-in\phi}\partial_\phi + inre^{-in\phi}\partial_r}$$

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$$g_{\mu\nu} \sim \bar{g}_{\mu\nu} + O(\dots), \qquad Q_{\partial_t} = 0$$

- Representation by covariant charges [Barnich, Brandt, 2001],

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$$l_n \to L_n[(g_{\mu\nu}, \dots); \mathcal{L}]$$

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and they admit a central term, with central charge

$$c_L \overset{Kerr}{=} \frac{12J}{\hbar}$$

The black hole entropy can then be reproduced by a CFT counting

$$S_{BH} = \frac{A}{4G\hbar} = \frac{2\pi J}{\hbar} \overset{!}{=} \frac{\pi^2}{3} c_L T_L = S_{CFT}$$

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Question : is this a numerical coincidence ?

Extensions of entropy matching [Many authors ...]

For any 4d black hole in Einstein gravity + matter fields, the near-horizon region is

$$ds^2 = \alpha(\theta)\left((-r^2dt^2 + \frac{dr^2}{r^2}) + d\theta^2 + \beta(\theta)(d\phi + k\, rdt)^2\right)$$

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and one can show

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Question : Does it depends on the field equations of Einstein gravity ?

What about other theories of gravity ?

Any diffeomorphic covariant Lagrangian

$$\mathbf{L}(f^*(\phi)) = f^*\mathbf{L}(\phi)$$

can be written as

$$\begin{aligned}\mathbf{L} \;=\; & d^Dx \; L(g_{ab}, R_{abcd}, \nabla_e R_{abcd}, \nabla_{(e}\nabla_{f)} R_{abcd}, \dots, \\ & \psi, \nabla_a \psi, \nabla_{(a}\nabla_{b)}\psi, \dots)\end{aligned}$$

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$$ds^2 = \alpha(\theta)\left((-r^2dt^2 + \frac{dr^2}{r^2}) + d\theta^2 + \beta(\theta)(d\phi + k\,rdt)^2\right)$$

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$$\frac{\delta L}{\delta R_{abcd}} \equiv \frac{\partial L}{\partial R_{abcd}} - \nabla_e \frac{\partial L}{\partial \nabla_e R_{abcd}} + \dots$$

Obtained from the black hole thermodynamics

$$T_{BH}\delta S_{BH} = \delta M - \Omega \delta J$$

[Wald 1993 ; Iyer, Wald 1994]

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The temperature can be derived from free quantum fields in the curved geometry $\Rightarrow T_L$ does not depend on gravitational field equations

$$T_L = \frac{1}{2\pi k}$$

Now, need some tricks

- Any *Diff*-invariant theory can be recasted in second order form using field redefinitions and auxiliary fields.
- The geometry has $SL(2,\mathbb{R}) \times U(1)$ and $t - \phi$ symmetry

Using asymptotic charge technology, we find

$$\boxed{c_L = -12k \int_S \frac{\delta L}{\delta R_{abcd}} \epsilon_{ab}\epsilon_{cd}\, vol(S) + 0}$$

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$$S_{BH} \stackrel{!}{=} \frac{\pi^2}{3} c_L T_L \boxed{+0} = S_{CFT}$$

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To be a CFT or not to be a CFT ?

- Universal matching

$$S_{BH} = \frac{\pi^2}{3} c_L T_L$$

  for all rotating extremal black holes.
- No $SL(2,\mathbb{R})_L \times SL(2,\mathbb{R})_R$ invariant ground state
- Only one Virasoro algebra appears in the asymptotic symmetry group. No other solutions than NHEK.

  $\Rightarrow$ Only one chiral sector.
- The central charge depends on the parameters of the black hole

Question : What is the nature of what looks like a chiral sector of a CFT ?

This is an open question.

There are several embeddings in string/M-theory.
[Song, Strominger ; G.C., Song, Virmani ; Guica, El-Showk ; . . .]

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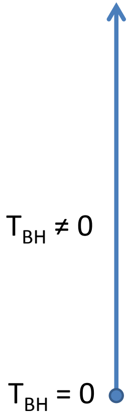
Second step of the Kerr/CFT correspondence



❷ Close to extremality :
Dynamics of probes can be obtained from a CFT two-point correlation function.

❶ At extremality :
Temperatures T_L, T_R, Virasoro algebra,
Match $S_{BH} = S_{CFT}$!

Scattering “close to extremality”

A near-extremal rotating black hole is defined from the condition

$$M\, T_H \ll 1, \qquad \Leftrightarrow \qquad \tau_H \equiv \frac{r_+ - r_-}{r_+} \ll 1.$$

We will look at the scattering of a probe field $\Box\Phi = 0$,

$$\Phi = e^{-i\omega t} e^{im\phi} \Theta(\theta) R(r)$$

close to the superradiant bound

$$\omega \sim m\Omega_H + q_e \Phi_e$$

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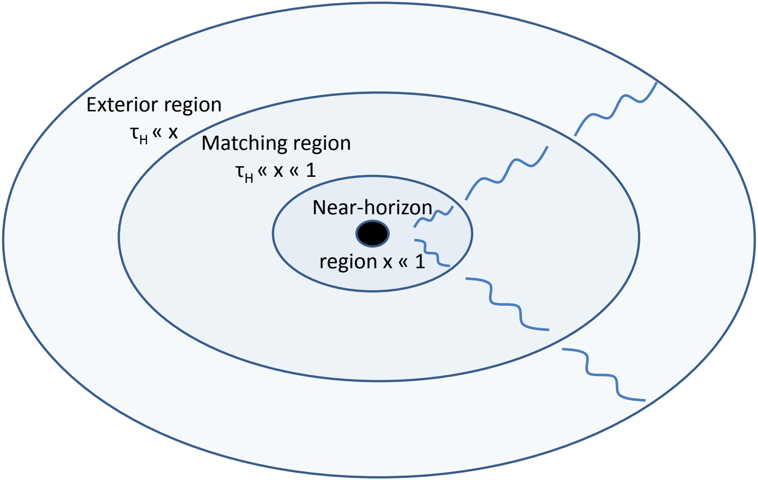

$$x \equiv \frac{r - r_+}{r_+}, \qquad \tau_H = \frac{r_+ - r_-}{r_+} \ll 1$$

Superradiant scattering

Define the absorption probability

$$\sigma_{abs}(\omega, m, l; M, J) \equiv \frac{dE_{abs}/dt}{dE_{in}/dt}.$$

[Computed by Press and Teukolsky in the 70s]
What we really want to match with a CFT computation is

$$\sigma_{abs}^{near} = \frac{dE_{abs}/dt}{|\Phi(x = x_B)|^2}, \qquad \tau_H \ll x_B \ll 1.$$

Result :

$$\sigma_{abs}^{near}(\omega, m, l; M, J) = \text{Gamma functions}$$

Feature : Superradiance ($\sigma_{abs} < 0$) occurs when

$$\omega < m\Omega_H + q_e\Phi_e$$

Spontaneous emission also occurs : extremal Kerr-Neumann black holes spontaneously decay.

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Dual CFT picture

The process is controlled by the two-point function

$$G(t^+, t^-) = \langle \mathcal{O}^\dagger(t^+, t^-)\mathcal{O}(0)\rangle \,,$$

where $t^\pm$ are the coordinates of the CFT.
From the so-called Fermi's golden rule, the absorption probability σ_{abs} is given by

$$\sigma_{abs} \sim \int dt^+ dt^- e^{-i\omega_R t^- - i\omega_L t^+}[G(t^+ - i\epsilon, t^- - i\epsilon) - G(t^+ + i\epsilon, t^- + i\epsilon)]$$

Dual CFT picture : Matching

More precisely, the form of the thermal two-point function is dictated by conformal invariance to be

$$G(t^+, t^-) \sim \left(\frac{\pi T_L}{\sinh(\pi T_L t^+)}\right)^{2h_L} \left(\frac{\pi T_R}{\sinh(\pi T_R t^-)}\right)^{2h_R} e^{iq_L\mu_L t^+ + iq_R\mu_R t^-}$$

at temperatures (T_L, T_R) and chemical potentials (μ_L, μ_R) when the operator has conformal dimensions (h_L, h_R) and charges (q_L, q_R).

Matching the gravity result around the Kerr black hole gives

$h_L = h_L(m, l, a\omega)$	$h_R = h_R(m, l, a\omega)$
$T_L = \frac{1}{2\pi}$,	$T_R = T_R^{ext}$
$\omega_L = m$	$\frac{\omega_R - q_R\mu_R}{T_R} = \frac{\omega - m\Omega_H}{2\pi T_H} - 2r_+\omega$
$q_L = e$,	$q_R = m$
$\mu_L = -\frac{Q^3}{2J}$	$\mu_R = \Omega_H$

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$h_L = h_L(m, l, a\omega)$	$h_R = h_R(m, l, a\omega)$
$T_L = \frac{1}{2\pi}$,	$T_R = T_R^{ext}$
$\omega_L = m$	$\frac{\omega_R - q_R \mu_R}{T_R} = \frac{\omega - m\Omega_H}{2\pi T_H} - 2r_+\omega$
$q_L = e$,	$q_R = m$
$\mu_L = -\frac{Q^3}{2J}$	$\mu_R = \Omega_H$

Features

- CFT description forms a representation under the symmetries

$$\mathit{Virasoro}_L \times \mathit{Current}_L \times (\mathit{Virasoro}_R \times \mathit{Current}_R)$$

  while only $\mathit{Virasoro}_L$ belongs to the asymptotic symmetry group of the near-horizon geometry.
- Real and complex weights

$$h_L(m,l,a\omega) = h_R(m,l,a\omega) \in \mathbb{R} \text{ or } i\mathbb{R}$$

  - Imaginary conformal weights are related to Schwinger pair production.
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- Many features to be better understood !

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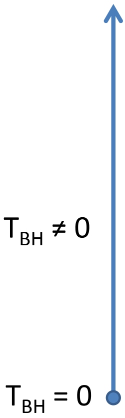
Third and last step of the Kerr/CFT correspondence



❸ Far from extremality :
Dynamics of probes has broken
$SL(2,\mathbb{R}) \times SL(2,\mathbb{R})$ symmetry.
Entropy still matches !

❷ Close to extremality :
Dynamics of probes can be obtained from a CFT two-point correlation function.

❶ At extremality :
Temperatures T_L, T_R, Virasoro algebra,
Match $S_{BH} = S_{CFT}$!

Conformal invariance away from extremality

- Away from extremality, there is no decoupled near-horizon geometry with conformal invariance.
- Yet, conformal invariance can be present in the solution space of fields propagating on the geometry and control the scattering amplitudes and the asymptotic growth of states.

Scattering Process

Let's look again at a scalar probe Φ obeying

$$\Box\Phi = 0.$$

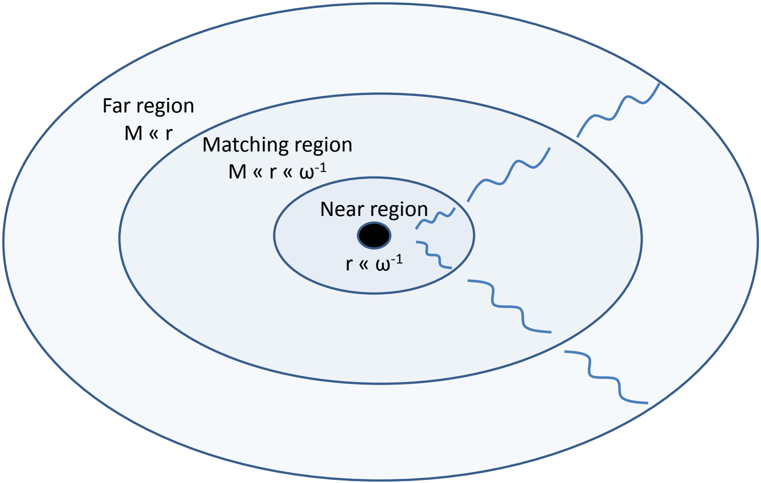
We have

$$\Phi = e^{-i\omega t + im\phi}\Theta(\theta)R(r)$$

We look only in the low-energy regime,

$$M\omega \ll 1$$

Scattering Process



In the near region $r \ll \omega^{-1}$,

$$\Theta(\theta) = Y_{lm}(\theta), \qquad R(r) = \text{hypergeometric}$$

$SL(2,\mathbb{R})_L \times SL(2,\mathbb{R})_R$ symmetry

Trick : define new coordinates

$$(w^+, w^-, y, \theta)$$

related to (t, r, ϕ, θ) as

$$\begin{aligned} w^+ &= \sqrt{\frac{r-r_+}{r-r_-}} e^{2\pi T_R \phi} \\ w^- &= \sqrt{\frac{r-r_+}{r-r_-}} e^{2\pi T_L \phi - \frac{t}{2M}} \\ y &= \sqrt{\frac{r_+ - r_-}{r-r_-}} e^{\pi(T_L+T_R)\phi - \frac{t}{4M}} \end{aligned}$$

where

$$T_R = \frac{T_H}{\Omega_H}, \qquad T_L = \frac{1}{2\pi}\left(1 + \frac{M^2 - J}{J}\right).$$

$SL(2,\mathbb{R})_L \times SL(2,\mathbb{R})_R$ symmetry

The radial part of the wave equation can then be written as

$$\mathcal{H}^2\Phi \equiv \left(-H_0^2 + \frac{1}{2}(H_1 H_{-1} + H_{-1}H_1)\right)\Phi = l(l+1)\Phi$$

where $\Phi = e^{-i\omega t}R(r)$ and

$$\begin{aligned} H_1 &= i\partial_+, \\ H_0 &= i(w^+\partial_+ + \frac{1}{2}y\partial_y) \\ H_{-1} &= i(w^{+2}\partial_+ + w^+ y\partial_y - y^2\partial_-) \end{aligned}$$

obey the $SL(2,\mathbb{R})_L$ algebra. There is also a second set of generators $SL(2,\mathbb{R})_R$ such that

$$\bar{\mathcal{H}}^2\Phi = l(l+1)\Phi\,.$$

Identification breaks symmetry to $U(1)_L \times U(1)_R$

At fixed r, the change of coordinates is

$$\begin{aligned} w^+ &= e^{2\pi T_R \phi} \\ w^- &= e^{2\pi T_L \phi - \frac{t}{2M}} \end{aligned}$$

where

$$T_R = \frac{T_H}{\Omega_H}, \qquad T_L = \frac{1}{2\pi}\left(1 + \frac{M^2 - J}{J}\right)$$

The identification

$$\phi \sim \phi + 2\pi$$

breaks $SL(2,\mathbb{R})_L \times SL(2,\mathbb{R})_R \to U(1)_L \times U(1)_R$.

T_L, T_R can be interpreted as temperatures

Assume that (w^+, w^-) plane describes a $SL(2,\mathbb{R}) \times SL(2,\mathbb{R})$ invariant CFT vacuum.
We define the coordinates t^+ and t^- as

$$\begin{aligned} w^+ &\equiv e^{t_+} = e^{2\pi T_R \phi} \\ w^- &\equiv e^{-t_-} = e^{2\pi T_L \phi - \frac{t}{2M}} \end{aligned}$$

Then the identification

$$\phi \sim \phi + 2\pi$$

can be written as

$$t_+ \sim t_+ + 4\pi^2 T_R, \qquad t_- \sim t_- - 4\pi^2 T_L$$

Tracing over the region outside the strip leads to a density matrix with temperatures T_L, T_R.

Entropy matching

Remarkably, the entropy of the Kerr black hole

$$S_{BH} = \frac{2\pi}{\hbar G}\left(M^2 + \sqrt{M^4 - J^2}\right)$$

agrees with the Cardy formula

$$S_{CFT} = \frac{\pi^2}{3}(c_L T_L + c_R T_R)$$

with

$$c_L = c_R = \frac{12J}{\hbar}.$$

Conclusions

- Remarkable agreement between black hole entropy and Cardy formula

$$S_{BH} = \frac{\pi^2}{3}(c_L T_L + c_R T_R)$$

  where c_L computed for extremal black holes, $c_R = c_L$, and T_L, T_R computed independently for extremal black holes and non-extremal black holes.
- Two regimes where the dynamics of probes is governed by conformal symmetry :
  - at near-extremality close to superradiant bound
  - away from extremality at low energy
- Open question : what is the nature of this “CFT” ?
- Open question : Are there other regimes where the CFT description is valid ?

For further reading : check the reviews

- *“Cargèse Lectures on the Kerr/CFT correspondence”*, Bredberg, Keeler, Lysov, Strominger, **1103.2355**
- *“The Kerr/CFT correspondence, a bird-eye view”*, G.C., **1202.XXXX**