The general gaugings of maximal $d = 9$ supergravity

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Gauged Supergravities

1. Relation between
   - RR $(p+1)$-form potentials in $d=10$ II SUGRA
   - D-branes in String Theory (Polchinski '95)
2. Find new $\left\{ \begin{array}{l} \text{SUGRA fields} \\ \text{string configurations} \end{array} \right\}$ from $\left\{ \begin{array}{l} \text{string configurations} \\ \text{SUGRA fields} \end{array} \right\}$
3. Non trivial to find higher-rank fields in SUGRA
4. U-duality $\Rightarrow$ new fields belonging to the same orbits as the known fields
5. Study of all the possible consistent SUSY transf. for p-forms in $d=10$ (Bergshoeff *et al.* '01, '05, '06; Greitz *et al.* '11)
6. E_{11}
   - Bosonic field content of SUGRA for every dimension (Julia '98; West '11; Riccioni *et al.* '09)
   - Covariant WZ-terms of all possible branes in all dimensions (U-duality) (Bergshoeff, Riccioni '10, '11)

Embedding tensor formalism

1. Study of the most general deformations of field theories (De Wit, Samtleben, Trigiante '03, '04)
2. Requirement: tensor hierarchy. Introduce new higher-rank potentials (Stückelberg gauge transformations) (De Wit, Samtleben, Trigiante '05)
3. Additional constraints arising from gauge and SUSY invariance
4. Interesting cases
   - $d = 11$ $N = 1$, No $1-$forms
   - $d = 10$ $N = 2B$, No $1-$forms
   - $d = 10$ $N = 2A$, The $1-$form transforms under the only (abelian) global symmetry
   - $d = 9$ $N = 2$? 3 vectors ET formalism?
5. Maximal $d = 9$ supergravity
   - Generalized dimensional reduction and rescaling symmetries (Bergshoeff *et al.* '02)
   - What about..?
     - Any other deformation arising from dimensional reduction
     - Possible combinations of known deformations
     - Other deformations with no higher-dim origin

Maximal $d = 9$ (Symmetries)

1. ONE undeformed maximal theory (Gates *et al.* '86)
2. Global symmetry: $SL(2,\mathbb{R}) \times (\mathbb{R}^+)^2$
   - α: acts on the metric and leaves invariant the eom's
   - β: leaves invariant metric and action (trombone symmetry)
3. Field content

$$\left\{ e_\mu{}^a, \varphi, \tau \equiv \chi + ie^{-\phi}, A^I{}_{(1)}, B^i{}_{(2)}, C_{(3)}, \psi_\mu, \tilde{\lambda}, \lambda \right\}$$

4. Complex scalar $SL(2,\mathbb{R})/U(1)$ coset parameterized by an $SL(2,\mathbb{R})$ matrix $\mathcal{M}$

$$\mathcal{M} \equiv e^\phi \begin{pmatrix} |\tau|^2 & \chi \\ \chi & 1 \end{pmatrix}, \qquad \mathcal{M}^{-1} \equiv e^\phi \begin{pmatrix} 1 & -\chi \\ -\chi & |\tau|^2 \end{pmatrix}.$$

Maximal $d = 9$

1. Gauge trasformations

$$
\begin{aligned}
\delta_\Lambda A^I &= -d\Lambda^I \\
\delta_\Lambda B^i &= -d\Lambda^i + \delta^i{}_{\mathbf{i}}\left[\Lambda^{\mathbf{i}} F^0 + \Lambda^0 F^{\mathbf{i}} + \tfrac{1}{2}\left(A^0 \wedge \delta_\Lambda A^{\mathbf{i}} + A^{\mathbf{i}} \wedge \delta_\Lambda A^0\right)\right] \\
\delta_\Lambda[C - \tfrac{1}{6}\varepsilon_{\mathbf{ij}} A^{0\mathbf{ij}}] &= -d\Lambda - \varepsilon_{\mathbf{i}j}\left(F^{\mathbf{i}}\Lambda^j + \Lambda^{\mathbf{i}} H^j - \delta_\Lambda A^{\mathbf{i}} B^j + \tfrac{1}{2}\delta^j{}_{\mathbf{j}} A^{0\mathbf{i}} \delta_\Lambda A^{\mathbf{j}}\right)
\end{aligned}
$$

2. Field strengths

$$
\begin{aligned}
F^I &= dA^I \\
H^i &= dB^i + \tfrac{1}{2}\delta^i{}_{\mathbf{i}}(A^0 \wedge F^{\mathbf{i}} + A^{\mathbf{i}} \wedge F^0) \\
G &= d[C - \tfrac{1}{6}\varepsilon_{\mathbf{ij}} A^{0\mathbf{ij}}] - \varepsilon_{\mathbf{i}j} F^{\mathbf{i}} \wedge \left(B^j + \tfrac{1}{2}\delta^j{}_{\mathbf{j}} A^{0\mathbf{j}}\right)
\end{aligned}
$$

Equations of motion

1. eom's of the scalars

$$d\star d\varphi - \frac{2}{\sqrt{7}} e^{\frac{4}{\sqrt{7}}\varphi} F^0 \wedge \star F^0 - \frac{3}{2\sqrt{7}} e^{\frac{3}{\sqrt{7}}\varphi} (\mathcal{M}^{-1})_{\mathbf{ij}} F^{\mathbf{i}} \wedge \star F^{\mathbf{j}} + \frac{1}{2\sqrt{7}} e^{-\frac{1}{\sqrt{7}}\varphi} (\mathcal{M}^{-1})_{ij} H^i \wedge \star H^j - \frac{1}{\sqrt{7}} e^{\frac{2}{\sqrt{7}}\varphi} G \wedge \star G = 0$$

$$d\left[\star \frac{d\bar{\tau}}{(\Im\mathrm{m}\tau)^2}\right] - i\frac{d\tau \wedge \star d\bar{\tau}}{(\Im\mathrm{m}\tau)^3} - \partial_\tau (\mathcal{M}^{-1})_{\mathbf{ij}} \left[F^{\mathbf{i}} \wedge \star F^{\mathbf{j}} + H^i \wedge \star H^j\right] = 0$$

2. eom's for the $p-$forms

$$d\left(e^{\frac{4}{\sqrt{7}}\varphi} \star F^0\right) = -e^{-\frac{1}{\sqrt{7}}\varphi} \mathcal{M}^{-1}_{\mathbf{ij}} F^{\mathbf{i}} \wedge \star H^j + \frac{1}{2} G \wedge G$$

$$d\left(e^{\frac{3}{\sqrt{7}}\varphi} \mathcal{M}^{-1}_{\mathbf{ij}} \star F^{\mathbf{j}}\right) = -e^{\frac{3}{\sqrt{7}}\varphi} \mathcal{M}^{-1}_{\mathbf{ij}} F^0 \wedge \star H^j + \varepsilon_{\mathbf{i}j} e^{\frac{2}{\sqrt{7}}\varphi} H^j \wedge \star G$$

$$d\left(e^{-\frac{1}{\sqrt{7}}\varphi} \mathcal{M}^{-1}_{ij} \star H^j\right) = \varepsilon_{\mathbf{i}j} e^{\frac{2}{\sqrt{7}}\varphi} F^{\mathbf{j}} \wedge \star G - \varepsilon_{ij} H^j \wedge G$$

$$d\left(e^{\frac{2}{\sqrt{7}}\varphi} \star G\right) = F^0 \wedge G + \frac{1}{2}\varepsilon_{ij} H^i \wedge H^j$$

Magnetic duals

1. Explicit dual field strengths

$$\tilde{G} = d\tilde{C} + C \wedge F^0 - \frac{1}{24}\varepsilon_{\mathbf{ij}} A^{0\mathbf{ij}} \wedge F^0 - \varepsilon_{ij}\left(H^i - \frac{1}{2}dB^i\right) \wedge B^j$$

$$\tilde{H}_i = d\tilde{B}_i - \delta_{ij} B^j \wedge G + \delta_{\mathbf{ij}} \tilde{C} \wedge F^{\mathbf{j}} + \frac{1}{2}\delta_{\mathbf{ij}}\left(A^0 \wedge F^{\mathbf{j}} + A^{\mathbf{j}} \wedge F^0\right) \wedge C + \ldots$$

$$\tilde{F}_0 = d\tilde{A}_0 + \frac{1}{2} C \wedge G - \varepsilon_{\mathbf{i}j} F^{\mathbf{i}} \wedge \left(\delta^{jk}\tilde{B}_k - \frac{2}{3} B^j \wedge C\right) + \ldots$$

$$\tilde{F}_{\mathbf{i}} = d\tilde{A}_{\mathbf{i}} + \delta_{\mathbf{i}j}\left(B^j + \frac{7}{18}\delta^j{}_{\mathbf{k}} A^{0\mathbf{k}}\right) \wedge \tilde{G} - \delta_{\mathbf{i}}{}^j F^0 \tilde{B}_j - \frac{1}{9}\delta_{\mathbf{ij}}\left(8A^0 F^{\mathbf{j}} + A^{\mathbf{j}} F^0\right)\tilde{C} + \ldots$$

are in agreement with the following duality relations

$$\tilde{G} = e^{\frac{2}{\sqrt{7}}\varphi} \star G$$

$$\tilde{H}_i = e^{-\frac{1}{\sqrt{7}}\varphi} \mathcal{M}^{-1}_{ij} \star H^j$$

$$\tilde{F}_0 = e^{\frac{4}{\sqrt{7}}\varphi} \star F^0$$

$$\tilde{F}_{\mathbf{i}} = e^{\frac{3}{\sqrt{7}}\varphi} \mathcal{M}^{-1}_{\mathbf{ij}} \star F^{\mathbf{j}}$$

2. eom's for $\left\{\begin{array}{c}\text{magnetic}\\ \text{electric}\end{array}\right\}$ fields $\equiv$ Bianchi's for $\left\{\begin{array}{c}\text{electric}\\ \text{magnetic}\end{array}\right\}$ fields

Magnetic duals

1. What about the scalars?
   - Global symmetry group acts on the manifold
   - 1-form Noether current j_A associated with each generator T_A of the global symmetry and satisfying

$$d \star j_A = 0$$

   - define $(d-2)-$form potential $\tilde{A}^A_{(d-2)}$, such that

$$J_A \equiv d\tilde{A}^A_{(d-2)} = G^{AB} \star j_B$$

   - Expectation: 1 triplet associated to $SL(2,\mathbb{R})$ and 2 singlets associated to $\mathbb{R}^+$

Magnetic duals

1. $(d-1)-$forms
   - ET formalism $\Rightarrow$ as many as deformation parameters we have
   - lagrange multipliers enforcing their constancy

$$\sum_{\sharp} dm_{\sharp} \wedge \tilde{A}^{\sharp}_{(d-1)}$$

   - Field strengths $\tilde{F}^{\sharp}_{d} = \frac{1}{2} \star \frac{\partial V}{\partial m_{\sharp}}$

2. d-forms
   - ET formalism $\Rightarrow$ as many as constraints in deformation parameters
   - lagrange multipliers enforcing their validity

$$\sum_{\flat} \mathcal{Q}_{\flat} \tilde{A}^{\flat}_{(d)}$$

Bianchi identities

$$
\begin{aligned}
dF^I &= 0 \\
dH^i + F^0 F^i &= 0 \\
dG - F^i H_i &= 0 \\
d\tilde{G} + F^0 G + \tfrac{1}{2}\epsilon_{ij} H^i H^j &= 0 \\
d\tilde{H}_i + F_i \tilde{G} - H_i G &= 0 \\
d\tilde{F}_0 + F^j \tilde{H}_j - \tfrac{1}{2} G G &= 0 \\
d\tilde{F}_i + F^0 \tilde{H}_i - H_i \tilde{G} &= 0 \\
dJ_A &= 0
\end{aligned}
$$

Deforming the theory

1. Consistent deformation of the theory
   - Definition of a suitable covariant derivative acting on any field (gauge generators)

$$X_I^{(r)} = \vartheta_I{}^A T_A^{(r)}$$

   - Modified gauge transformations
   - Field strengths are modified!! Addition of Stückelberg deformation parameters, Z's
   - Bianchi and Ricci identities will be modified!!
2. Deformation of SUSY transformation rules:
   - Replace derivatives by covariant ones
   - Replace field strenghts by the deformed ones
   - Fermion shifts
3. Action and/or eom's modified (fermion mass terms, scalar potential V).

The embedding tensor

1. Gauge generators for a given representation (r)

$$X_I^{(r)} = \vartheta_I{}^A T_A^{(r)}$$

2. For a given field, we had (undeformed case)

$$\delta_\alpha = \alpha^A T_A^{(r)},$$

and we will ask the theory to be invariant under

$$\delta_\Lambda = \Lambda^I X_I^{(r)} = \Lambda^I \vartheta_I{}^A T_A^{(r)}$$

3. Quadratic constraint (gauge invariance condition)

$$\vartheta_I{}^A T_{A\,J}{}^K \vartheta_K{}^C - \vartheta_I{}^A \vartheta_J{}^B f_{AB}{}^C = 0$$
$$X_{IJ}{}^K \vartheta_K{}^C - \vartheta_J{}^A X_{IA}{}^C = 0$$

Covariant derivatives

1. Covariant derivatives

$$\mathfrak{D}\varphi = d\varphi + A^I \vartheta_I^A k_A{}^\varphi \,, \qquad \mathfrak{D}\tau = d\tau + A^I \vartheta_I^A k_A{}^\tau \,,$$

2. Covariant derivative of a p-form:

$$\mathfrak{D}\eta^{(p)} = d\eta^{(p)} + A^I \wedge \delta_I(\eta^{(p)}) = d\eta^{(p)} + A^I \wedge X_I \eta^{(p)} \qquad (1)$$

3. Basic property: Leibnitz rule (Jacobi identities not satisfied)

$$\mathfrak{D}(XY) = \mathfrak{D}XY + \epsilon X \mathfrak{D} Y$$

Field strengths and gauge variations

1. Gauge variations

$$\delta_\Lambda A^I = -\mathfrak{D}\Lambda^I + Z^I{}_i \Lambda^i$$
$$\delta_\Lambda B^i = -\mathfrak{D}\Lambda^i + \left[\Lambda^i F^0 + \Lambda^0 F^i + \tfrac{1}{2}\left(A^0 \delta_\Lambda A^i + A^i \delta_\Lambda A^0\right)\right] + Z^i \Lambda$$
$$\delta_\Lambda [C - \tfrac{1}{6}\varepsilon_{ij} A^{0ij}] = -\mathfrak{D}\Lambda - \varepsilon_{ij}\left(F^i \Lambda^j + \Lambda^i H^j - \delta_\Lambda A^i \wedge B^j + \tfrac{1}{2} A^{0i} \delta_\Lambda A^j\right) + Z\tilde{\Lambda}$$
$$\delta_\Lambda \tilde{C} = -\mathfrak{D}\tilde{\Lambda} + (OLD) + Z^i \tilde{\Lambda}_i$$

2. Field strengths

$$F^I = dA^I + \tfrac{1}{2} X_{JK}{}^I A^J \wedge A^K + Z^I{}_i B^i$$
$$H^i = \mathfrak{D}B^i + (OLD) + (XA^{012}) + Z^i C$$
$$G = \mathfrak{D}C + (OLD) + Z^i_j B^{ij} + Z\tilde{C}$$
$$\tilde{G} = \mathfrak{D}\tilde{C} + (OLD) + Z^0_j B^j C + (XA^J B^i B^j) + Z^i \tilde{H}_i$$

Tensor hierarchies

$$
\begin{aligned}
dF^I &= 0, && 3 && \mathfrak{D}F^I = Z^I_i H^i, \\
dH^i + F^0 F^i &= 0, && 4 && \mathfrak{D}H^i + F^0 F^i = Z^i G, \\
dG - F^i H_i &= 0, && 5 && \mathfrak{D}G - F^i H_i = Z\tilde{G}, \\
d\tilde{G} + F^0 G + \tfrac{1}{2}\epsilon_{ij} H^i H^j &= 0, && 6 && \mathfrak{D}\tilde{G} + F^0 G + \tfrac{1}{2} H^i H_i = Z^i H_i, \\
d\tilde{H}_i + F_i \tilde{G} - H_i G &= 0, && 7 && \mathfrak{D}\tilde{H}_i + F_i \tilde{G} - H_i G = Z^I_i \tilde{F}^0_I, \\
d\tilde{F}_0 + F^j \tilde{H}_j - \tfrac{1}{2} GG &= 0, && 8 && \mathfrak{D}\tilde{F}_0 + F^i \tilde{H}_i - \tfrac{1}{2} GG = \vartheta^A_0 J_A, \\
d\tilde{F}_i + F^0 \tilde{H}_i - H_i \tilde{G} &= 0, && && \mathfrak{D}\tilde{F}_i + F^0 \tilde{H}_i - H_i \tilde{G} = \vartheta^A_i J_A. \\
dJ_A &= 0 && 9 && \mathfrak{D}J_A + \ldots = \ldots F^\sharp \\
& && 10 && \mathfrak{D}F^\sharp + \ldots = \ldots G^\flat \\
& && 11 && \mathfrak{D}G^\flat + \ldots = \ldots
\end{aligned}
$$

Deformed SUSY transformations

1. Replace derivatives and field strengths by covariant ones where

$$\mathfrak{D}_\mu\epsilon \equiv \left\{\nabla_\mu + \tfrac{i}{2}\left[\tfrac{1}{2}e^{\phi}\mathfrak{D}^5_\mu\chi + A^I{}_\mu\vartheta_I{}^m\mathcal{P}_m\right] + \tfrac{9}{14}\gamma_\mu \not{A}^I\vartheta_I{}^4\right\}\epsilon$$

2. Add fermion shifts $f, k, g, h, \tilde{g}, \tilde{h}$

$$\begin{aligned}
\delta_\epsilon\psi_\mu &= \mathfrak{D}_\mu\epsilon + f\gamma_\mu\epsilon + k\gamma_\mu\epsilon^* + \tfrac{i}{8\cdot 2!}e^{-\frac{2}{\sqrt{7}}\varphi}\left(\tfrac{5}{7}\gamma_\mu\gamma^{(2)} - \gamma^{(2)}\gamma_\mu\right)F^0\epsilon + \ldots \\
\delta_\epsilon\tilde{\lambda} &= i\,\not{\mathfrak{D}}\varphi\epsilon^* + \tilde{g}\epsilon + \tilde{h}\epsilon^* - \tfrac{1}{\sqrt{7}}e^{-\frac{2}{\sqrt{7}}\varphi}\,\not{F}^0\epsilon^* + \ldots \\
\delta_\epsilon\lambda &= -e^{\phi}\,\not{\mathfrak{D}}\tau\epsilon^* + g\epsilon + h\epsilon^* - \tfrac{i}{2\cdot 2!}e^{\frac{3}{2\sqrt{7}}\varphi + \frac{1}{2}\phi}(\not{F}^1 - \tau\,\not{F}^2)\epsilon + \ldots
\end{aligned}$$

3. keep the boson rules
4. We will impose algebra closure

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}] = \delta_{gct} + \delta_\Lambda + (duality)$$

- Constraints on ET and fermion shifts (linear constraints!!)

Constraints summary

1. Quadratic constraints
   - Gauge invariance of the embedding tensor

$$\vartheta X + X\vartheta = 0$$

   - Gauge invariance of the Stückelberg shifts

$$XZ + XZ = 0$$

   - Orthogonality constraints (tensor hierarchy)

$$\vartheta Z = 0$$
$$ZZ = 0$$

2. Linear constraints
   - Leibnitz product condition (group representation consistency)

$$X + Z + Z = 0$$

   - Closure of the algebra (SUSY)

Reduction of parameters

1. From 24 free parameters.....
   - 15 from the ET $\vartheta_I{}^A$
   - 9 from the Stückelberg shifts $Z_i{}^I$, Z^i, Z
2. We get 8 independent deformations and $Z = Z(\vartheta)$

$$\begin{aligned} \vartheta_0{}^m &= m_m\,, \ (m=1,2,3) & \vartheta_{\mathbf{1}}{}^4 &= -m_{11}\,, & \vartheta_{\mathbf{1}}{}^5 &= \tilde{m}_4\,, \\ \vartheta_0{}^5 &= -\tfrac{16}{3}m_{\mathrm{IIB}}\,, & \vartheta_{\mathbf{2}}{}^4 &= m_{\mathrm{IIA}}\,, & \vartheta_{\mathbf{2}}{}^5 &= m_4\,. \end{aligned}$$

$\mathbb{R}^+$	$\vartheta_0{}^1$	$\vartheta_0{}^2 - \vartheta_0{}^3$	$\vartheta_0{}^2 + \vartheta_0{}^3$	$\vartheta_{\mathbf{1}}{}^4, \vartheta_{\mathbf{1}}{}^5$	$\vartheta_{\mathbf{1}}{}^4, \vartheta_{\mathbf{2}}{}^5$	$\vartheta_0{}^5$
α	-3	-3	-3	0	0	-3
β	$-1/2$	$-5/4$	$1/4$	$3/4$	0	$-1/2$
γ	0	2	-2	-1	1	0
δ	0	0	0	-2	-2	0

3. Total agreement with E_{11}

Quadratic constraints

1. Final set

$$\vartheta_0{}^m \left(12\vartheta_{\mathbf{i}}{}^4 + 5\vartheta_{\mathbf{i}}{}^5\right) \equiv \mathcal{Q}^m{}_{\mathbf{i}} = 0\,,$$
$$\vartheta_{\mathbf{i}}{}^4\vartheta_0{}^5 \equiv \mathcal{Q}^4{}_{\mathbf{i}} = 0\,,$$
$$\vartheta_{\mathbf{i}}{}^5\vartheta_0{}^5 \equiv \mathcal{Q}^5{}_{\mathbf{i}} = 0\,,$$
$$\vartheta_{\mathbf{j}}{}^4\left(\vartheta_0^m T_m\right)_{\mathbf{i}}{}^{\mathbf{j}} \equiv \mathcal{Q}_{\mathbf{i}} = 0\,,$$
$$\varepsilon^{\mathbf{ij}}\vartheta_{\mathbf{i}}{}^4\vartheta_{\mathbf{j}}{}^5 \equiv \mathcal{Q} = 0\,,$$

2. If we set $\vartheta_I{}^5 = 0$ (trombone symmetry)
3. E_{11} predicts an additional doublet of 9-forms. Is this contradictory??
4. Apparently not $\Rightarrow$ new Stückelberg shifts $\propto \vartheta$ can eliminate one of these doublets in the undeformed case (Huebscher *et al.* '10)

Results

1. Noether currents ($\tilde{A}_A^{(d-2)}$ $(d-2)$-forms)
   - **3** of $SL(2,R)$
   - $\mathbf{1}+\mathbf{1}$ corresponding to $(R^+)^2$
2. Mass parameters ($\tilde{A}_\sharp^{(d-1)}$ $(d-1)$-forms)
   - **3** $\vartheta_0{}^m$
   - $\mathbf{2}+\mathbf{2}$ $\vartheta_i{}^4$, $\vartheta_i{}^5$
   - **1** $\vartheta_0{}^5$
3. Quadratic constraints ($\tilde{A}_\flat^{(d)}$ d-forms)
   - **6** $\mathcal{Q}_{\mathbf{i}}{}^m$
   - $\mathbf{2}+\mathbf{2}+\mathbf{2}$ $\mathcal{Q}_{\mathbf{i}}{}^4$, $\mathcal{Q}_{\mathbf{i}}{}^5$, $\mathcal{Q}_{\mathbf{i}}$
   - **1** $\mathcal{Q}$

Conclusions

1. Application of ET formalism to the study of the most general deformations of maximal $d = 9$ SUGRA
2. Constraints on the deformation parameters imposed by gauge and SUSY invariance
3. Independent set of deformation parameters $(8 = \mathbf{3} + \mathbf{2} + \mathbf{2} + \mathbf{1})$
4. Analysis of the gauged theory: field strengths, gauge and SUSY transformations
5. Determination of the 7-, 8- and 9-forms, which are dual to j_A, ${\vartheta_I}^A$ and QC, respectively.
6. All the higher-rank fields have an interpretation in terms of gaugings and symmetries.
7. Good agreement with E_{11} level decomposition

Prospectives

1. $d = 9$
   - Detailed expressions for higher rank forms
   - Stückelberg shifts
   - Scalar potential V and solutions
2. Gauged supergravities
   - Study of general gaugings in other dimensions
   - Classification and origin of gaugins
   - Interplay between new gaugins and ST interpretation (double field theory, generalized complex geometry, ...)
3. ....

Thanks!!