A simple model with asymptotically Lifshitz BHs

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based on: arXiv:1201.5480
arXiv:1105.6335 w/ S. Vandoren

FROGS
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FOM

Lifshitz metric

- The final aim is extend AdS/CFT to study non-isotropic, strongly coupled, field th.

$$t \to \lambda^z t \qquad x \to \lambda x$$

- Lifshitz algebra generators identified with isometries of the Lifshitz metric

$$ds^2 = \frac{\ell^2}{r^2} dr^2 - \frac{r^{2z}}{\ell^{2z}} dt^2 + \frac{r^2}{\ell^2} d\vec{x}^2_{d-1}$$

Kachru
Liu
Mulligan

- Want to study asymptotically Lifshitz charged BHs, to mimic finite temperature and chemical potential

z=1 (AdS) case

- In AAdS case there is the known AdS-RN

$$S = -\frac{1}{16\pi G_{d+1}} \int d^{d+1}x \sqrt{-g} \left(R - 2\Lambda - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4} e^{\lambda\phi} F^2 \right)$$

- Where the solution reads

$$ds^2 = \frac{\ell^2}{r^2} \frac{dr^2}{b_k(r)} - b_k(r) \frac{r^2}{\ell^2} dt^2 + r^2 d\Omega^2_{k,d-1}$$

$$b_k \simeq 1 + \frac{k}{r^2} - m r^{-d} + \rho^2 r^{-2(d-1)}$$

$$F_{rt} = \rho\, r^{1-d} \qquad e^{\lambda\phi} = 1$$

Chamblin
Emparan
Johnson
Myers

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Anisotropic case

- Let's play the same game with different asymptotics (*z!=1*)

$$S = -\frac{1}{16\pi G_{d+1}} \int d^{d+1}x \sqrt{-g} \left(R - 2\Lambda - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4} e^{\lambda\phi} F^2 \right)$$

- Now the solution is no longer charged nor with spherical topology

Taylor

$$b_k \simeq 1 - m r^{-(d+z-1)}$$

$$F_{rt} = f r^{d+z-2} \qquad e^{\lambda\phi} = \mu r^{2(1-d)}$$

$$\mu f^2 = 2(z-1)(d+z-1)$$

Anisotropic case

- One may try to introduce charge by adding an extra field

$$S = -\frac{1}{16\pi G_{d+1}} \int d^{d+1}x\sqrt{-g}\left(R - 2\Lambda - \frac{1}{2}(\partial\phi)^2 - \sum_{i=1}^{2}\frac{1}{4}e^{\lambda_i\phi}F_i^2\right)$$

- Actually there are two possible solutions now

$$b_k \simeq \begin{cases} 1 + \boxed{\frac{k}{r^2}} - mr^{-(d+z-1)} \\ 1 - mr^{-(d+z-1)} + \boxed{\rho^2 r^{-2(d+z-2)}} \end{cases}$$

Anisotropic case

- Natural question: do we have spherical topology and charge with *U(1)*^{3} theory?

$$S = -\frac{1}{16\pi G_{d+1}} \int d^{d+1}x\sqrt{-g}\left(R - 2\Lambda - \frac{1}{2}(\partial\phi)^2 - \sum_{i=1}^{3}\frac{1}{4}e^{\lambda_i \phi}F_i^2\right)$$

- The answer is yes, for the blackening function we obtain

$$b_k \simeq 1 + \boxed{\frac{k}{r^2}} - mr^{-(d+z-1)} + \boxed{\rho^2 r^{-2(d+z-2)}}$$

The solution with U(1)^3

$$ds^2 = \frac{\ell^2}{r^2}\frac{dr^2}{b_k(r)} - b_k(r)\frac{r^{2z}}{\ell^{2z}}dt^2 + r^2 d\Omega^2_{k,d-1}$$

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$$e^{\phi} = \mu\, r^{\sqrt{2(d-1)(z-1)}}$$

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$$\downarrow$$

$$\mathcal{A}_t^{(z)\prime} \simeq f_1(d,z) r^{d+z-2}$$

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$$\mathcal{A}_t^{(z)\prime} \simeq f_1(d,z) r^{d+z-2}$$

These fields diverge at the boundary!!

$$A_t' \simeq \rho\, r^{2-d-z}$$

$$e^\phi = \mu\, r^{\sqrt{2(d-1)(z-1)}}$$

Cons

- Divergent fields at the boundary spoil Lifshitz symmetry

$$\mathcal{L}_\xi \mathcal{A} \neq 0$$

$$\xi = zt\partial_t - r\partial_r + x^i\partial_i$$

- Model cannot be obtained from string theory due to the constant potential, and for generic potential no Lifshitz solution is known

Charmousis, Gouteraux,
Kim, Kiritsis, Meyer

Pros

- Analytic for generic $d>2$ and $z\geq 1$
- Can be understood as effective IR theories that must be UV completed
- Great toy model: matter fields not coupling to $\mathcal{A}$ do not know about their divergent behaviour!
- Infinities can be removed with (almost) standard holographic renormalization

Gursoy
Plauschinn
Stoof
Vandoren

Holo-ren

- Prior to an on-shell evaluation: the variational problem (*k=1* from now on)
  - add Gibbons-Hawking term
  - consider badly behaved gauge fields

$$\mathcal{A}_t^{(z)\prime} \simeq f_1(d,z) r^{d+z-2} \qquad \mathcal{A}_t^{(k)\prime} \simeq f_2(d,z) r^{d+z-4}$$

- Correct variational problem implies

$$\tilde{S} = S + S_{GH} + \frac{1}{2\kappa_2} \int_\partial \sqrt{-h} \frac{1}{2} n_\mu \left(e^{\lambda_z \phi} \mathcal{A}_\nu^{(z)} \mathcal{F}^{\mu\nu}_{(z)} + e^{\lambda_k \phi} \mathcal{A}_\nu^{(k)} \mathcal{F}^{\mu\nu}_{(k)} \right)$$

Holo-ren

- Recall the AAdS counterterm

$$S_{ct} = -\frac{1}{\kappa^2}\int_\partial d^dx\sqrt{-h}\left(\frac{d-1}{\ell} + \frac{\ell}{2(d-2)}R + \frac{\ell^3}{2(d-2)^2(d-4)}\left[R_{ab}R^{ab} - \frac{d}{4(d-1)}R^2\right] + \cdots\right)$$

- The series truncates depending on the number of dimensions, and for fixed d one needs the term with R^n, where

$$n = \left\lfloor \frac{d-1}{2} \right\rfloor$$

- For the non-AdS case this relation reads

$$n = \left\lfloor \frac{d+z-2}{2} \right\rfloor$$

Holo-ren

- Can we avoid restricting to specific *d* and *z*? There are infinite counterterms, but for the most general boundary metric!

- Notice that the asymptotic metric is static with topology $\mathbb{R}_t \times S^{d-1}$, then

$$R^{(h)}_{t\alpha\beta\gamma} = 0$$

$$R^{(h)}_{ijkl} \sim h_{ik}h_{jl} - h_{il}h_{jk}$$

- In concrete, any contraction of curvature tensors is proportional to a power of $R_{(h)}$

Holo-ren

- One method to obtain the counterterm: consider

$$S_{ct} = \frac{1}{\kappa^2} \int_\partial \sqrt{-h} \sum_{n=0}^{n_{max}} c_n R_{(h)}^n$$

- For several values of *d* and *z* fix c_0 to cancel divergences, repeat with c_1, c_2...
- Find each $c_{i \leq n_{max}}$ as a function of *d* and *z*, and generalize to arbitrary power *n*
- Resum the series (and check for other *d*'s and *z*'s)

Holo-ren

- A second method to write down the counterterm: calculate on-shell action in the neutral case with no black hole
- Factor out $\sqrt{-h}$ and trade all factors of radial coordinate for the boundary Ricci scalar
- Subtract precisely that counterterm: equivalent to backgound subtraction!

Kraus
Larsen
Siebelink

On-shell evaluation

- The on-shell, renormalized, action

$$\tilde{S} - \frac{1}{\kappa^2}\int_\partial d^d x\sqrt{-h}\frac{d-1}{\ell}\sqrt{1+\frac{(d-2)\ell^2 R_{(h)}}{(d-1)(d+z-3)^2}} = \beta W$$

On-shell evaluation

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- What about the energy?

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- What about the energy?

$$T_{mn} = \frac{1}{\kappa^2}\left[K_{mn} - K\,h_{mn} + \frac{d-1}{\ell}\sqrt{1+\frac{(d-2)\ell^2 R_{(h)}}{(d-1)(d+z-3)^2}}h_{mn}\right]$$

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- There are corrections, but *M* is accesible

$$M = \int d^{d-1}x\sqrt{\det h_{ij}}k^m\xi^n T_{mn} = \frac{V_{d-1}}{16\pi G_{d+1}}\frac{m(d-1)}{\ell^{1+z}}$$

Thermodynamics

- Defining charge and chemical potential in the standard way

$$\tilde{Q} = \frac{1}{16\pi G_{d+1}} \int e^{\lambda\phi} * F_2 = \frac{V_{d-1}\ell^{z-1}\rho}{16\pi G_{d+1}}$$

$$\tilde{\Phi} = A_t(\infty) = \frac{\rho\,\mu^{-\sqrt{2\frac{z-1}{d-1}}}}{d+z-3} r_h^{3-d-z}$$

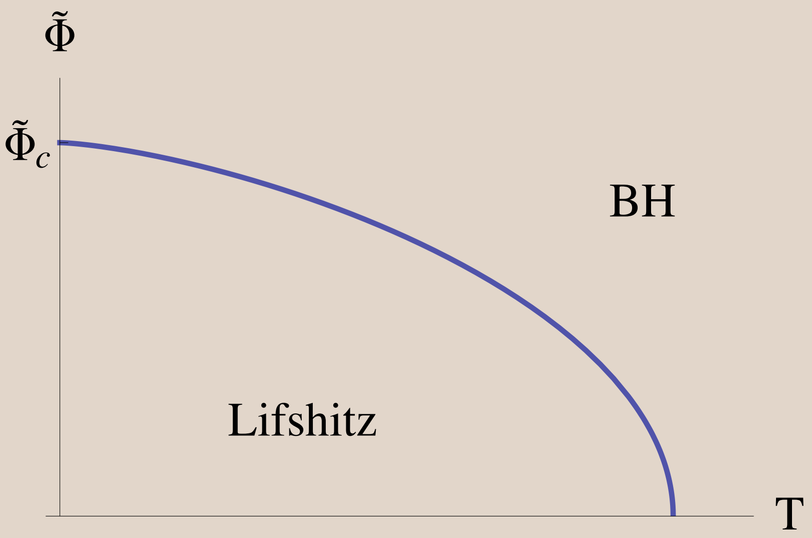
- Thermodynamic relations are satisfied, in particular

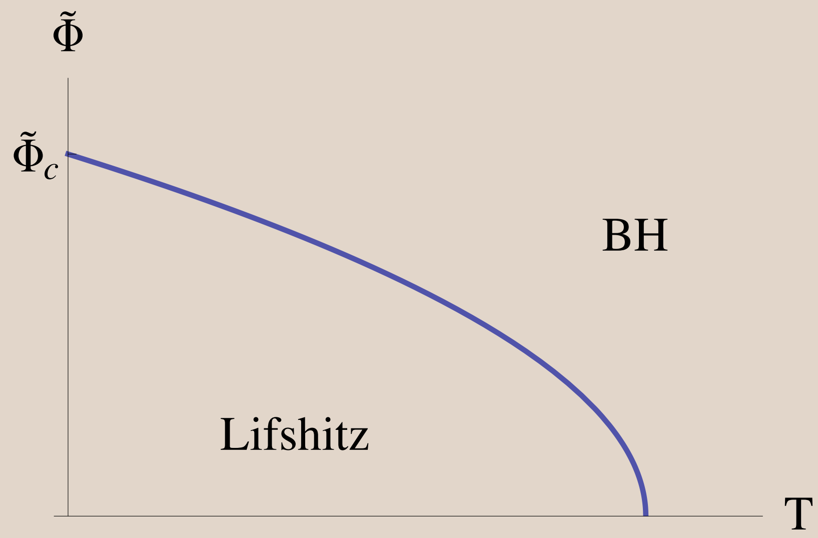
$$W = M - TS - \tilde{\Phi}\tilde{Q}$$

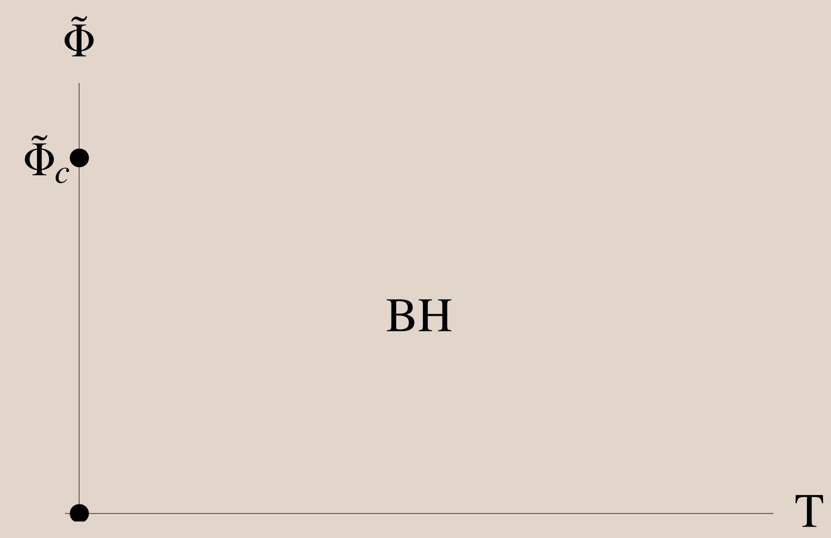
$$F = \Delta M - TS$$

Phase diagrams

- Grand-canonical ensemble





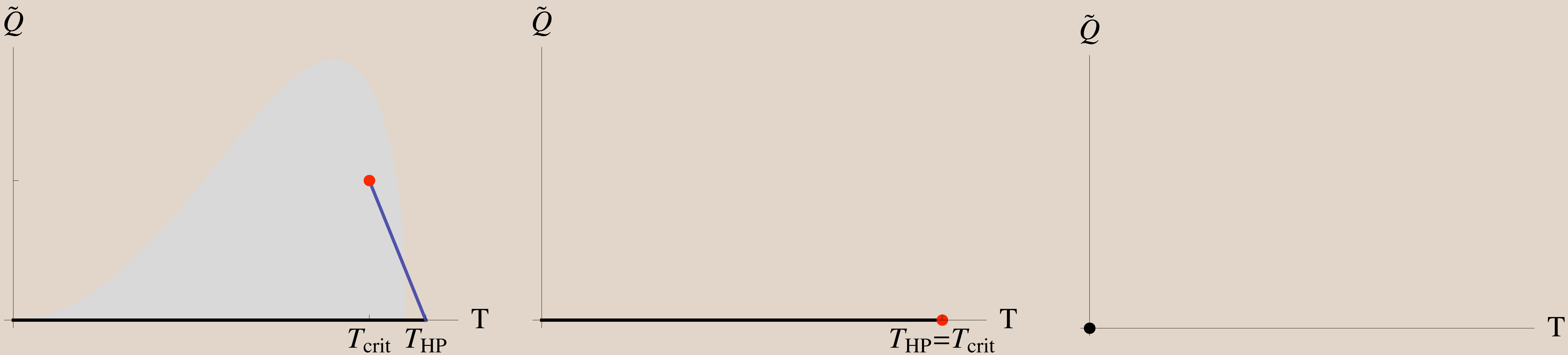


1<=z<2 z=2 z>2

$$\tilde{\Phi}_c^2 = k\frac{2(d-1)(d-2)^2}{(d+z-3)^3}\ell^{2(1-z)}\mu^{-\sqrt{2\frac{z-1}{d-1}}} \ ,$$

Phase diagrams

- Canonical ensemble



1<=z<2 z=2 z>2

$$T_{HP} = \frac{d-1}{2\pi(2-z)\ell}\left[\frac{(2-z)(d-2)^2}{z(d+z-3)^2}\right]^{z/2}$$

Summary and outlook

- We presented a solution for $U(1)^3$ theory with dilatonic couplings; generic z and d
  - Some vector fields just support geometry via Neumann boundary conditions!
  - The same fields have a bad behaviour at the boundary, however...
- Finite on-shell action and mass of b.h. with holographic renormalization: phase diagrams
- If no matter couples to the $\mathcal{A}$ fields and quantites are renormalized, can we make sense of this theory anyway?