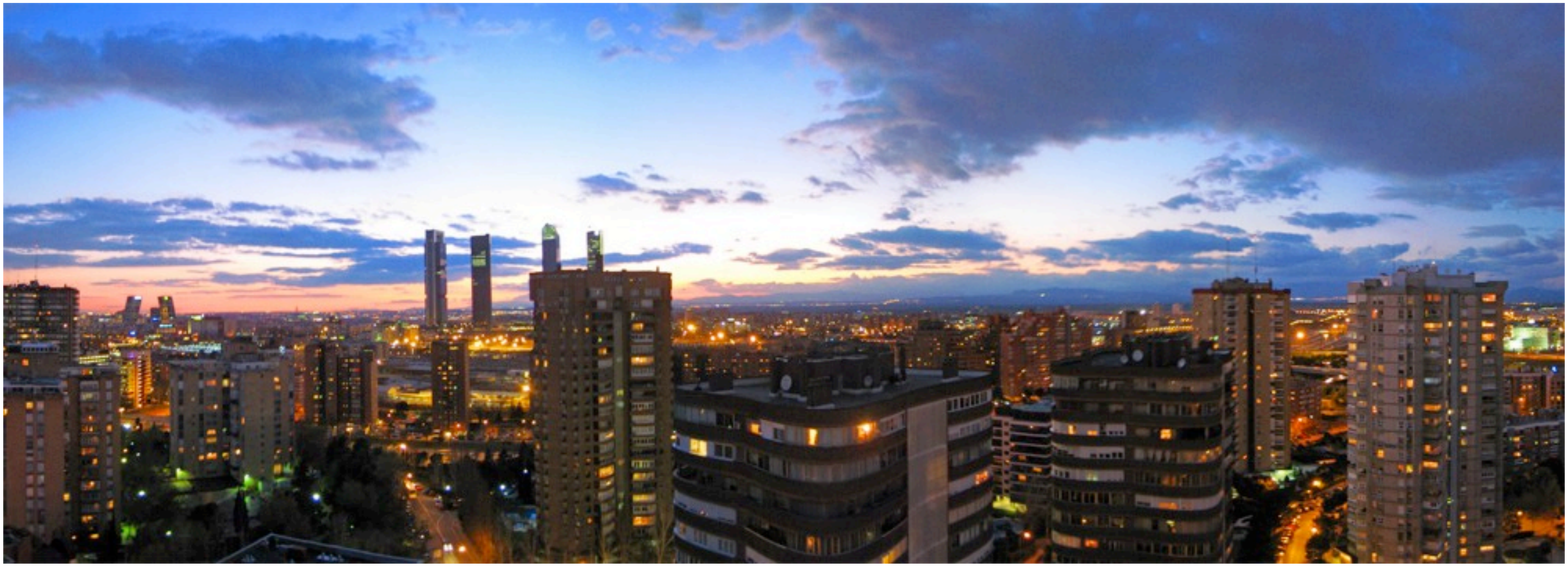
Kaixo, egunon!

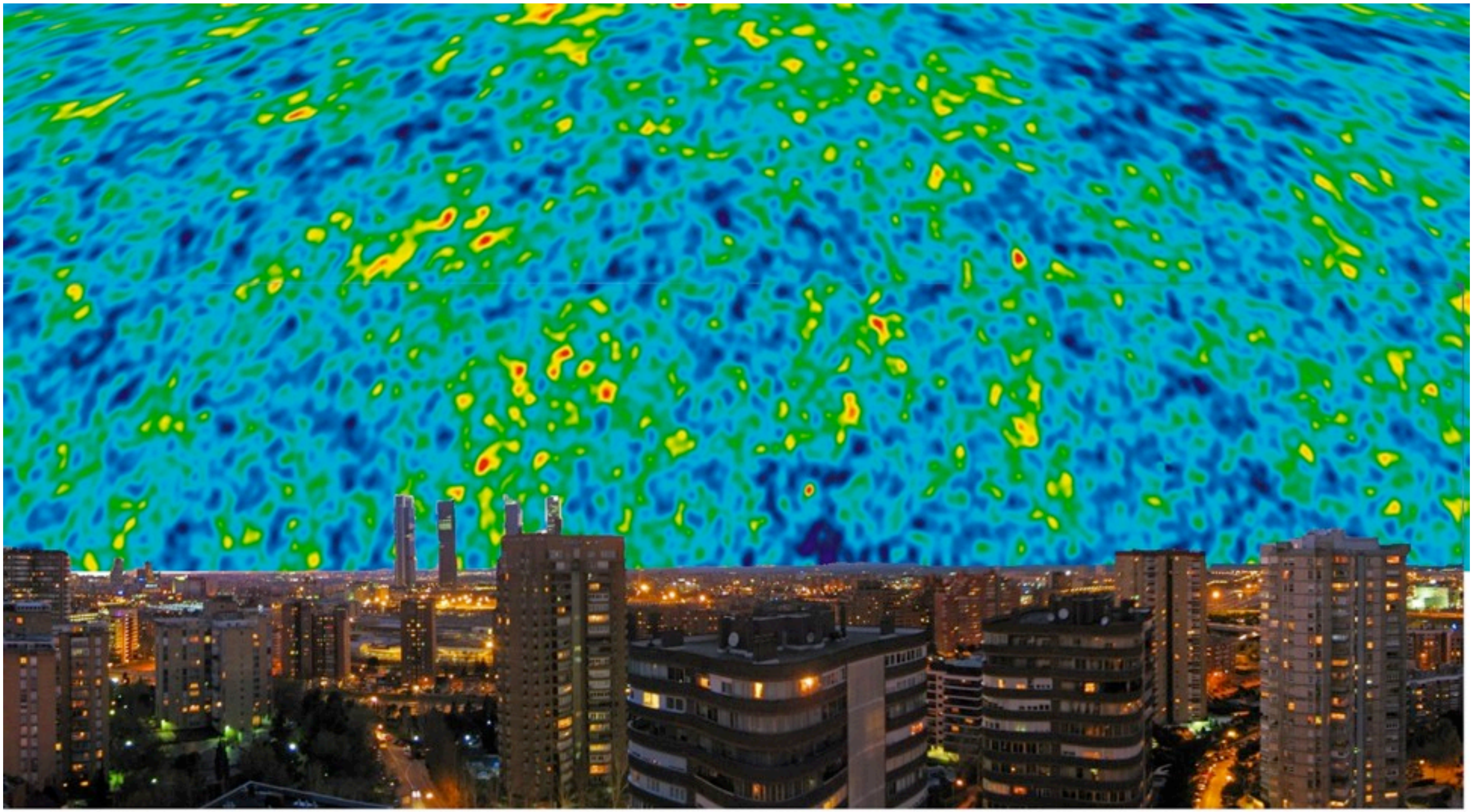
Cosmology strings and supergravity

With Cesar Gomez and Raul Jimenez

A pleasant Madrid evening...

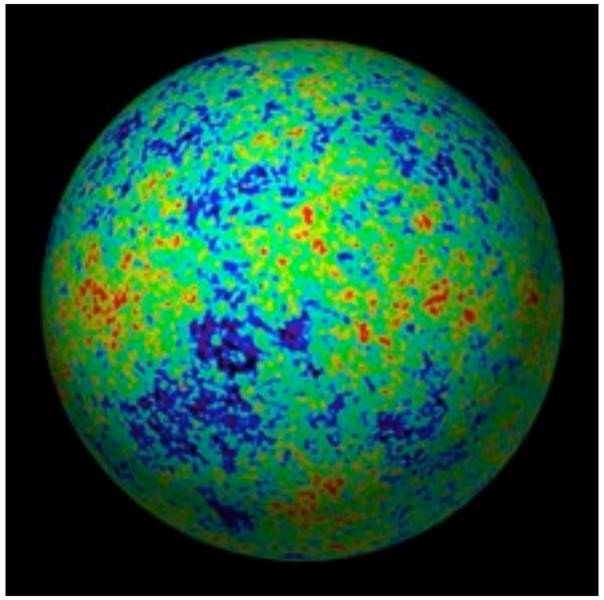


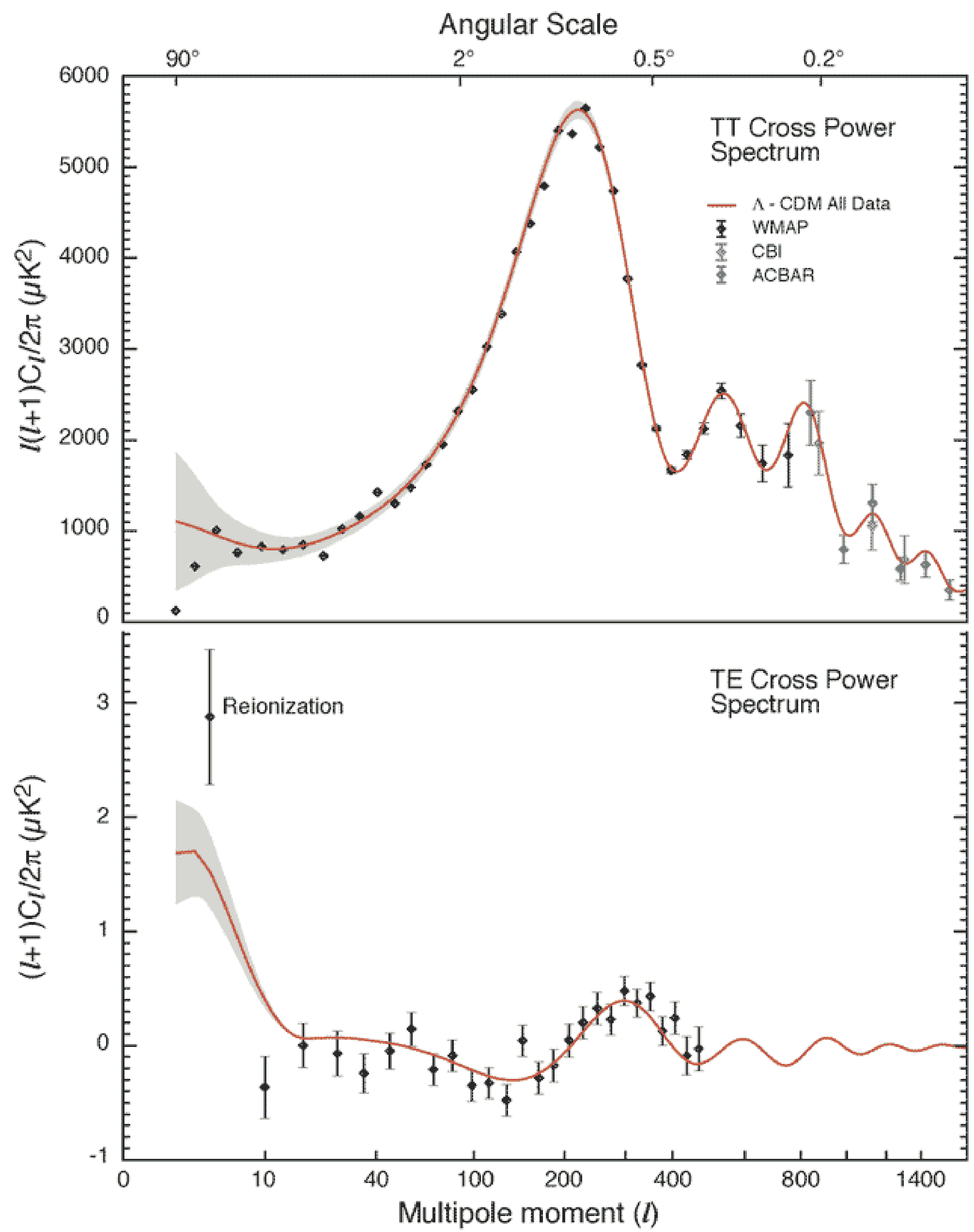
A psychedelic view instead...



Picture created with Raul Jimenez

Baryon acoustic oscillations

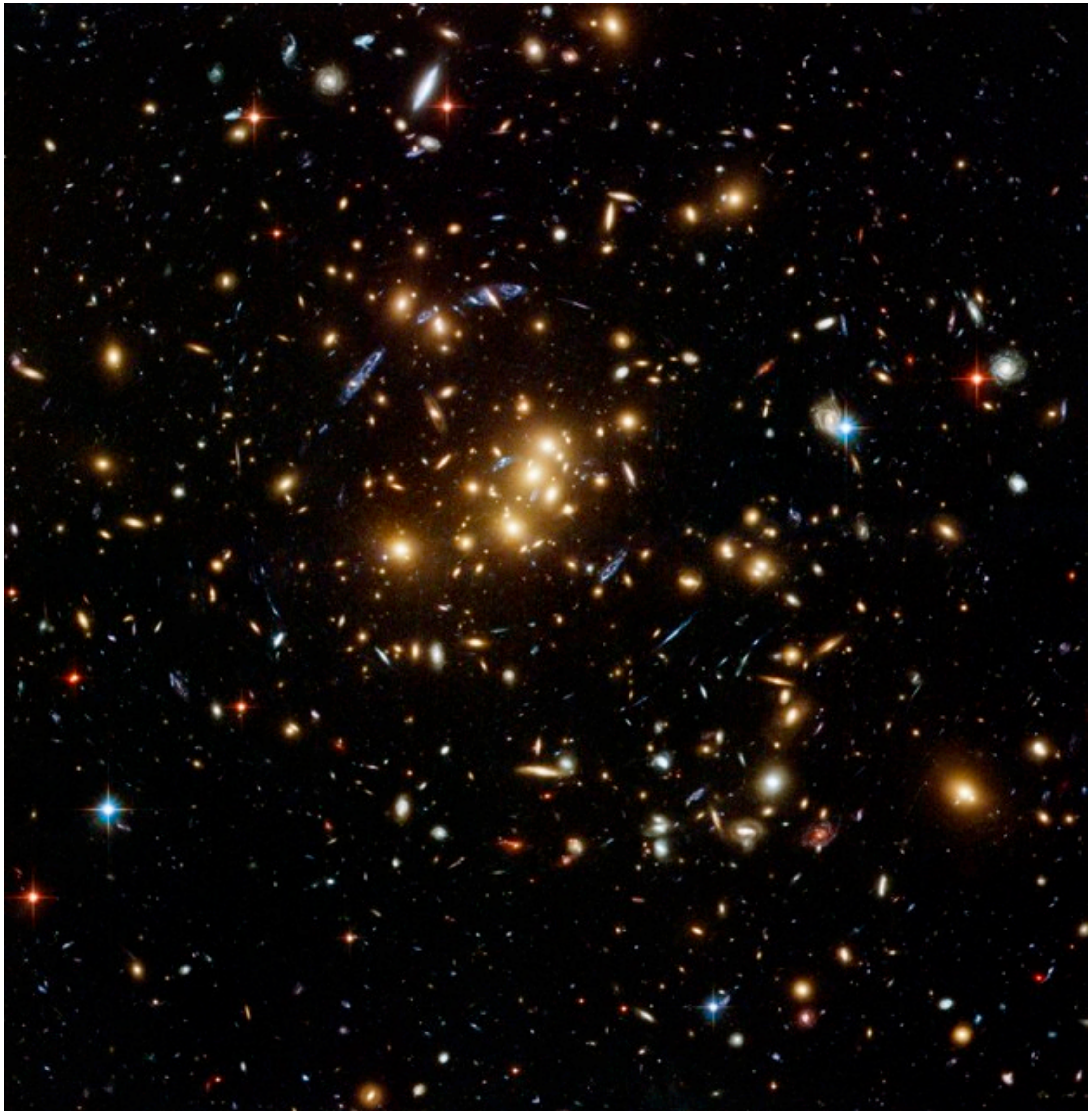




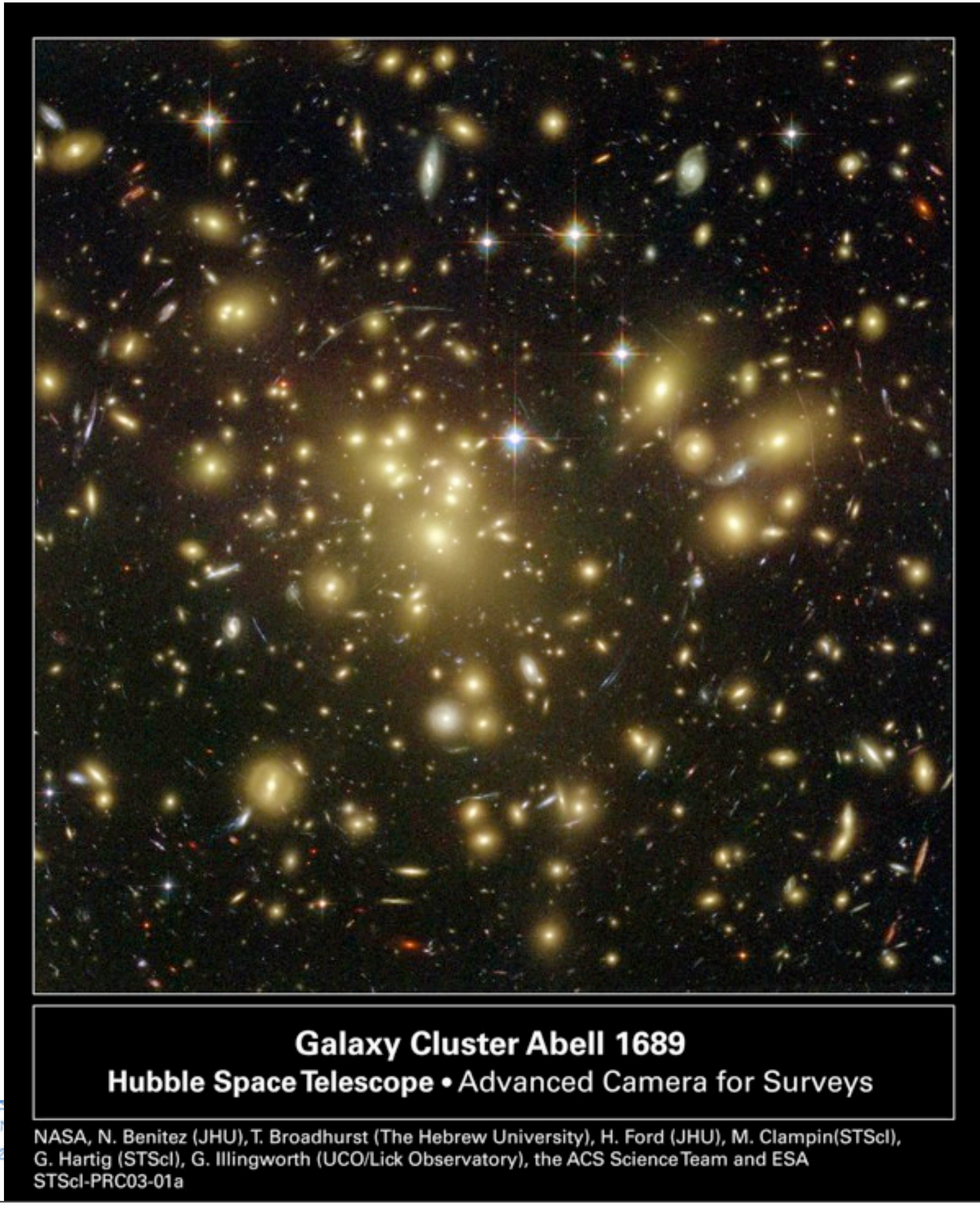
$$\Omega_\Lambda + \Omega_{DM} + \Omega_B = 1$$

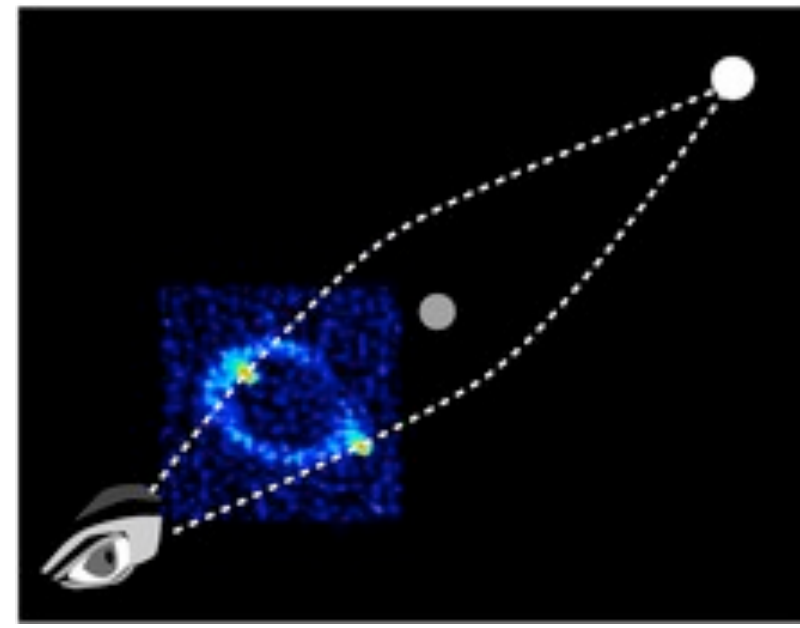
$$73\% + 22.6\% + 4.4\% = 100\%$$

Einstein rings

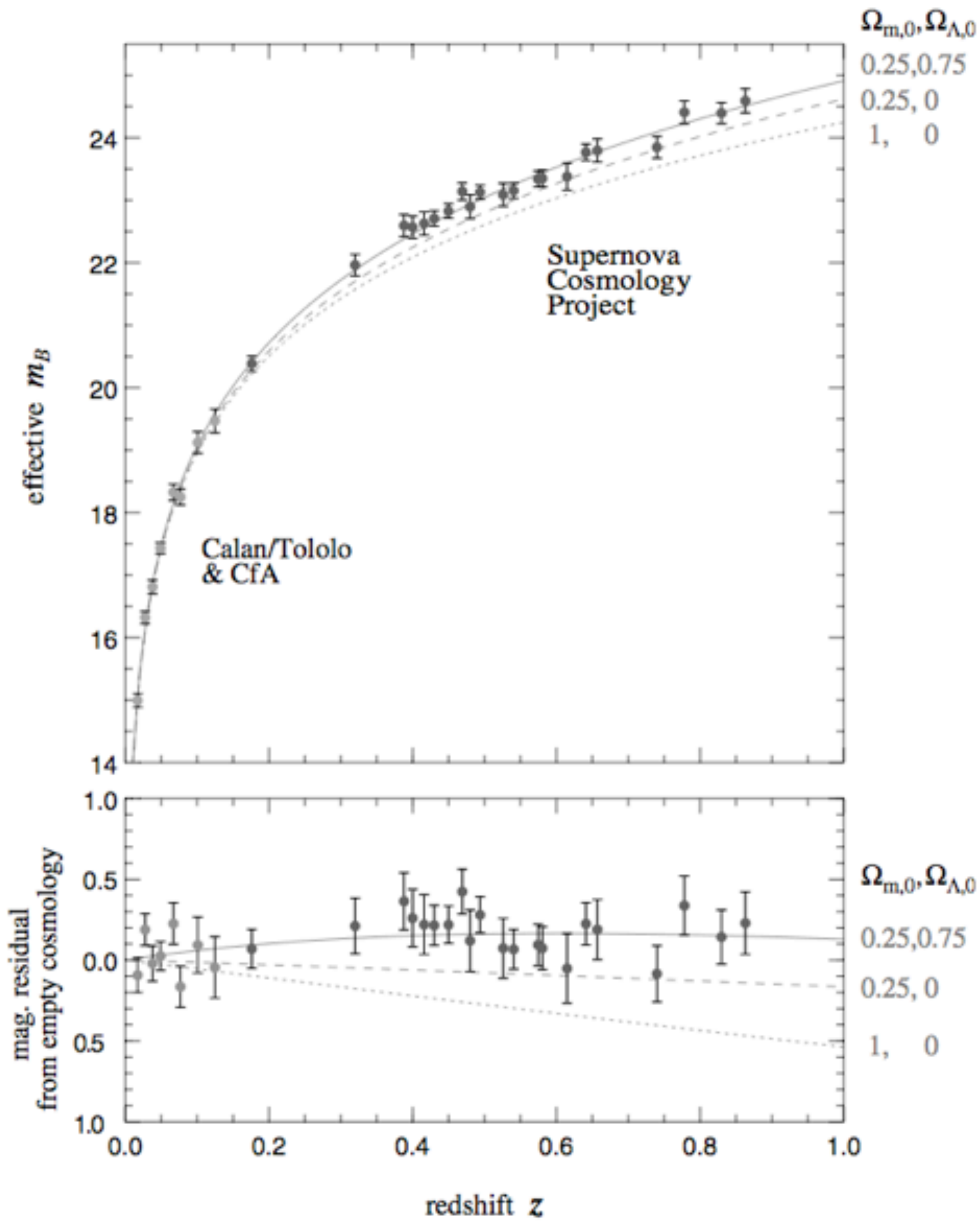


More rings


Galaxy Cluster Abell 1689
Hubble Space Telescope • Advanced Camera for Surveys
NASA, N. Benitez (JHU), T. Broadhurst (The Hebrew University), H. Ford (JHU), M. Clampin(STScI), G. Hartig (STScI), G. Illingworth (UCO/Lick Observatory), the ACS Science Team and ESA
STScI-PRC03-01a



Dark energy



BBN $$\eta_b = \frac{n_b - n_{\bar{b}}}{n_\gamma} = (5.1 \pm 0.5) \times 10^{-10}$$

String theory is ambitious

Resolution of the initial singularity (AdS/CFT ?)

Pre-BB scenarios

Cancellation of the cosmological constant: Brown-Teitelboim Bousso-Polchinski

Brane-inflation: radions and tachyons

Moduli and dilaton stabilisation

Trans-Planckian physics

RS scenarios

Large extra-dimensions…

We still do not have a string-cosmology paradigm

String theory can generate many inflation scenarios and much more...

Inflation is 30 years old

Originally Inflation was related to the horizon, flatness and relic problems

Nowadays, its major claim to fame is seeds of structure. There is more and more evidence that the general philosophy has some elements of truth, and it is remarkably robust...

Matter is well represented by a perfect fluid

$$T_{ab} = (p+\rho)u_a u_b + p g_{ab}$$

The Einstein equations are

$$G_{ab} = 8\pi G\, T_{ab}$$

$$k = \pm 1, 0$$

These are the possible curvatures distinguishing the space sections

$$ds^2 = -dt^2 + a(t)^2\, ds_3^2$$

$$H \equiv \frac{\dot a(t)}{a(t)} \qquad \Omega \equiv \frac{\rho}{\rho_c} \qquad 1-\Omega = \frac{k}{a^2 H^2}$$

$$\rho_c = \frac{3H^2}{8\pi G}$$

$$\left(\frac{\dot a}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho$$

$$\frac{\ddot a}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

$$\dot\rho + 3H(\rho+p) = 0$$

$$p = w\,\rho$$

$$\dot\rho + 3\frac{\dot a(t)}{a(t)}(1+w)\rho = 0$$

$$\rho = \rho_0\left(\frac{a_0}{a}\right)^{3(1+w)}$$

$$S = \frac{1}{16\pi G}\int\sqrt{-g}(R-2\Lambda) + \int\sqrt{-g}\left(-\frac{1}{2}g^{ab}\partial_a\phi\partial_b\phi - V(\phi)\right)$$

$$\rho = \frac{1}{2}\dot\phi^2 + V(\phi) \qquad p = \frac{1}{2}\dot\phi^2 - V(\phi)$$

Accelerating the Universe

We need to violate the dominant energy condition for a sufficiently long time.

If we take during inflation H approx. constant, the number of e-foldings:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

$$p < -\frac{1}{3}\rho \quad \Rightarrow \quad \boxed{\ddot{a}(t) > 0}$$

$$|1-\Omega| \propto \dot{a}^{-2}$$

$$N = \log\left(\frac{a(t_f)}{a(t_i)}\right) = H(t_f - t_i)$$

$$|1-\Omega(t_f)| = e^{-2N}|1-\Omega(t_i)|$$

Number of e-foldings could be 50-100, making a huge Universe.

We can use QFT to construct some models

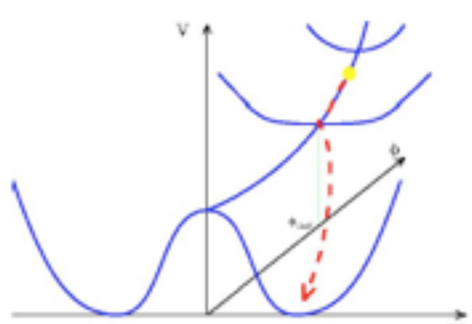
$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) \qquad p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

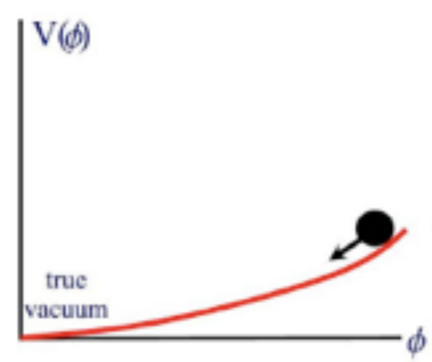
$$S = \frac{1}{16\pi G}\int\sqrt{-g}(R-2\Lambda) + \int\sqrt{-g}\left(-\frac{1}{2}g^{ab}\partial_a\phi\partial_b\phi - V(\phi)\right)$$

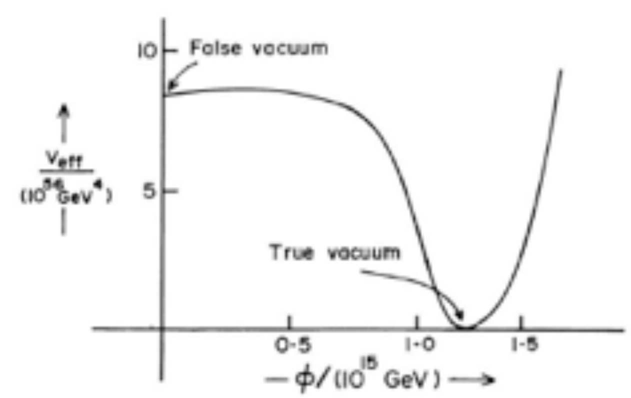
Slow roll paradigm
Hybrid inflation

$$\dot{\phi}^2 << V(\phi)$$

$$\frac{V''}{V} << 1, \qquad \frac{V'}{V} << 1$$







(Courtesy of Licia Verde)

Origins of Inflation

The number of models trying generating inflation is enormous. Frequently they are not very compelling and with large fine tunings.

Different UV completions of the SM provide alternative scenarios for cosmology, and it makes sense to explore their cosmic consequences.

Basic properties

Enough slow-roll to generate the necessary number of e-foldings and the necessary seeds for structure.

A (not so-) graceful exit from inflation, otherwise we are left with nothing.

A way of converting "CC" into useful energy: reheating.

Everyone tries to find "natural" mechanisms within its favourite theory.

SSB Scenarios

Observable Sector

Gauge mediation

Gravity mediation

Anomaly mediation

MEDIATOR

Hidden Sector

It is normally assumed that SSB takes places at scales well below the Planck scale. The universal prediction is then the existence of a massless goldstino that is eaten by the gravitino. However in the scenario considered, the low-energy gravitino couplings are dominated by its goldstino component and can be analyzed also in the global limit.

This often goes under the name of the Akulov-Volkov lagrangian, or the non-linear realization of SUSY

$$m_{3/2} = \frac{f}{M_p} = \frac{\mu^2}{M_p}$$

$$\mu \to \infty$$
$$M \to \infty \qquad m_{3/2} \text{ fixed}$$

Flat directions

One reason to use SUSY in inflationary theories is the abundance of flat directions. Once SUSY breaks most flat directions are lifted, sometime by non-perturbative effects. However, the slopes in the potential can be maintained reasonably gentle without excessive fine-tuning.

For flat Kahler potentials, and F-term breaking, there is always a complex flat direction in the potential. A general way of getting PSGB, the key to most susy models.The property below holds for any W breaking SUSY.

Most models of supersymmetric inflation are hybrid models (multi-field models, chaotic, waterfall...)

$$\begin{aligned} F &= -\partial W(\phi) \\ &\;. \\ V &= \partial W(\phi)\,\overline{\partial W(\phi)} \end{aligned}$$

$$V(\phi + z\langle F\rangle) = V(\phi)$$

Important properties of SSB

The Akulov-Volkov-type actions provide the correct framework to analyse the general properties of SSB. We can use the recent Komargodski-Seiberg presentation.

The starting point of their analysis is the Ferrara-Zumino (FZ) multiplet of currents that contains the energy-momentum tensor, the supercurrent and the R-symmetry current

$$J_\mu = j_\mu + \theta^\alpha S_{\mu\alpha} + \overline{\theta}_{\dot{\alpha}} \overline{S}^{\dot{\alpha}}_\mu + (\theta\sigma^\nu\overline{\theta})\, 2T_{\nu\mu} + \ldots$$

$$X = x(y) + \sqrt{2}\theta\psi(y) + \theta^2 F(y)$$

$$\psi_\alpha = \frac{\sqrt{2}}{3}\sigma^\mu_{\alpha\dot{\alpha}}\overline{S}^{\dot{\alpha}}_\mu, \qquad F = \frac{2}{3}T + i\partial_\mu j^\mu$$

$$\overline{D}^{\dot{\alpha}} J_{\alpha\dot{\alpha}} = D_\alpha X$$

General Lagrangian

$$\mathrm{S} = \int d^4\theta K(\Phi^i, \bar{\Phi}^{\bar{i}}) + \int d^2\theta W(\Phi^i) + \int d^2\bar{\theta}\bar{W}(\bar{\Phi}^{\bar{i}})$$

$$\mathrm{J}_{\alpha\dot{\alpha}} = 2g_i(D_\alpha\Phi^i)(\bar{D}_{\dot{\alpha}}\bar{\Phi}) - \tfrac{2}{3}[D_\alpha, \bar{D}_{\dot{\alpha}}]K + i\partial_\alpha(Y(\Phi) - \bar{Y}(\bar{\Phi}))$$

$$X = 4W - \frac{1}{3}\overline{D}^2 K - \frac{1}{2}\overline{D}^2 Y(\Phi)$$

X is a chiral superfield, microscopically it contains the conformal anomaly (the anomaly multiplet), hence it contains the order parameter for SUSY breaking as well as the goldstino field. It may be elementary in the UV, but composite in the IR. Generically its scalar component is a PSGB in the UV. This is our inflaton. The difficulty with this approach is that WE WANT TO BREAK SUSY ONLY ONCE! unlike other scenarios in the literature

The key observation is: X is essentially unique, and:

$$X \to X_{NL} \qquad X_{NL}^2 = 0$$

$$UV \to IR$$

$$SPoincare/Poincare$$

Some IR consequences

$$L = \int d^4\theta \, X_{NL} \overline{X}_{NL} + \int d^2\theta \, f \, X_{NL} + c.c.$$

$$X_{NL} = \frac{G^2}{2F} + \sqrt{2}\theta G + \theta^2 F$$

This is precisely the Akulov-Volkov Lagrangian

Coupling goldstinos to other fields: reheating

We can have two regimes of interest. Recall that a useful way to express SUSY breaking effects in Lagrangians is the use of spurion fields. The gluino mass can also be included...

$$m_{soft} << E << \Lambda$$

The goldstino superfield is the spurion

$$\int d^4\theta \left|\frac{X_{NL}}{f}\right| m^2 \, Q e^V \overline{Q} + \int d^2\theta \frac{X_{NL}}{f}(B\, Q\, Q + AQ\, Q\, Q\,) + c.c.$$

$$E << m_{soft}$$

Integrate out the massive superpartners adding extra non-linear constraints

$$X_{NL}^2 \; = \; 0, \qquad X_{NL}\, Q_{NL} \; = \; 0$$

For light fermions, and similar conditions for scalars, gauge fields,...

Reheating depends very much on the details of the model, as does CP violation, baryogenesis...

Some details

An important part of our analysis is the fact that the graceful exit is provided by the Fermi pressure in the Landau liquid in which the state of the X-field converts once we reach the NL-regime. This is a little crazy, but very minimal however...

$$K(X,\bar{X}) \;=\; X\bar{X}\left(1+\frac{a(X+\bar{X})}{2M}-\frac{bX\bar{X}}{6M^2}-\frac{c\left(X^2+\bar{X}^2\right)}{9M^2}+\dots\right)-2M^2\log\left(\frac{X+\bar{X}}{M}+1\right)$$

$$W(X) \;=\; f_0 \,+\, f\,X$$

$$V = e^{\frac{K}{M^2}}(K^{-1}_{X,\bar{X}}DW\bar{D}W - \frac{3}{M^2}|W|^2) \qquad\qquad D\,W \;=\; \partial_X\,W \,+\, \frac{1}{M^2}\partial_X K\,W$$

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The full lagrangian

$$-\frac{1}{2}R + \frac{1}{2}\varepsilon^{mnpq}\left(\bar\psi_m \bar\sigma_n \nabla_p \psi_q - \psi_m \sigma_n \nabla_p \bar\psi_q\right)$$

$$-G_{i\bar j}\partial^m A^i \partial_m \bar A^{\bar j} - \frac{i}{2}G_{i\bar j}\left(\chi^i\sigma^m \nabla_m \bar\chi^{\bar j} + \bar\chi^{\bar j}\bar\sigma^m\nabla_m\chi^i\right)$$

$$-\frac{1}{\sqrt2}\left(\bar\psi_m\bar\sigma^n\sigma^m\bar\chi^{\bar j}\right)G_{i\bar j}\partial_n A^i - \frac{1}{\sqrt2}\left(\psi_m\sigma^n\bar\sigma^m\chi^i\right)G_{i\bar j}\partial_n\bar A^{\bar j}$$

$$+e^{K/2}\Big[\frac{i}{\sqrt2}\left(\bar\psi_m\bar\sigma^m\chi^k\right)D_kW + \frac{i}{\sqrt2}\left(\psi_m\sigma^m\bar\chi^{\bar k}\right)\overline{D_kW}$$

$$-\bar\psi_m\bar\sigma^{mn}\bar\psi_n W + \psi_m\sigma^{mn}\psi_n\bar W\Big]$$

$$-\frac{i}{2}G_{i\bar j}\varepsilon^{klmn}\left(\chi^i\sigma_k\bar\chi^{\bar j}\right)\left(\psi_l\sigma_m\bar\psi_n\right) - \frac{1}{2}G_{i\bar j}g^{mn}\left(\psi_m\chi^i\right)\left(\bar\psi_n\bar\chi^{\bar j}\right)$$

$$-\frac{1}{3}M\bar M - e^{K/2}\left(M\bar W + \bar M W\right) + \frac{1}{3}b^a b_a - \frac{b_a}{2}G_{i\bar j}\chi^i\sigma^a\bar\chi^{\bar j}$$

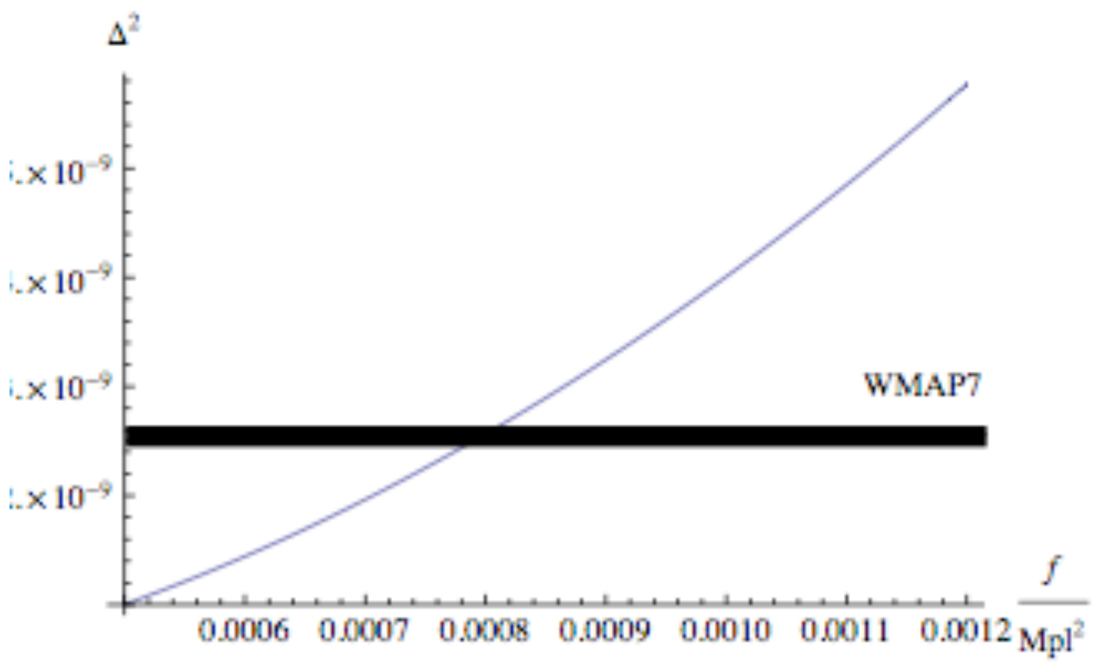
$$-\frac{1}{2}e^{K/2}\left(\partial_i + K_i\right)D_jW\,\chi^i\chi^j + h.c.$$

$$+\frac{1}{4}\partial^2_{m\bar k}G_{n\bar l}\,\chi^m\chi^n\bar\chi^{\bar k}\bar\chi^{\bar l} + G_{i\bar j}F^i\bar F^{\bar j}$$

$$+e^{K/2}F^iD_iW - \frac{1}{2}F^i\Gamma_{i,\bar j\bar k}\bar\chi^{\bar j}\bar\chi^{\bar l}$$

$$+e^{K/2}\bar F^{\bar i}\,\overline{D_iW} - \frac{1}{2}\bar F^{\bar i}\Gamma_{\bar i,jk}\chi^j\chi^k$$

Primordial density fluctuations



$$\sqrt{f} \sim 10^{11-13}\,\mathrm{GeV}$$

$$\epsilon = \frac{M_{pl}^2}{2}\left(\frac{V'}{V}\right)^2, \qquad \eta = M_{pl}^2 \frac{V''}{V};$$

$$\begin{aligned} n_S &= 1 - 6\epsilon + 2\eta, \\ r &= 16\epsilon \\ n_t &= -2\epsilon, \\ \Delta_R^2 &= \frac{V M_{pl}^4}{24\pi^2\epsilon}. \end{aligned}$$

Choosing useful variables

$$z = M(\alpha + i\beta)/\sqrt{2}$$

$$ds^2 = 2g_{z\bar{z}}dsd\bar{z} = \partial_z\partial_{\bar{z}}K(\alpha,\beta)M^2(d\alpha^2 + d\beta^2)$$

$$S = L^3 \int dta^3 \left(\frac{1}{2}g(\alpha,\beta)M^2(\dot{\alpha}^2 + \dot{\beta}^2) - f^2V(\alpha,\beta)\right)$$

$$t = \tau M/f \qquad S = L^3 f^2 m_{3/2}^{-1} \int d\tau a^3 \left(\frac{1}{2}g(\alpha,\beta)(\alpha'^2 + \beta'^2) - V(\alpha,\beta)\right)$$

The full equations of motion, neglecting fermions for the moment are:

$$
\begin{aligned}
\alpha'' + 3\frac{a'}{a}\alpha' + \frac{1}{2}\partial_\alpha \log g(\alpha'^2 - \beta'^2) + \partial_\beta \log g\alpha'\beta' + g^{-1}V'_\alpha &= 0 \\
\beta'' + 3\frac{a'}{a}\beta' + \frac{1}{2}\partial_\beta \log g(\beta'^2 - \alpha'^2) + \partial_\alpha \log g\alpha'\beta' + g^{-1}V'_\beta &= 0 \\
\frac{a'}{a} = \frac{H}{M_{3/2}} = \frac{1}{\sqrt{3}}\left(\frac{1}{2}g(\alpha'^2 + \beta'^2) + V(\alpha,\beta)\right)
\end{aligned}
$$

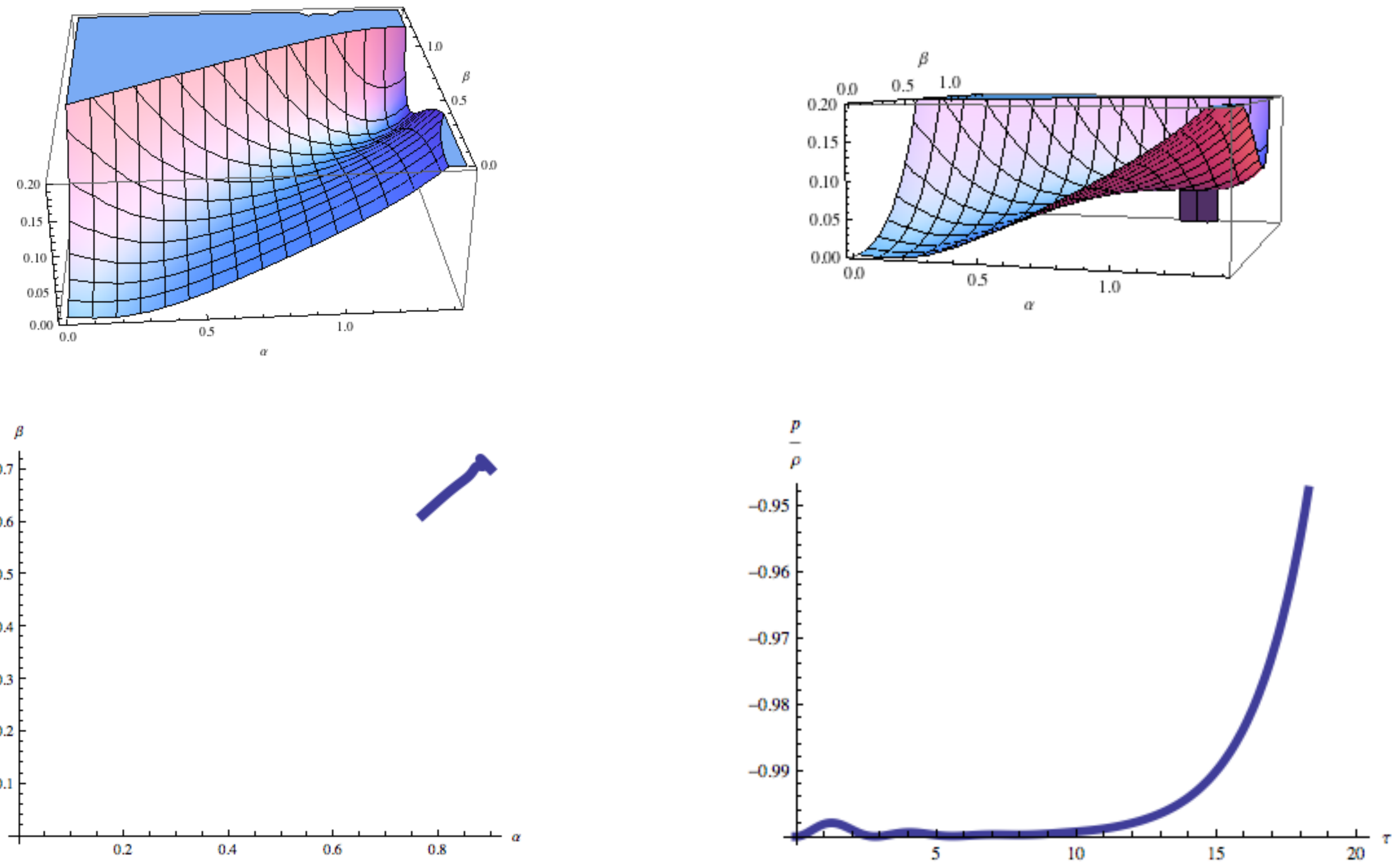
Plus fermion terms on the RHS

Looking for the attractor and slow roll implies that the geodesic equation on the target manifold is satisfied for a particular set of initial conditions. This determines the attractor trajectories in general for any model of hybrid inflation. Numerical integration shows how it works. We have not tried to prove "theorems' but there should be general ways of showing how the attractor is obtained this way

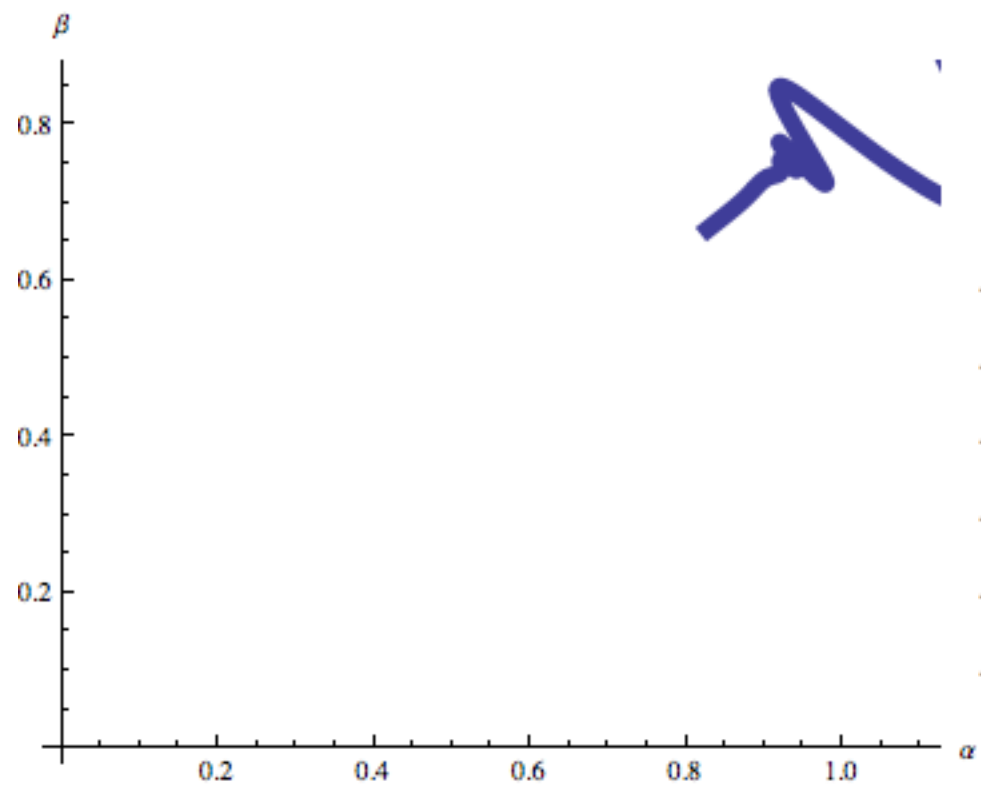
$$D\dot{\Phi}^i/dt \sim 0$$

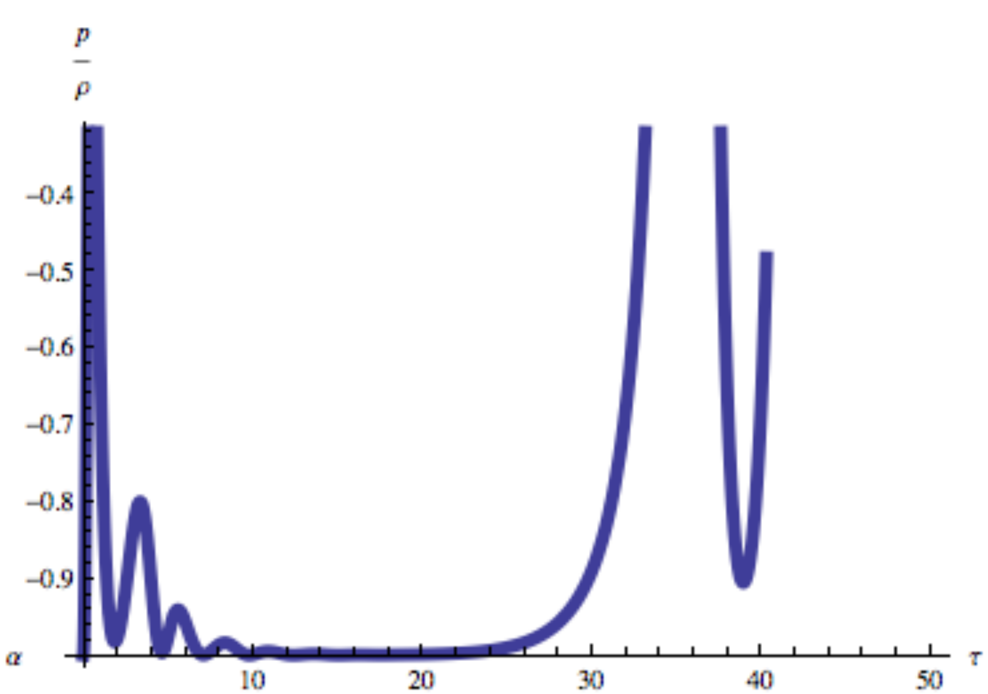
$$H = \sqrt{\frac{1}{18}}\left(3V + \sqrt{6V' + 9V^2}\right)$$

Trajectories with more than the observed 12 e-folding a=0, b=1, c=-1.5

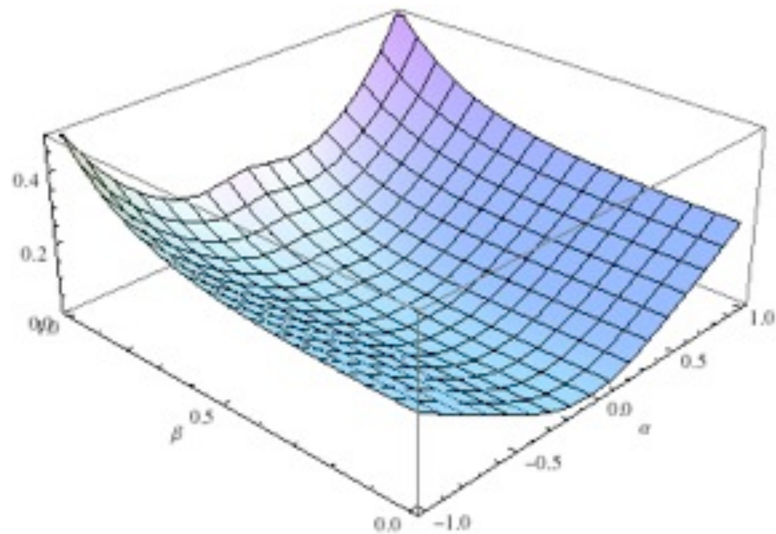


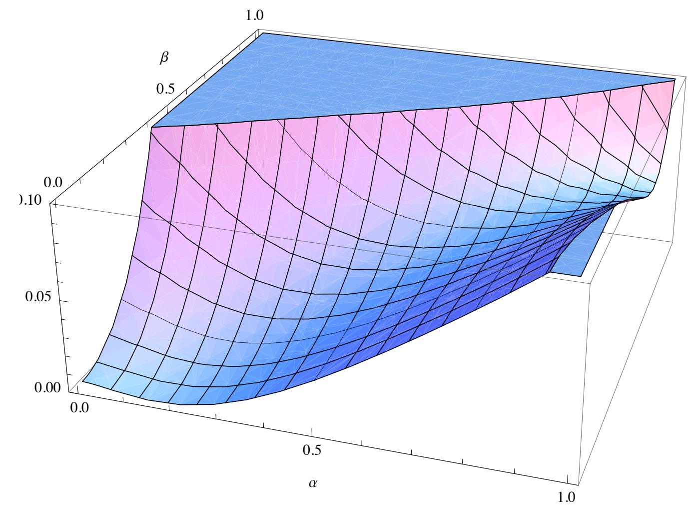
One more example





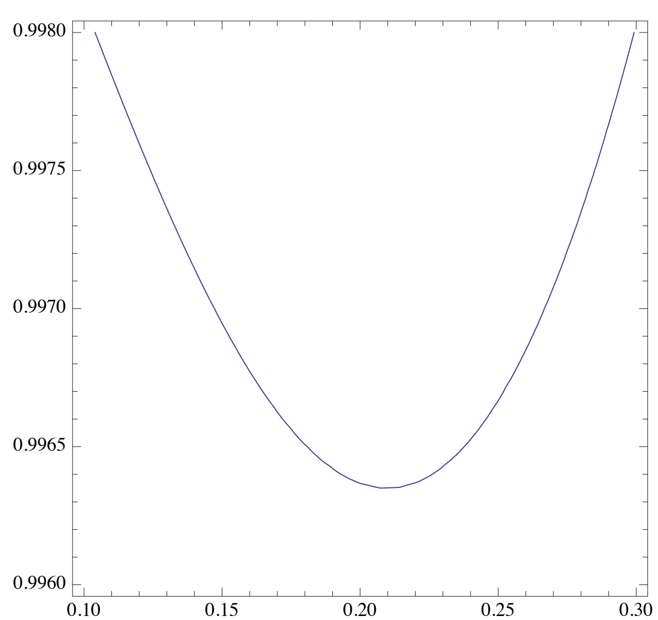
The scalar potential

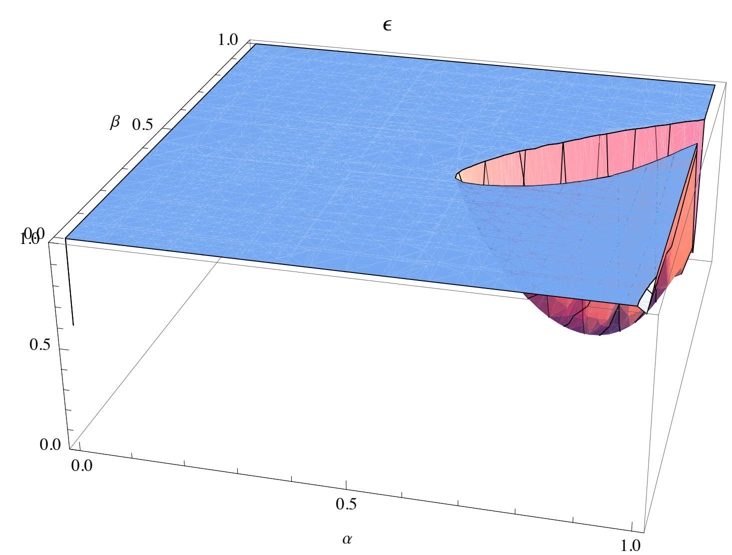




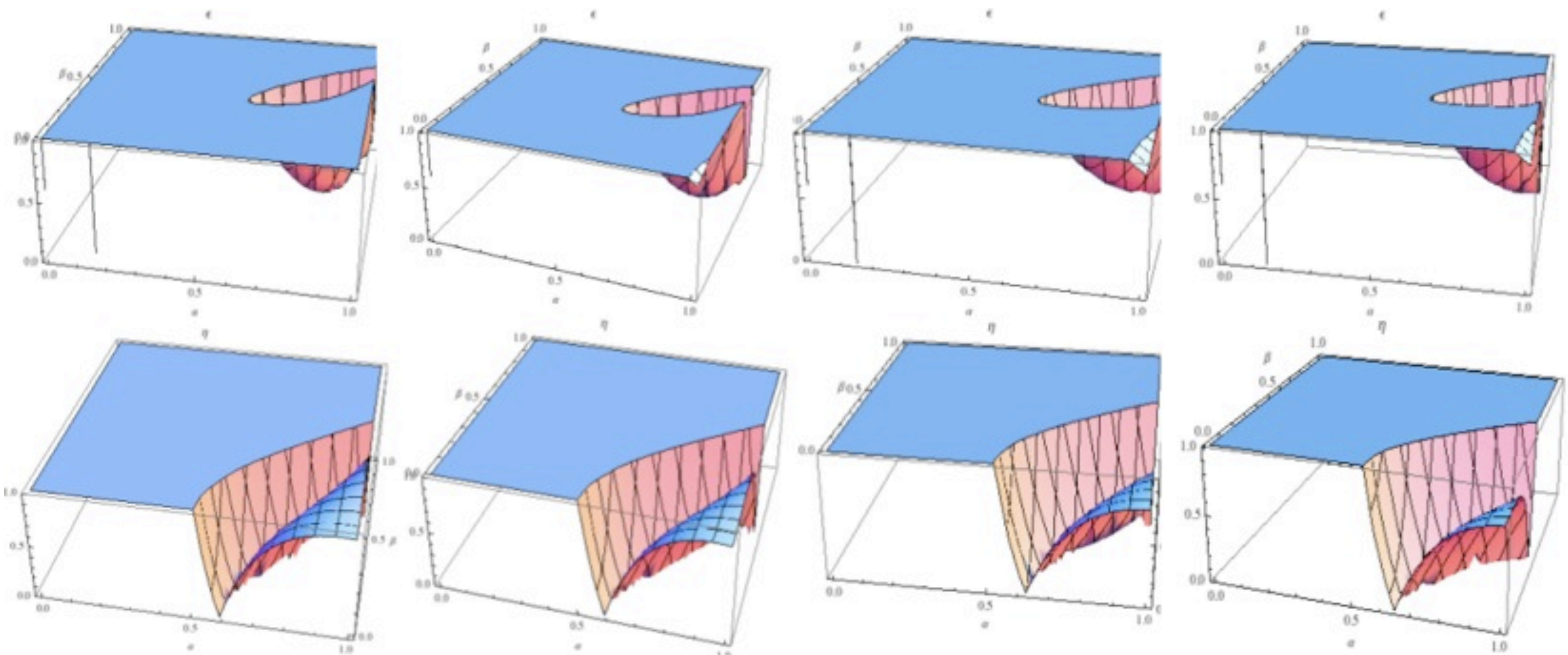
a=0, b=1, c=-1.7

a=0, b=1, c=0

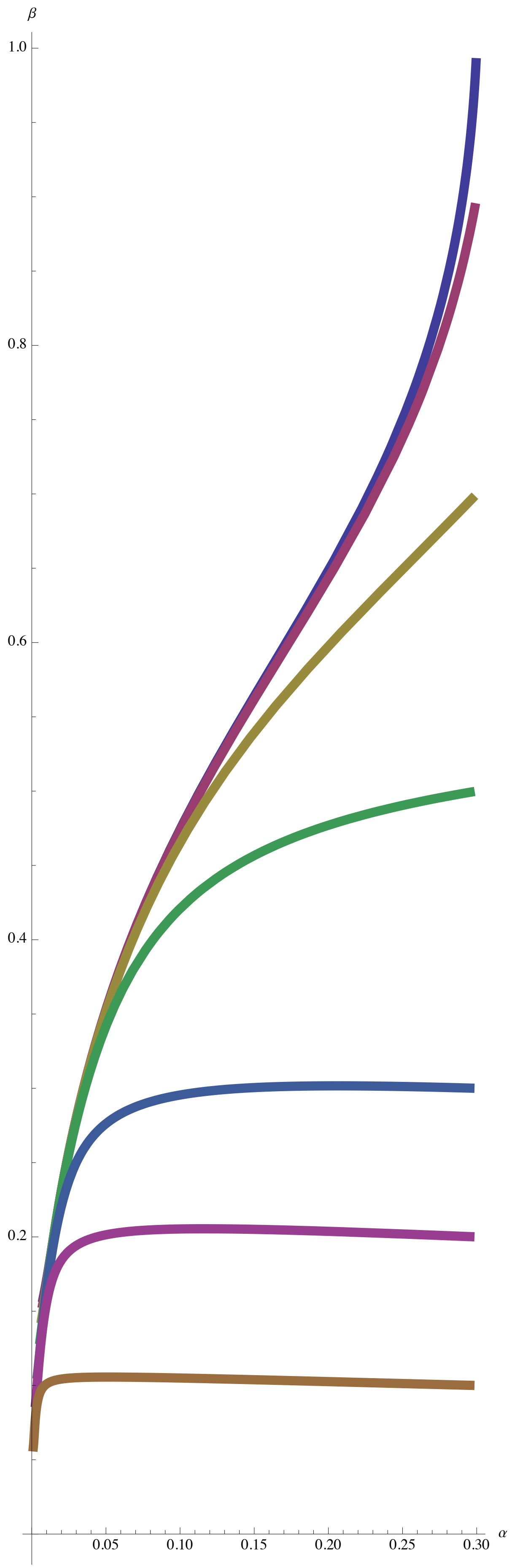


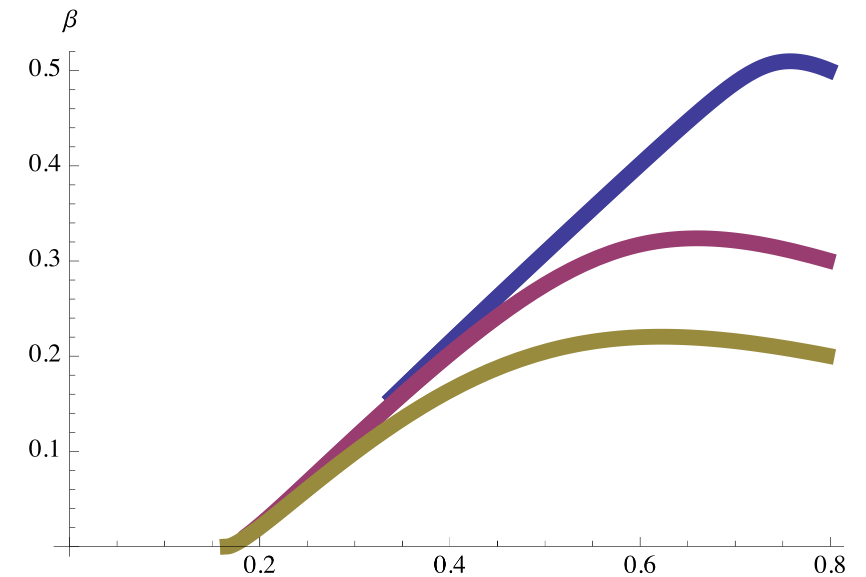


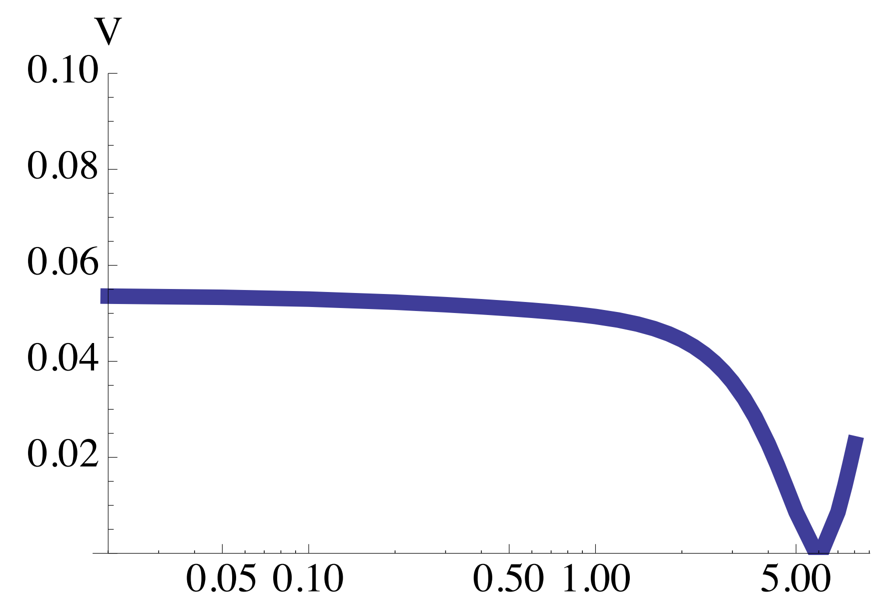
Epsilon and eta



Attractor and inflationary trajectories







Nearly a textbook example of inflationary potential

Decoupling and the Fermi sphere

$$\eta = (\frac{m_{\rm INF}}{m_{3/2}})^2$$

The energy density in the universe (f^2) contained in the coherent X-field quickly transforms into a Fermi sea whose level is not difficult to compute, we match the high energy theory dominated by the X-field and the Goldstino Fock vacuum into a theory where effectively the scalar has disappeared and we get a Fermi sea, whose Fermi momentum is

$$q_F = \sqrt{\frac{f}{\eta}}$$

To produce the observed number of particles in the universe leads to gravitino masses in the 10-100 TeV region.

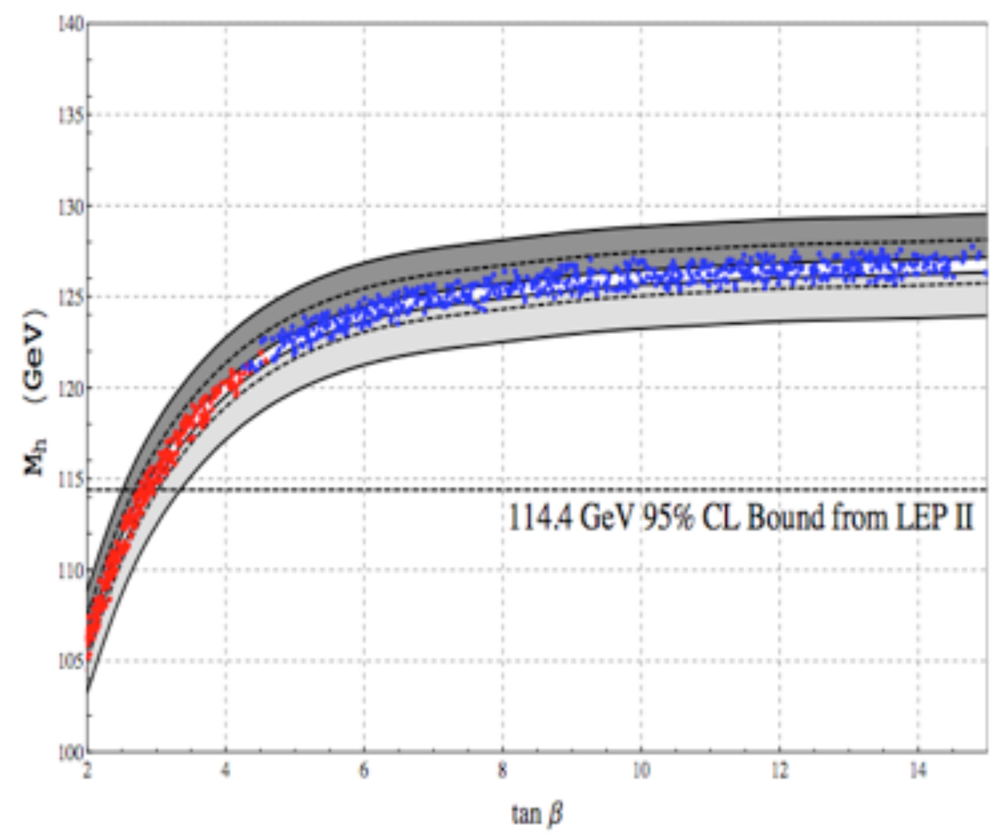
Summary of our scenario

We take as the basic object the X field containing the Goldstino. Its scalar component above SSB behaves like a PSGB and drives inflation

Its non-linear conversion into a Landau liquid in the NL regime provides an original graceful exit, in our case the conversion is not complete and we get a dark universe with goldstinos and inflatons

Reheating can be obtained through the usual Goldstino coupling to low energy matter

In the simplest of all possible such scenarios, the Susy breaking scale is fitted to be of the order of 10^{13-14} GeV, m of the order of 10-100 TeV (the plot from Kane et al)



Luis Alvarez-Gaume, Bilbao 1 Feb. 2012

Eskerrik asko!