Deeply Virtual Compton Scattering in Gauge/Gravity Duality

Marko Djurić

Centro de Física do Porto

work with Miguel S. Costa

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U.PORTO
FACULDADE DE CIÊNCIAS
UNIVERSIDADE DO PORTO

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- To do this we will use string theory.
- More specifically, a recent conjecture by Maldacena relating string theory on $AdS_5 \times S_5$ to $\mathcal{N} = 4SYM$ allows us to study QCD at strong coupling.

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- To $\mathcal{O}(\lambda)$

$$\alpha(0) \simeq 1 + \frac{\log 2}{\pi^2}\lambda$$

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- Correspondence works in the limit

$$N_C \to \infty, \quad \lambda = g^2 N_C = R^4/\alpha'^2 \gg 1, \text{fixed}$$

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where

$$Im\ \chi(\tau, L) = g_0^2 e^{(1-\rho)\tau} \frac{L}{\sinh L} \frac{\exp(\frac{-L^2}{\rho\tau})}{(\rho\tau)^{3/2}}$$

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- In the limit $\tau \gg 1$, $\lambda \gg 1$ and $\lambda/\tau \to 0$

$$\Re\chi \approx \cot(\frac{\pi\rho}{2})\Im\chi$$

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- At $t = 0$
  Weak coupling:

$$\mathcal{K}(k_\perp, k'_\perp, s) = \frac{s^{j_0}}{\sqrt{4\pi\mathcal{D}\log s}} e^{-(\log k_\perp - \log k'_\perp)^2/4\mathcal{D}\log s}$$

$$j_0 = 1 + \frac{\log 2}{\pi^2}\lambda, \quad \mathcal{D} = \frac{14\zeta(3)}{\pi}\lambda/4\pi^2$$

  Strong coupling:

$$\mathcal{K}(z, z', s) = \frac{s^{j_0}}{\sqrt{4\pi\mathcal{D}\log s}} e^{-(\log z - \log z')^2/4\mathcal{D}\log s}$$

$$j_0 = 2 - \frac{2}{\sqrt{\lambda}}, \quad \mathcal{D} = \frac{1}{2\sqrt{\lambda}}$$

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- We can expand the exponential to get

$$A(s, -\mathbf{q}_\perp{}^2) = -2is \int d^2l e^{-i\mathbf{l}_\perp \cdot \mathbf{q}_\perp} (i\chi + \frac{(i\chi)^2}{2} + \cdots) \, .$$

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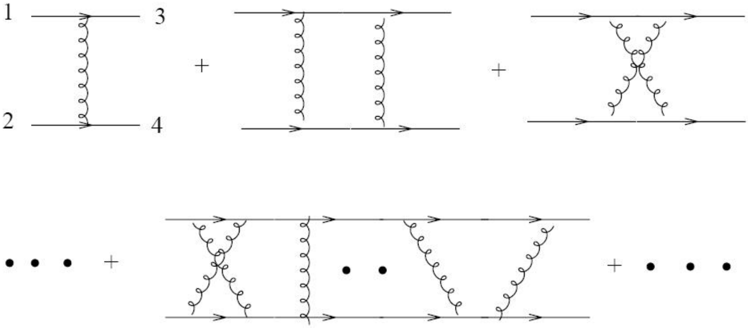

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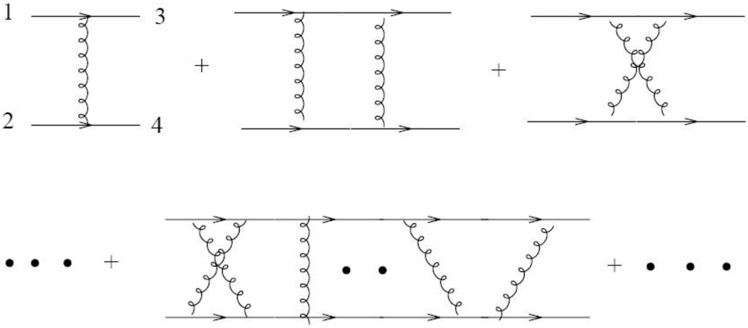

- Eikonal approximation in AdS space (Brower, Strassler, Tan; Cornalba,Costa,Penedones)

$$A(s, -\mathbf{q}_\perp{}^2) = 2is \int d^2 l e^{-i\mathbf{l}_\perp \cdot \mathbf{q}_\perp} \int dz d\bar{z}\, P_{13}(z) P_{24}(\bar{z})(1 - e^{i\chi(s,b,z,\bar{z})})$$

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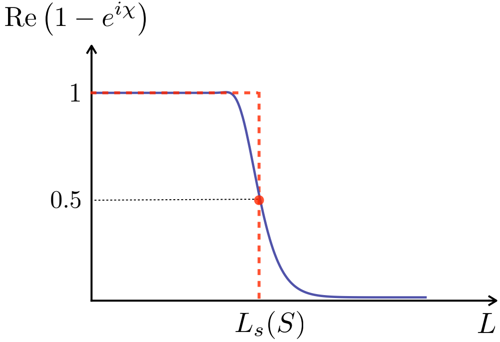

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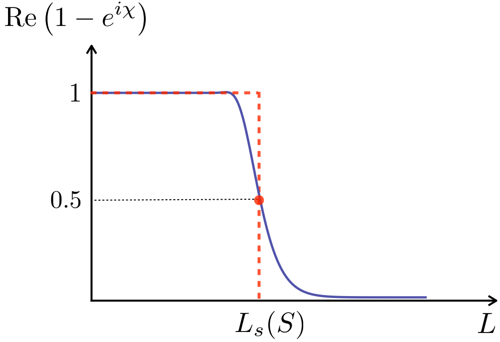

- The first approximation we'll use is a black disk [Cornalba, Costa, Penedones]

$$1 - e^{i\chi(\tau,L)} = \Theta(L_s(\tau) - L)$$

where the saturation radius L_s of the disk increases with energy as

$$L_s(\tau) \approx \omega\,\tau \ .$$

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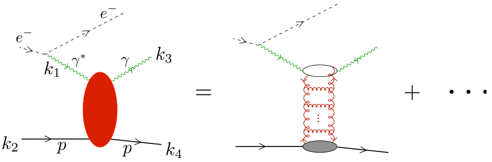

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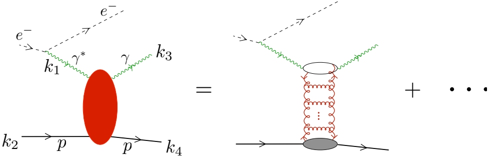

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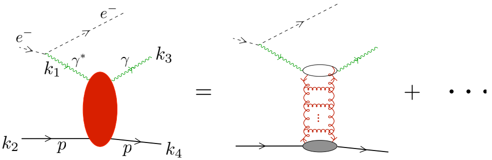

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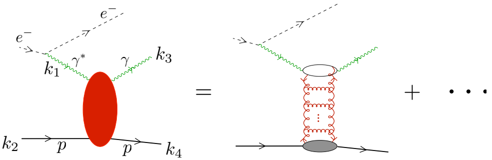

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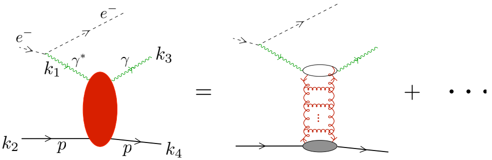

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- the scaling variable

$$x \approx \frac{Q^2}{s}$$

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$$\frac{d\sigma}{dt}(x, Q^2, t) = \frac{|W|^2}{16\pi s^2},$$

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- This has the previously mentioned form, we just need to supply the wavefunctions $\Psi(z)$ and $\Phi(\bar{z})$ for the photon and the proton.
- This is similar to what was done by Brower, MD, Sarčević and Tan for DIS, but now we cannot use the same approximation for $\Psi(z)$.

For the current analysis, we will assume that the proton wave function is sharply peaked near the IR boundary z_0, with $z_* \leq z_0$, with $z_\star$ of the order of the inverse proton mass. For simplicity, we will simply replace $\Phi(\bar{z})$ by a delta-function

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C is a normalization constant that can be calculated at weak coupling

$$C = \alpha \, \frac{10}{3 \, \pi^3} = 7.845 \times 10^{-4}$$

This is a useful reference value, but we will leave C as a parameter.

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- This expression depends on the three parameters C, ω and z_* (no real dependence on z_0).
- The lower limit of integration arises from a change of variable and is

$$z_- = \left(z_*(z_*s)^{-w}\right)^{1/1+w}\,,$$

and corresponds to $l_\perp = 0$.

Conformal Pomeron

- The next model we consider is the conformal pomeron, with

$$1 - e^{i\chi} \approx -i\chi = -i(\cot(\frac{\pi\rho}{2}) + i) g_0^2 e^{(1-\rho)\tau} \frac{L}{\sinh L} \frac{\exp(\frac{-L^2}{\rho\tau})}{(\rho\tau)^{3/2}}$$

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$$1 - e^{i\chi} \approx -i\chi = -i(\cot(\frac{\pi\rho}{2}) + i) g_0^2 e^{(1-\rho)\tau} \frac{L}{\sinh L} \frac{\exp(\frac{-L^2}{\rho\tau})}{(\rho\tau)^{3/2}}$$

- Depends on 3 parameters:

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- Note that they cannot be fit separately in the single pomeron model!

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- Similarly, the $t = 0$ result for the hard-wall model can also be written explicitly

$$\chi_{hw}(\tau, t=0, z, \bar{z}) = \chi(\tau, 0, z, \bar{z}) + \mathcal{F}(\tau, z, \bar{z})\, \chi(\tau, 0, z, z_0^2/\bar{z})\,.$$

- The function

$$\mathcal{F}(\tau, z, \bar{z}) = 1 - 4\sqrt{\pi\tau}\, e^{\eta^2}\, \mathrm{erfc}(\eta)\,, \qquad \eta = \frac{-\log(z\bar{z}/z_0^2) + 4\tau}{\sqrt{4\tau}}$$

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- For the data here analysed, the size of $\mathcal{F}$ will roughly vary between -0.1 and -0.4.

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- It can be shown that at large l the eikonal for the hard-wall model has a cut-off

$$\chi_{hw}(\tau, l, z, \bar{z}) \sim \exp[-m_1 l - (m_0 - m_1)^2\, l^2/4\rho\tau]$$

where m_1 and m_0 are solutions of

$$\partial_z(z^2 J_0(m_1 z))\Big|_{z=z_0} = 0\,, \qquad \partial_z(z^2 J_2(m_0 z))\Big|_{z=z_0} = 0.$$

At small l we assume χ still has the $t = 0$ form of a sum of two conformal kernels, but now with l dependence as well

$$\chi^{(0)}_{hw}(\tau, l, z, \bar{z}) \sim \chi(\tau, l, z, \bar{z}) + \mathcal{F}(\tau, z, \bar{z})\, \chi(\tau, l, z, z_0^2/\bar{z}).$$

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- Putting it together, the approximation we use for χ in the hard-wall model is

$$\chi_{hw}(\tau, l, z, \bar{z}) = C(\tau, z, \bar{z})\, D(\tau, l)\, \chi_{hw}^{(0)}(\tau, l, z, \bar{z}),$$

where

$$D(\tau, l) = \min\left(1, \frac{\exp[-m_1 l - (m_0 - m_1)^2 l^2/4\rho\tau]}{\exp[-m_1 z_0 - (m_0 - m_1)^2 z_0^2/4\rho\tau]}\right)$$

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- Note that the hard wall model has one more parameter, z_0.

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- We will look at both the differential and total exclusive cross sections.
- We have 52 points for the differential and 44 points for the cross section.

DIS

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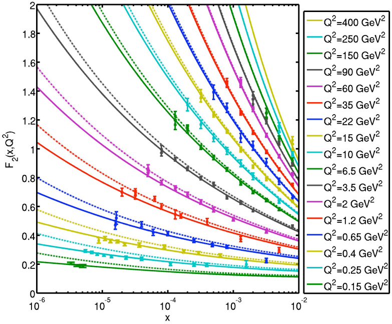

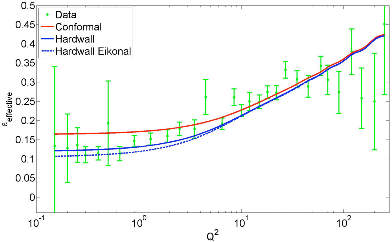

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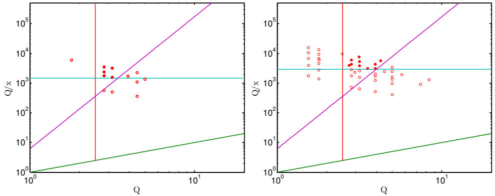

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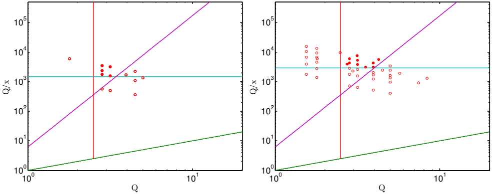

- The parameters we get from the differential cross section data are

$$\omega = 0.243 \pm 0.045\,, \quad z_\star = 3.50 \pm 0.89\,\mathrm{GeV}^{-1}\,, \quad C = 0.0013 \pm 0.0010\,,$$

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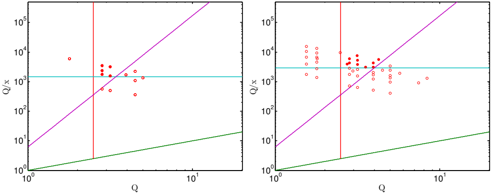

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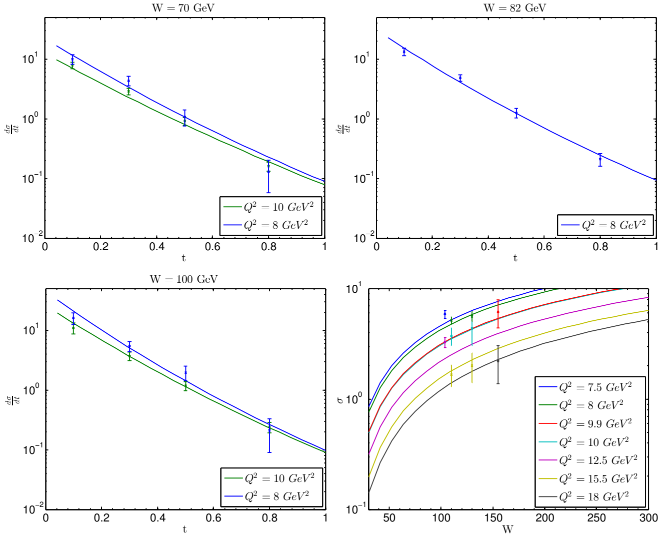
- The parameters from the total cross section data are

$$\begin{aligned} \omega &= 0.275 \pm 1.581\,, \; z_\star = 1.39 \pm 124.80\ \mathrm{GeV}^{-1}\,,\,, \\ C &= 0.94 \pm 5.30 \times 10^{-3}\,, \chi^2_{d.o.f.} = 1.22\,. \end{aligned}$$

Here are the plots corrseponding to these parameters:



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- Fitting the differential cross section to the data, we get

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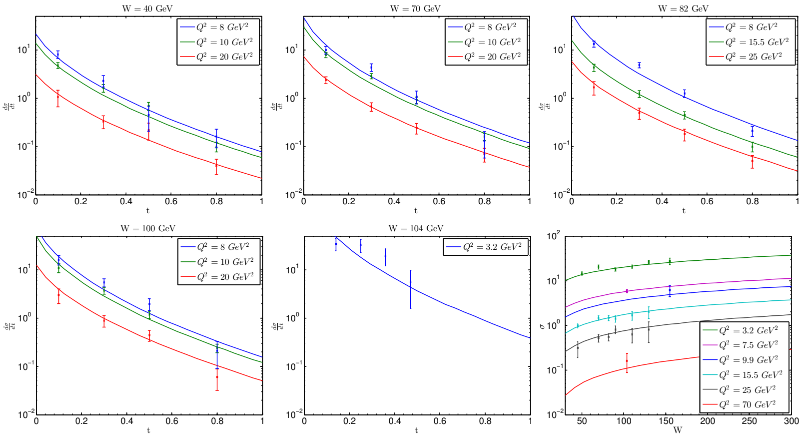

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σ
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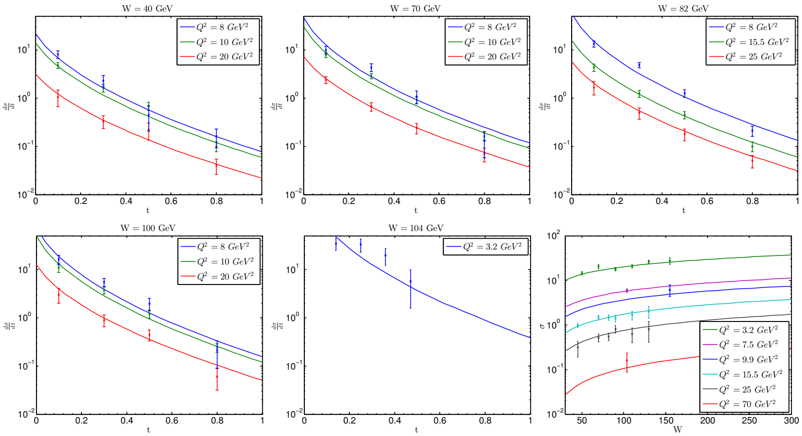

Running the same fit using the eikonal approximation, instead of just keeping single pomeron exchange, does not improve the fits, due to the fact that the size of χ is small in this kinematical region.

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- The parameters we obtain by fitting are

$$g_0^2 = 2.46 \pm 0.70\,, \quad z_* = 3.35 \pm 0.41 \text{ GeV}^{-1}, \quad \rho = 0.712 \pm 0.038\,,$$
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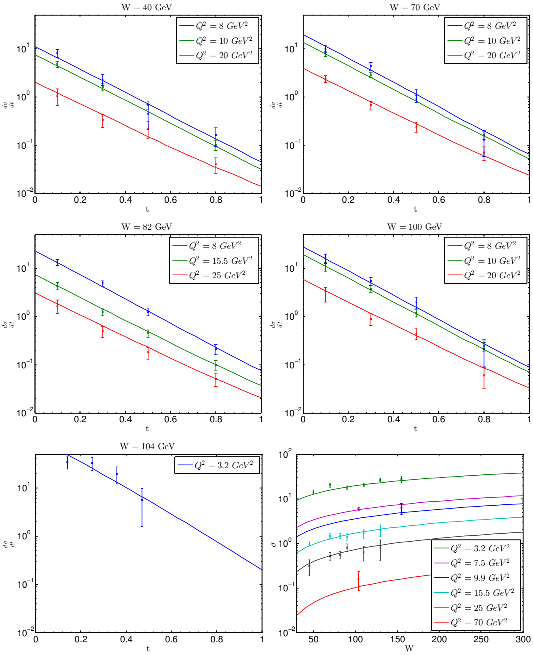

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- It might therefore be possible to extend some of the insights we gain even into the weak coupling regime.
- The hard wall model, although a simple modification of AdS, seems to capture effects of confinement well. Interesting to repeat some of the calculations using a different confinement model to identify precisely what features are model independent.

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- It is also interesting to extend these methods beyond $2 \to 2$ scattering.
- Indeed, work is under way in studying $2 \to 3$ scattering, which includes double pomeron exchange, and which would be useful for example for the study of the production of the Higgs boson (also known as the SX boson) at the LHC.

Thank you!