Supergravity geometries dual to $\mathcal{N} = 2$ SCFTs

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1^{st} Feb, IStr 2012

Recipe for $\mathcal{N} = 2$ SCFT duals

- AdS/CFT: $D = 10$ (IIB), $D = 11$ supergravity
- $D = 4$ SCFTs $\longleftrightarrow$ AdS_5
- half-maximal SUSY - 16 supercharges
- Killing spinors charged under $SU(2) \times U(1)$ R-symmetry

$$ds^2_D = e^{2\lambda}[ds^2(AdS_5) + e^{2A}ds^2(S^2) + ds^2(M_{D-7})]$$

What to expect in next 25 mins?

- Review various constructions
- Killing spinor bilinears
- Introduce Lin, Lunin, Maldacena (LLM)

  "The most general geometry dual to $\mathcal{N} = 2$ SCFTs"

- Small loophole? - ÓC, Wu, Yavartanoo JHEP 1104 (2011) 002
- LLM in type IIB - ÓC, Stefanski JHEP 1110 (2011) 061

Some common constructions

- Orientifold of $AdS_5 \times S^5$ Aharony, Fayyazuddin, Maldacena (1998)

$$ds^2_{S^5} = d\theta^2 + \sin^2\theta d\phi^2 + \cos^2\theta ds^2_{S^3}$$

  $\phi \sim \phi + 2\pi(1 - \frac{\alpha}{2})$, singularity depends on α

- Dimensional reduction and twist in $D = 7$ sugra Maldacena, Nuñez (2000)
- Killing spinor bilinears LLM (2004) . Also Gaiotto, Maldacena (2009), Stefanski, Reid-Edwards (2010)
- Non-Abelian T-duality Sfetsos, Thompson (2010) - talk at IStr 2011. S^3 fibration of S^5, where $SO(4) \simeq SU(2) \times SU(2)$. More generally, Lozano, ÓC, Sfetsos, Thompson (2011)

Killing Spinor (KS) Bilinears

- Preserved SUSY $\longleftrightarrow$ KS
- Via $\bar{\epsilon}\gamma^{(n)}\epsilon$, convert KSE into differential conditions
- $\mathcal{N}=1$ SCFT duals $ds^2 = e^{2\lambda}[ds^2_{AdS_5} + ds^2_{M_6}]$ Gauntlett, Martelli, Sparks, Waldram (2004), $Y^{p,q}$ discovered
- $U(1)$ R-symmetry isometry "emerges" $\nabla_{(m}\tilde{K}_{n)} = 0$
- More concretely...

$$\begin{aligned} \nabla_m \xi &= -\frac{1}{2} i m \gamma_m \gamma_7 \xi + \frac{1}{24} e^{-3\lambda} \gamma^{n_1 n_2 n_3} G_{m n_1 n_2 n_3} \xi, \\ \tilde{K}_m &= \frac{1}{2} \bar{\xi} \gamma_m \gamma_7 \xi, \quad e^{-6\lambda} d(e^{6\lambda} \tilde{K}) = e^{-3\lambda} * G + 4 m Y \end{aligned}$$

$\mathcal{N} = 1 \to \mathcal{N} = 2$ SCFT duals

- Further S^2 ansatz LLM (2004) (*)

$$\begin{aligned} ds^2_{M_6} &= e^{2A} ds^2(S^2) + ds^2(M_4), \\ G_4 &= e^{-3\lambda-2A}\mathcal{I}\mathrm{vol}(M_4) + \mathcal{F}\wedge \mathrm{vol}(S^2). \end{aligned}$$

- Split spinor across S^2 $\xi = \chi_+ \otimes \epsilon_+ + \chi_- \otimes \epsilon_-$, $\nabla_\alpha \chi_\pm = \frac{i}{2}\sigma_\alpha \chi_\pm$, $\chi_- = i\sigma_3\chi_+$

$$\begin{aligned} \nabla_\mu \epsilon_\pm &= \tfrac{i}{4}\gamma_5\gamma_\mu e^{-6\lambda-2A}\mathcal{I}\epsilon_\pm \\ &\mp \left(\tfrac{m}{2}\gamma_\mu\gamma_5 + \tfrac{1}{4}e^{-3\lambda-2A}\gamma^\nu \mathcal{F}_{\mu\nu}\right)\epsilon_\mp. \end{aligned}$$

- LLM noted if $\mathcal{I} = 0$, then KSEs decouple using $\epsilon_- = \pm\gamma_5\epsilon_+$. Show LLM solutions are not perturb. connected to $\mathcal{I} \neq 0$ solutions.

Solutions with $\mathcal{I} \neq 0$? (arXiv:1010.5982)

- Motivation: Large class of new $\mathcal{N} = 2$ SCFT quivers Gaiotto (2009); gravity duals based on LLM Gaiotto, Maldacena (2009)
- Good point to look for generalisation
- Using algebraic constraints can show $\epsilon_- = \gamma_5 \epsilon_+ \Rightarrow \mathcal{I} = 0$.
- Find two $U(1)$ Killing vectors X, Y s.t. $\mathcal{L}_X Y = 0$.
- $\mathcal{I} \neq 0$ implies $\epsilon_+ = \epsilon_- = 0$ no solution
- Could imagine more general $\epsilon_- = a_1 \epsilon_+ + a_2 \gamma_5 \epsilon_+ + a_3 \epsilon_+^c + a_4 \gamma_5 \epsilon_+^c$ generic ϵ_+, a_i functions. However $X \propto Y$ implies $\epsilon_- = \gamma_5 \epsilon_+$
- Interpolation to $AdS_4 \times S^7$, $AdS_7 \times S^4$?

LLM ansatz in type IIB (arXiv:1107.5763)

- $\mathcal{N} = 1$ SCFT duals classified Gauntlett, Martelli, Sparks, Waldram (2005)
- Recall solutions: i) SE_5, $(S^5, S^2 \times S^3)$ ii) Pilch-Warner (S^5)
- Starting point

$$
\begin{aligned}
ds^2_{10} &= e^{2\Delta}\left[ds^2(AdS_5) + ds^2(M_5)\right], \\
F_5 &= (\mathrm{vol}_{AdS_5} + \mathrm{vol}_{M_5}) f.
\end{aligned}
$$

In addition

$$
\begin{aligned}
D_M \epsilon &= \left(\nabla_M - \tfrac{i}{2} Q_M\right)\epsilon, \\
P &= -iQ + \frac{1}{2} d\phi = \frac{i}{2} e^{\phi} dC^{(0)} + \frac{1}{2} d\phi, \\
G &= i e^{\phi/2}(\tau dB - dC^{(2)}).
\end{aligned}
$$

KSE in $D = 5$

- Differential

$$D_m \xi_1 + \frac{i}{4}\left(e^{-4\Delta} f - 2m\right)\gamma_m \xi_1 + \frac{1}{8} e^{-2\Delta} G_{mnp}\gamma^{np}\xi_2 = 0,$$
$$\bar{D}_m \xi_2 - \frac{i}{4}\left(e^{-4\Delta} f + 2m\right)\gamma_m \xi_2 + \frac{1}{8} e^{-2\Delta} G^*_{mnp}\gamma^{np}\xi_1 = 0.$$

- Algebraic

$$\gamma^m \partial_m \Delta \xi_1 - \frac{1}{48} e^{-2\Delta}\gamma^{mnp} G_{mnp}\xi_2 - \frac{i}{4}\left(e^{-4\Delta} f - 4m\right)\xi_1 = 0,$$
$$\gamma^m \partial_m \Delta \xi_2 - \frac{1}{48} e^{-2\Delta}\gamma^{mnp} G^*_{mnp}\xi_1 + \frac{i}{4}\left(e^{-4\Delta} f + 4m\right)\xi_2 = 0,$$
$$\gamma^m P_m \xi_2 + \frac{1}{24} e^{-2\Delta}\gamma^{mnp} G_{mnp}\xi_1 = 0,$$
$$\gamma^m P^*_m \xi_1 + \frac{1}{24} e^{-2\Delta}\gamma^{mnp} G^*_{mnp}\xi_2 = 0.$$

SE_5 Killing vectors

- Set $\xi_2 = 0$, then $e^{-4\Delta} f = 4m$, $G = 0$, P constant if M_5 compact
- $ds^2(M_5) = e^{2B} ds^2(S^2) + ds^2(M_3)$, $\xi_i = \chi_+ \otimes \epsilon_{i+} + \chi_- \otimes \epsilon_{i-}$.
- KSE become

$$\begin{aligned} \nabla_m \epsilon_\pm + \frac{i}{2} m \sigma_m \epsilon_\mp &= 0, \\ \left[\pm i e^{-B} + im\right] \epsilon_\pm + \sigma^m \partial_m B \epsilon_\mp &= 0. \end{aligned}$$

- Find 6 real Killing directions: K^1, $\Re(L^1)$, M^1, N^1

$$i_{K^1} dB = -e^{-B} \Im(T_1), \quad dT_1 = -imK^2$$

- Separately $\Re(T_1) = 0$. So K^1 zero or global $U(1) \Rightarrow K^1 = K^2 = 0$ $\Rightarrow$ no solution

SE_5 Killing vectors

- Shown there is no global $U(1)$
- Relax condition that warp B independent of Killing
- Suitable ansatz + solving constraints

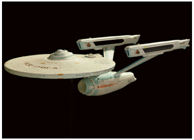
$$\epsilon_+ = \sin\frac{\theta}{2}\left(\begin{array}{c} e^{i\phi_1} \\ e^{i\phi_2} \end{array}\right), \quad \epsilon_- = -i\cos\frac{\theta}{2}\left(\begin{array}{c} e^{i\phi_2} \\ e^{i\phi_1} \end{array}\right).$$

- Redefine $\phi = \phi_1 - \phi_2$, $\psi = \phi_1 + \phi_2$ to get

$$\begin{aligned} ds^2(M_3) &= d\theta^2 + \sin^2\theta(d\phi^2 + \sin^2\phi d\psi^2), \\ e^B &= \cos\theta. \end{aligned}$$

- Located fibration S^5 only example!

Away from SE_5...to boldly go...



- More generally have to decompose G. Assume P, Q just along M_3.

$$G = \mathcal{A} \wedge \text{vol}(S^2) + g\text{vol}(M_3).$$

- With full fields - 2 Killing directions X, Y by trial + error. As a check

$$K_5^{GMSW} = (\bar{\chi}_+ \sigma_3 \chi_+) X + (\bar{\chi}_+ \chi_+) Y.$$

- Good! For Killing global $U(1)$ K require $\mathcal{L}_K \mathcal{F} = 0$. After some work can show $f = g = 0$ necessary. Ansatz now similar to LLM.

Away from SE_5

- Without f and g can show $\mathcal{L}_X Y = 0$ and $X \cdot Y = 0$.
- Further can show

$$dX = 2m * Y, \quad dY = 2m * X.$$

- Only one dimension left if X and Y are non-zero.
- However, possible to show that whether X or Y are chosen then $X = Y = 0$ is implied.
- No suitable $U(1)$'s. Thus realising $SU(2) \times U(1)$ is problematic!

R-symmetry caveat

- Back in $D = 5$ R-symmetry K_5 emerges from KSE.
- This assumes all fields non-zero.
- Hidden $U(1)$'s that would become Killing just as in SE_5?
- Using results of Gauntlett, Martelli, Sparks, Waldram (2005) possible to show $K_5 = 0$ leads to no solution.
- No extra $U(1)$'s in $D = 5$.

Non-geometric $SU(2)$

- Geometry corresponding to pure $\mathcal{N} = 2$ $SU(N)$ SYM Gauntlett, Kim, Martelli, Waldram (2001)
- $SU(2)$ acts on the KS of type IIB - non-geometric
- Study of $\mathcal{N} = 2$ SQCD, sub-critical $AdS_5 \times S^1$ Gadde, Pomoni, Rastelli (2009)
- Return to $\mathcal{N} = 1$ SCFT duals, set $f = 0$, find singular solutions with $U(1) \times U(1)$ isometry Gabella, ÓC, Martelli, Sparks.
- Continue "hide-n-seek" for IIB $\mathcal{N} = 2$ SCFT dual geometries.

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Gracias

Obrigado

Gràcies

Grazes