Non-extremal black holes as interpolating solutions
in 4-dimensional N=2 Supergravity

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Proposal (for static spherically symmetric geometries) based on the study of several examples:

One can obtain non-extremal solutions by deforming susy extremal ones. Then by taking the extremal limit both supersymmetric and non-supersymmetric extremal BHs can be smoothly recovered.

For each case first order flow equations exist and for some of them the generalized superpotential can be explicitly written.

The macroscopic thermodynamical properties can be fully analyzed.

Outline

- Black holes in Einstein-Maxwell theories.
- Black holes in N=2 four-dimensional supergravity.
- The deformation procedure: non-extremal black holes from an ansatz.

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- Example: the $\overline{\mathbb{CP}}^n$ model.

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Black hole basics

Charged black holes

They appear in Einstein-Maxwell theories where gravity is coupled to e.m. fields:

$$\mathcal{L} = R - \frac{1}{4}\mathcal{F}^{\mu\nu}\mathcal{F}_{\mu\nu}$$

Charges: $\quad p \propto \int_{S^2} \mathcal{F} \qquad q \propto \int_{S^2} \star\mathcal{F}$

For static spherically symmetric asymptotically flat solutions the ansatz is:

$$\begin{aligned} ds^2 &= e^{2U}dt^2 - e^{-2U}\gamma_{mn}dx^m dx^n \\ \gamma_{mn}dx^m dx^n &= \frac{c^4}{\sinh^4 c\tau}d\tau^2 + \frac{c^2}{\sinh^2 c\tau}d\Omega^2_{(2)} \end{aligned}$$

Charged black holes

$$\mathrm{d}s^2 = \mathrm{e}^{2U(\tau)}\mathrm{d}t^2 - \mathrm{e}^{-2U(\tau)}(c^4 \sinh^{-4}(c\tau)\mathrm{d}\tau^2 + c^2 \sinh^{-2}(c\tau)\mathrm{d}\Omega^2)$$

c is extremality parameter and it holds [GIBBONS,KALLOSH,KOL]: $c^2 = 2ST$.

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The relation $\tau \leftrightarrow r$ is: $\sinh^{-2}(c\tau) = (r - r^-)(r - r^+)$

$$r^\pm = r_\mathrm{h} \pm c$$

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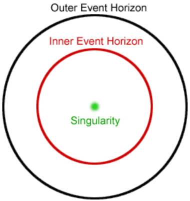
- General (non-extremal) Reisnerr-Nordström solution:

$$c = \sqrt{M^2 - (q^2 + p^2)}\,, \qquad e^{2U} = \frac{(r - r^-)(r - r^+)}{r^2}$$



outer: $r^+ \leftrightarrow \tau \to -\infty$
inner: $r^- \leftrightarrow \tau \to +\infty$

Charged black holes

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- Static extremal BHs: $c = 0$, $e^U = 1 - \frac{\sqrt{q^2 + p^2}}{r}$, $\tau \sim -\frac{1}{r}$
  - $M^2 = q^2 + p^2$
  - Finite non-vanishing entropy but zero temperature
  - $S^2 \otimes \mathrm{AdS}_2$ near horizon geometry

An extremal static BH is utterly defined by $\mathbf{Q} = (\mathbf{q}, \mathbf{p})$

Black holes in N=2, D=4 ungauged supergravity

N=2 Supergravity in 4D

- Multiplet content of the full theory:
  - Supergravity multiplet: $(e^i_\mu,\ \psi^A_\mu,\ \mathcal{A}^0_\mu)$ $\quad A = 1, 2$
  - n_v Vector multiplets: $(A^a_\mu,\ \lambda^{a\,A},\ z^a)$ $\quad a = 1, .., n_V$
  - n_H Hypermultiplets: (χ^α, ϕ^u) $\quad \alpha = 1, .., 2n_H, \quad u = 1, .., 4n_H$

  Since irrelevant in our discussion, we put to zero the fermion fields and omit the hypermultiplets

- The Lagrangian we deal with is (ungauged theory):

$$\mathcal{L} = -R(G) + 2g_{a\bar{b}}(z)\partial_\mu z^a \partial^\mu \bar{z}^{\bar{b}} + \mathrm{Im}\,\mathcal{N}_{IJ}(z)\mathcal{F}^I{}_{\mu\nu}\mathcal{F}^{J\,\mu\nu} + \mathrm{Re}\,\mathcal{N}_{IJ}(z)\epsilon^{\mu\nu\rho\sigma}\mathcal{F}^I{}_{\mu\nu}\mathcal{F}^J{}_{\rho\sigma}$$

$\Rightarrow$ Structure of a Maxwell-Einstein-scalars theory

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N=2 Supergravity in 4D

$$I = (0, a)$$
$$a = 1, \dots, n_V$$

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- Geometry of the scalar manifold: (very) special

$$F = F(X^I) \qquad\qquad z^a = \frac{X^a}{X^0}$$

$$g_{a\bar{b}} = \frac{\partial}{\partial z^a} \frac{\partial}{\partial \bar{z}^{\bar{b}}} K \qquad\qquad K = -\ln \left[\mathrm{i} (X^I, \partial_{X^I} F) \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \overline{\begin{pmatrix} X^I \\ \partial_{X^I} F \end{pmatrix}} \right]$$

$$\mathcal{N}_{IJ} = \overline{\partial_{X^I} \partial_{X^J} F} + 2\mathrm{i} \frac{\operatorname{Im}(\partial_{X^I} \partial_{X^K} F) \operatorname{Im}(\partial_{X^J} \partial_{X^M} F) X^M X^K}{\operatorname{Im}(\partial_{X^M} \partial_{X^K} F) X^M X^K}$$

N=2 Supergravity in 4D

$$I=(0,a)$$
$$a=1,\ldots,n_V$$

$$\mathcal{L}=-R(G)+2g_{a\bar{b}}(z)\partial_\mu z^a\partial^\mu\bar{z}^{\bar{b}}+\mathrm{Im}\,\mathcal{N}_{IJ}(z)\mathcal{F}^{\mathrm{I}}{}_{\mu\nu}\mathcal{F}^{\mathrm{J}\,\mu\nu}+\mathrm{Re}\,\mathcal{N}_{IJ}(z)\epsilon^{\mu\nu\rho\sigma}\mathcal{F}^{\mathrm{I}}{}_{\mu\nu}\mathcal{F}^{\mathrm{J}}{}_{\rho\sigma}$$

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$$\mathcal{N}_{\mathrm{IJ}}=\overline{\partial_{X^{\mathrm{I}}}\partial_{X^{\mathrm{J}}}F}+2\mathrm{i}\frac{\mathrm{Im}(\partial_{X^{\mathrm{I}}}\partial_{X^K}F)\,\mathrm{Im}(\partial_{X^{\mathrm{J}}}\partial_{X^M}F)X^MX^K}{\mathrm{Im}(\partial_{X^M}\partial_{X^K}F)X^MX^K}$$

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- By assuming spherical symmetry and staticity, solving for the vectors and integrating $\Rightarrow$ effective 1D Lagrangian: [FERRARA,GIBBONS,KALLOSH]

$$\boxed{\mathcal{L}_{\text{eff}} = (\dot{U}(\tau))^2 + g_{a\bar{b}}\dot{z}^a(\tau)\dot{\bar{z}}^{\bar{b}}(\tau) - \mathrm{e}^{2U} V_{\text{bh}}(z, \Gamma) + c^2}$$

$$\dot{} = \frac{\mathrm{d}}{\mathrm{d}\tau} \qquad \Gamma = (p^I\ q_I) \propto \int_{S^2} \mathcal{F}^I \oplus \frac{\partial \mathcal{L}}{\partial \mathcal{F}^I}$$

$$-V_{\text{bh}} = -\tfrac{1}{2}\Gamma^\Lambda\Gamma^\Sigma\mathcal{M}_{\Lambda\Sigma} = \tfrac{1}{2}(p^I\ q_I)\begin{pmatrix} (\mathfrak{I} + \mathfrak{R}\mathfrak{I}^{-1}\mathfrak{R})_{IJ} & -(\mathfrak{R}\mathfrak{I}^{-1})_I{}^J \\ -(\mathfrak{I}^{-1}\mathfrak{R})^I{}_J & (\mathfrak{I}^{-1})^{IJ} \end{pmatrix}\begin{pmatrix} p^J \\ q_J \end{pmatrix}$$
$$= |\mathcal{Z}|^2 + 4g^{a\bar{b}}\partial_{z^a}|\mathcal{Z}|\partial_{\bar{z}^{\bar{b}}}|\mathcal{Z}|$$

$$\mathfrak{R}_{IJ} = \mathrm{Re}\,\mathcal{N}_{IJ}\,, \qquad \mathfrak{I}_{IJ} = \mathrm{Im}\,\mathcal{N}_{IJ}$$

central charge: $\mathcal{Z} = \mathrm{e}^{K/2}(p^I\partial_{X^I}F - q_IX^I)$

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Field equations (second order):

$$\text{eqm}\begin{cases} \ddot{U} + \mathrm{e}^{2U}V_{\text{bh}} = 0 \\ \ddot{z}^a + g^{a\bar{b}}\partial_{z^c}g_{d\bar{b}}\dot{z}^c\dot{z}^d + \mathrm{e}^{2U}g^{a\bar{b}}\partial_{\bar{z}^{\bar{b}}}V_{\text{bh}} = 0 \end{cases}$$

$$\text{constraint}\left\{ \ddot{U}^2 + g_{a\bar{b}}\dot{z}^a\dot{\bar{z}}^{\bar{b}} + e^{2U}V_{\text{bh}} = c^2 \right.$$

Important to know

- First-order formalism: [CERESOLE, DALL'AGATA & PERZ ET AL.]

$$-\mathrm{e}^{2U} V_{\mathrm{bh}} = (\partial_U Y)^2 + 4 g^{a\bar{b}} \partial_{z^a} Y \partial_{\bar{z}^{\bar{b}}} Y - c^2$$

Generalized Superpotential $Y = Y(U, z; \Gamma) > 0$

$$\Rightarrow \quad \mathcal{L}_{\mathrm{eff}} = \dot{U}^2 + g_{a\bar{b}} \dot{z}^a \dot{\bar{z}}^{\bar{b}} - \mathrm{e}^{2U} V_{\mathrm{bh}} + c^2$$

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Sum of squares $\left(\dot{U} \pm \partial_U Y\right)^2 + \left| \dot{z}^a \pm 2 g^{a\bar{b}} \partial_{\bar{z}^{\bar{b}}} Y \right|^2$

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$$\Rightarrow \text{Extremizing:} \quad \begin{cases} \dot{U} = \pm Y \\ \dot{z}^a = \pm 2 g^{a\bar{b}} \partial_{\bar{z}^{\bar{b}}} Y \end{cases}$$

First-order flow equations

sign depends on conventions

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- $Y = \mathrm{e}^U |\mathcal{Z}(z;\Gamma)|$, $c = 0$: extremal susy BHs
- $Y = \mathrm{e}^U \mathcal{W}(z;\Gamma) \neq \mathrm{e}^U |\mathcal{Z}|$, $c = 0$: extremal non-susy BHs
- $Y \neq \mathrm{e}^U |\mathcal{Z}| \neq \mathrm{e}^U \mathcal{W}$, $c \neq 0$: non-extremal BHs

Important to know

- The eqm are in general difficult to solve but for supersymmetric

  BHs ($\Rightarrow$ extremal [KHURI,ORTÍN]) they are equivalent to:

$$2\partial_\tau \,\mathrm{Im}\left[\mathrm{e}^{-U-\mathrm{i}\alpha}\,\mathrm{e}^{-K/2}\begin{pmatrix} X^I \\ \partial_{X^I}F\end{pmatrix}\right] = -\begin{pmatrix} p^I \\ q_I\end{pmatrix}$$

l.h.s. total derivative, r.h.s. constant $\Rightarrow$ direct integration gives

Supersymmetric stabilization equation
[BEHRNDT,LÜST,SABRA & DENEF]

$$\boxed{2\,\mathrm{Im}(e^{-U}e^{-\mathrm{i}\alpha}\Omega) = \mathcal{H}}$$

$\mathcal{H} = -\Gamma\tau + 2\,\mathrm{Im}[\mathrm{e}^{\mathrm{i}\alpha}\Omega]_{\tau=0}$
vect of harmonic functions

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vect of harmonic functions

Once one has solved for the components of $\Omega' = \mathrm{e}^{-U-\mathrm{i}\alpha}\Omega$

the scalars are given by: $\quad z^a = \dfrac{\Omega'^{\,a}}{\Omega'^{\,0}}\,, \qquad \mathrm{e}^{-2U} = \mathrm{i}(\Omega'^{\,I}\bar{\Omega}'_I - \bar{\Omega}'^{\,I}\Omega'_I)$

Non-extremal black holes from deforming extremal ones

Non-extremal black holes

General prescription:

1. Consider the supersymmetric solution:

   $$U(\tau) = U_{\mathrm{e}}[H^I(\tau)] \qquad z^a(\tau) = z[H^I(\tau)]$$

   $$H^I(\tau) = h^I - \Gamma^I \tau \equiv \text{harmonic functions}$$

2. Make the ansatz:

   $$U(\tau) = U_{\mathrm{e}}[\hat{H}^I(\tau)] + c\tau \qquad z^a(\tau) = z[\hat{H}^I(\tau)]$$

   $$\hat{H}^I(\tau) = A^I + B^I \mathrm{e}^{2c\tau}$$

3. Determine the coefficients A^I, B^I by plugging the ansatz in the eqm and solving the resulting algebraic equations

Example: $\overline{\mathbb{CP}}^n$ *model*

- Prepotential $F = -\frac{\mathrm{i}}{4}\eta_{IJ}X^I X^J \qquad \eta_{IJ} = \mathrm{diag}(+ - \cdots -)$
- n scalars $z^a = \dfrac{X^a}{X^0}$, with the assumption $z^0 = 1$, defining:

  $Z^I = (1, z^a)\,, \qquad Z_I = (1, z_a) = (1, -z^a)$
- $n+1$ electric (q_I) and magnetic (p^I) charges combined in the complex quantity $\quad \gamma_I = q_I + \frac{\mathrm{i}}{2}\eta_{IJ}p^J$
- $K = -\log\left(\bar{z}^J z_J\right)\,, \qquad g_{a\bar{b}} = -e^K\left(\eta_{a\bar{b}} - e^K \bar{z}_a z_{\bar{b}}\right)$
- Holomorphic symplectic section: $\Omega = e^{\mathcal{K}/2}\begin{pmatrix} Z^I \\ -\frac{i}{2}Z_I \end{pmatrix}$
- BH potential: $\quad -V_{\mathrm{bh}} = 2e^{\mathcal{K}}|Z^I\gamma_I|^2 - \bar{\gamma}^I\gamma_I = |\mathcal{Z}|^2 + |\tilde{\mathcal{Z}}|^2$

  $\mathcal{Z} = e^{\mathcal{K}/2}Z^I\gamma_I\,, \qquad |\tilde{\mathcal{Z}}|^2 = e^{\mathcal{K}}|Z^I\gamma_I|^2 - \bar{\gamma}^I\gamma_I$

1. Consider the supersymmetric solution

Solving the stabilization equation $2\,\mathrm{Im}(\mathrm{e}^{-\mathrm{i}\alpha}\mathrm{e}^{-U}\Omega) = H$ yields:

$$z^a = \frac{\bar{\mathcal{H}}^a}{\bar{\mathcal{H}}^0} \qquad \mathrm{e}^{-2U_{\mathrm{e}}} = 4\bar{\mathcal{H}}^I\mathcal{H}_I$$

$$\mathcal{H}_I = h_I - \gamma_I\,\tau \qquad \gamma_I = q_I + \tfrac{i}{2}\eta_{IJ}p^J$$

2. Make the ansatz

$$z^a[\mathcal{H}] \;\to\; z^a[\hat{\mathcal{H}}] = \frac{\bar{\hat{\mathcal{H}}}^a}{\bar{\hat{\mathcal{H}}}^0} \qquad \hat{\mathcal{H}}^I = A^I + B^I\mathrm{e}^{2c\tau}$$

$$\mathrm{e}^{-2U_{\mathrm{e}}}[\mathcal{H}] \;\to\; \mathrm{e}^{-2U}[\hat{\mathcal{H}}] = 4\bar{\hat{\mathcal{H}}}^I\hat{\mathcal{H}}_I\,\mathrm{e}^{-2c\tau}$$

3. Determine the coefficients A^I, B^I by plugging the ansatz in the eqm and solving the resulting algebraic equations

Original field equations:

$$\begin{aligned} \ddot{U} + \mathrm{e}^{2U} V_{\text{bh}} &= 0 \\ \ddot{z}^a + g^{a\bar{b}} \partial_{z^c} g_{d\bar{b}} \dot{z}^c \dot{z}^d + \mathrm{e}^{2U} g^{a\bar{b}} \partial_{\bar{z}^{\bar{b}}} V_{\text{bh}} &= 0 \\ \ddot{U}^2 + g_{a\bar{b}} \dot{z}^a \dot{\bar{z}}^{\bar{b}} + e^{2U} V_{\text{bh}} &= c^2 \end{aligned}$$

For the non-extremal ansatz (generic case):

$$\begin{aligned} \ddot{U}_{\mathrm{e}} - (\dot{U}_{\mathrm{e}})^2 - g_{a\bar{b}} \dot{z}^a \dot{\bar{z}}^{\bar{b}} &= 0 \\ (2c)^2 \left[\mathrm{e}^{c\tau} \ddot{U}_{\mathrm{e}} + \dot{U}_{\mathrm{e}} \right] + e^{2U_{\mathrm{e}}} V_{\mathrm{bh}} &= 0 \\ (2c)^2 \left[\mathrm{e}^{c\tau} \left(\ddot{z}^a + g^{a\bar{b}} \partial_c g_{d\bar{b}} \dot{z}^c \dot{z}^d \right) + \dot{z}^a \right] + e^{2U_{\mathrm{e}}} g^{a\bar{b}} \partial_{\bar{b}} V_{\mathrm{bh}} &= 0 \end{aligned}$$

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And finally for our case ($\overline{\mathbb{CP}}^n$):

$$\begin{aligned}
\mathrm{Im}(\bar{B}^I A_I) =& 0 \\
\bar{A}^I A^J \xi_{IJ} =& 0 \\
(\bar{A}^I B^J + \bar{B}^I A^J)\xi_{IJ} =& 0 \\
\bar{B}^I B^J \xi_{IJ} =& 0 \\
(2c)^2(\bar{B}_a \bar{A}_0 - \bar{B}_0 \bar{A}_a)\bar{A}^I A_I + (\bar{\gamma}_a \bar{A}_0 - \bar{\gamma}_0 \bar{A}_a)\bar{A}^I \gamma_I =& 0 \\
-(2c)^2(\bar{B}_a \bar{A}_0 - \bar{B}_0 \bar{A}_a)\bar{B}^I B_I + (\bar{\gamma}_a \bar{B}_0 - \bar{\gamma}_0 \bar{B}_a)\bar{B}^I \gamma_I =& 0 \\
(\bar{\gamma}_a \bar{A}_0 - \bar{\gamma}_0 \bar{A}_a)\bar{A}^I \gamma_I + (\bar{\gamma}_a \bar{B}_0 - \bar{\gamma}_0 \bar{B}_a)\bar{B}^I \gamma_I =& 0
\end{aligned}$$

where $\quad \xi_{IJ} = 2\left(\gamma_I \bar{\gamma}_J + 8c^2 A_I \bar{B}_J\right) - \eta_{IJ}\left(\gamma^K \bar{\gamma}_K + 8c^2 A^K \bar{B}_K\right)$

3. Determine the coefficients A^I, B^I by plugging the ansatz in the eqm and solving the resulting algebraic equations

In addition, to fully express the coefficients in terms of the physical parameters, one imposes (~~NUT-charge~~ already implied by eqm):

- asymptotic flatness: $4(\bar{A}^I + \bar{B}^I)(A_I + B_I) = 1$
- definition of the mass: $4\,\mathrm{Re}[\bar{B}^I(A_I + B_I)] = 1 - M/c$
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$\Rightarrow$ Final solution:

$$A_I = \pm\frac{e^{\mathcal{K}_\infty/2}}{2}\left\{\bar{Z}_{I\,\infty}\left[1 + \frac{(M^2 - e^{\mathcal{K}_\infty}|\bar{Z}^J_\infty\bar{\gamma}_J|^2)}{M\,c}\right] + \frac{\gamma_I\bar{Z}^J_\infty\bar{\gamma}_J}{M\,c}\right\}$$

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Non-extremal BHs in the $\overline{\mathbb{CP}}^n$ model

Solutions:

$$z^a = \frac{\hat{\bar{\mathcal{H}}}^a}{\hat{\bar{\mathcal{H}}}^0} = \frac{\bar{A}^a + \bar{B}^a\,\mathrm{e}^{2c\tau}}{\bar{A}^0 + \bar{B}^0\,\mathrm{e}^{2c\tau}}$$

$$\mathrm{e}^{-2U} = 4\hat{\bar{\mathcal{H}}}^I\hat{\mathcal{H}}_I\mathrm{e}^{-2c\tau} = 4(\bar{A}^I + \bar{B}^I\,\mathrm{e}^{2c\tau})(A_I + B_I\,\mathrm{e}^{2c\tau})\,\mathrm{e}^{-2c\tau}$$

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Entropies: $\boxed{\frac{S_\pm}{\pi} = (M^2 - |\mathcal{Z}_\infty|^2) \pm (M^2 - |\tilde{\mathcal{Z}}_\infty|^2) \pm 2Mc}$

$+ \equiv$ outer horizon, $\tau \to -\infty$
$- \equiv$ inner horizon, $\tau \to +\infty$

Mass: $\boxed{M^2c^2 = (M^2 - |\mathcal{Z}_\infty|^2)(M^2 - |\tilde{\mathcal{Z}}_\infty|^2)}$

Non-extremal BHs in the $\overline{\mathbb{CP}}^n$ model

$$|\mathcal{Z}_\infty|^2 = e^{\mathcal{K}}|Z^I_\infty \gamma_I|^2$$
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Solutions:

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Extremal limits

From the expression of the mass

$$M^2c^2 = (M^2 - |\mathcal{Z}_\infty|^2)(M^2 - |\tilde{\mathcal{Z}}_\infty|^2)$$

possible to see two extremal limits in which $c \to 0$:

1. Supersymmetric: $M^2 \to |\mathcal{Z}_\infty|^2 = e^{\mathcal{K}_\infty}|Z^I_\infty \gamma_I|^2$

$$\hat{\mathcal{H}}_I \overset{M\to|\mathcal{Z}_\infty|}{\longrightarrow} \pm\frac{\bar{\mathcal{Z}}_\infty}{|\mathcal{Z}_\infty|}\mathcal{H}_I^{\rm susy} = \pm\frac{\bar{\mathcal{Z}}_\infty}{|\mathcal{Z}_\infty|}(h_I - \gamma_I\tau)$$

2. Non-supersymmetric: $M^2 \to |\tilde{\mathcal{Z}}_\infty|^2 = e^{\mathcal{K}_\infty}|Z^I_\infty \gamma_I|^2 - \bar{\gamma}^I\gamma_I$

$$\hat{\mathcal{H}}_I \overset{M\to|\tilde{\mathcal{Z}}_\infty|}{\longrightarrow} \pm\frac{e^{\mathcal{K}_\infty/2}}{2}\left\{\bar{Z}_{I\,\infty} - \frac{1}{|\tilde{\mathcal{Z}}_\infty|}\left[-\bar{Z}_{I\,\infty}\bar{\gamma}^J\gamma_J + \gamma_I\bar{Z}^J_\infty\bar{\gamma}_J\right]\tau\right\}$$

The non-extremal BH will evaporate until its mass equals the biggest between $|\mathcal{Z}_\infty|$ and $|\tilde{\mathcal{Z}}_\infty|$ $\Rightarrow$ depending on the charges the final extremal BH will be susy ($|\mathcal{Z}_\infty| > |\tilde{\mathcal{Z}}_\infty|$) or non-susy ($|\tilde{\mathcal{Z}}_\infty| > |\mathcal{Z}_\infty|$).

$\overline{\mathbb{CP}}^n$ *flow equations*

- First-order formalism:

$$-\mathrm{e}^{2U} V_{\mathrm{bh}} = (\partial_U Y)^2 + 4g^{a\bar{b}}\partial_{z^a} Y \partial_{\bar{z}^{\bar{b}}} Y - c^2$$

Generalized Superpotential $Y = Y(U, z; \Gamma) > 0$

$\Rightarrow \quad \mathcal{L}_{\mathrm{eff}} = \dot{U}^2 + g_{a\bar{b}}\dot{z}^a\dot{\bar{z}}^{\bar{b}} + (\partial_U Y)^2 + 4g^{a\bar{b}}\partial_{z^a} Y \partial_{\bar{z}^{\bar{b}}} Y \simeq$

Sum of squares $\left(\dot{U} \pm \partial_U Y\right)^2 + \left|\dot{z}^a \pm 2g^{a\bar{b}}\partial_{\bar{z}^{\bar{b}}} Y\right|^2$

$\Rightarrow$ Extremizing: $\begin{cases} \dot{U} = \pm Y \\ \dot{z}^a = \pm 2g^{a\bar{b}}\partial_{\bar{z}^{\bar{b}}} Y \end{cases}$

First-order flow equations

sign depends on conventions

- $Y = \mathrm{e}^U|\mathcal{Z}|(z;\Gamma)$, $c = 0$: extremal susy BHs
- $Y = \mathrm{e}^U\mathcal{W}(z;\Gamma) \neq \mathrm{e}^U|\mathcal{Z}|$, $c = 0$: extremal non-susy BHs
- $Y \neq \mathrm{e}^U|\mathcal{Z}| \neq \mathrm{e}^U\mathcal{W}$, $c \neq 0$: non-extremal BHs

$\overline{\mathbb{CP}}^n$ *flow equations*

- First-order formalism:

$$-\mathrm{e}^{2U} V_{\mathrm{bh}} = \Upsilon^2 + 4\, g^{a\bar{b}} \Psi_a \bar{\Psi}_{\bar{b}} - c^2$$

where:

$$\Upsilon = \frac{e^U}{\sqrt{2}} \sqrt{|\mathcal{Z}|^2 + |\tilde{\mathcal{Z}}|^2 + e^{-2U} c^2 + \sqrt{\left(|\mathcal{Z}|^2 + |\tilde{\mathcal{Z}}|^2 + e^{-2U} c^2\right)^2 - 4|\mathcal{Z}|^2 |\tilde{\mathcal{Z}}|^2}}$$

$$\Psi_a = e^{2U} \frac{\bar{\tilde{\mathcal{Z}}}\, \mathcal{D}_a \mathcal{Z}}{\Upsilon}$$

such that: $\partial_U \Psi_a - \partial_{z^a} \Upsilon = \partial_{z^a} \Psi_b - \partial_{z^b} \Psi_a = \partial_{\bar{z}^{\bar{a}}} \Psi_b - \partial_b \bar{\Psi}_{\bar{a}} = 0$

$\Rightarrow$ There exists a superpotential, whose gradient generates the vector field $(\Upsilon, \Psi_a, \bar{\Psi}_{\bar{b}})$ and the first-order equations:

$$\dot{U} = \Upsilon$$

$$\dot{z}^a = 2\, g^{a\bar{b}} \bar{\Psi}_{\bar{b}}$$

extremal limit: $\begin{cases} \text{susy:} & \Upsilon = \mathrm{e}^U |\mathcal{Z}|\,, \quad \Psi_a = \mathrm{e}^U \partial_{z^a} |\mathcal{Z}| \\ \text{non-susy:} & \Upsilon = \mathrm{e}^U |\tilde{\mathcal{Z}}|\,, \quad \Psi_a = \mathrm{e}^U \partial_{z^a} |\tilde{\mathcal{Z}}| \end{cases}$

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Non-extremal black holes

The following models have been studied:

- ✓ Axion-dilaton model: $F = -\mathrm{i}X^0X^1$
- ✓ $\overline{\mathbb{CP}}^n$ models: $F = -\frac{\mathrm{i}}{4}\eta_{IJ}X^IX^J$
- ✓ Axion free *stu* model: $F = -\frac{X^1X^2X^3}{X^0}$
- ✓ t^3 model: $F = -\frac{5}{6}\frac{(X^1)^3}{X^0}$
- ✓ Quantum corrected t^3 model (work in progress):
  $$F = -\frac{5}{6}\frac{(X^1)^3}{X^0} - \frac{11}{4}(X^1)^2 + \frac{25}{12}X^0X^1 - \mathrm{i}\frac{k}{2}(X^0)^2$$

The deformation procedure has been proved to work also in $N = 2$,

$D = 5$ supergravity [MEESSEN,ORTÍN].

Conclusions and outlook

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Now we should be convinced that (for all the model considered):

- There exists a procedure that allows to find non-extremal black holes by deforming (through an ansatz) susy solutions.
- Non-extremal BHs interpolate smoothly between supersymmetric and the non-supersymmetric extremal BHs.
- The macroscopic thermodynamical properties of non-extremal BH solutions can be fully analyzed.
- It is possible to write first order flow equations for the scalars and prove the existence of the generalized superpotential.

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BUT...

ARE WE SURE

THAT THE ANSATZ

(THE DEFORMATION PROCEDURE)

IS GENERAL ENOUGH?

Outlook

We know that :

- The extremal limits of $\hat{H}^I = A^I + B^I \mathrm{e}^{2c\tau}$ are harmonic functions.
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$\Rightarrow$ for more general solutions a different approach is needed

Outlook

In this direction:

- Rewrite the effective action in terms of real H-functions (no assumption on their form) and calculate the equivalents of the field eqm [MEESSEN,ORTÍN,PERZ,SHAHBAZI].
- Reformulate the Kähler geometry in terms of real variables, dimensional reduce the theory, find and solve the eqm and then uplift the solutions [MOHAUPT,VAUGHAN].