Relating the $\mathcal{N} = 4$ SYM low-energy effective action with the D3 brane on $AdS_5 \times S^5$ background

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References

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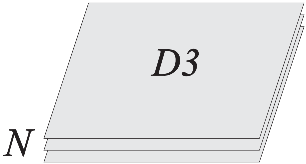
Motivations



$\mathcal{N} = 4$ SYM with gauge group $SU(N)$

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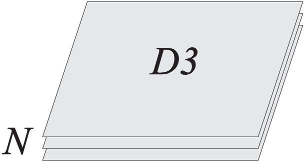

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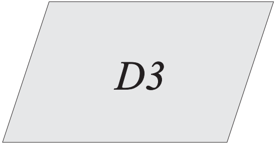


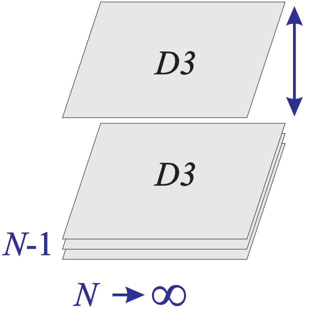
flat background

$\mathcal{N}=4$ Dirac-Born-Infeld action for
1 vector A_m,
6 scalars X^I, $I=1,\ldots,6$
4 spinors Ψ^A_α, $\bar\Psi_{A\dot\alpha}$, $A=1,2,3,4$

$$S_{D3}=\int d^4x\sqrt{-\det(\eta_{mn}+\partial_m X^I\partial_n X^I+F_{mn})}+\text{spinors}$$

- After fixing κ-symmetry
- In the static gauge

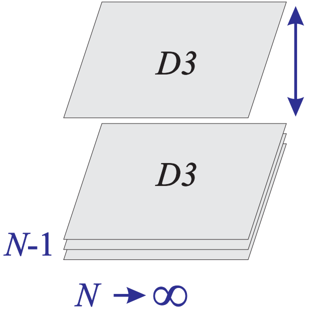
String theory point of view



- A stack of N D3 branes creates the $AdS_5 \times S^5$ background in the $N \to \infty$ limit
- The action of the probe-brane is given by Diarc-Born-Infeld-type action on the $AdS_5 \times S^5$ background $S = S_{D3,AdS}[F_{mn}, X^I, \Psi^A_\alpha]$

$$
\begin{aligned}
S &= S_{BI} + S_{WZ} + \text{spinors} \\
S_{BI} &= -T_3 \int d^4x \frac{|X|^4}{Q} \left[\sqrt{-\det(\eta_{mn} + \frac{Q}{|X|^4}\partial_m X^I \partial_n X^I + \frac{Q^{1/2}}{|X|^2} F_{mn})} - 1 \right]
\end{aligned}
$$

Field theory point of view



- $\mathcal{N}=4$ SYM in the Coulomb branch.
- The gauge symmetry is spontaneously broken $SU(N) \to U(1)\times SU(N-1)$.
- The massive fields are integrated out
- The effective action for massless fields

$$\Gamma = \Gamma[F_{mn}, \phi^I, \Psi^A_\alpha]$$

AdS/CFT correspondence

$$S_{D3,AdS}[F_{mn}, X^I, \Psi^A_\alpha] \longleftrightarrow \Gamma[F_{mn}, \phi^I, \Psi^A_\alpha]$$

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Aim of this talk

To review this relation for various leading terms

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Main problem

Only some leading contributions to Γ are known. Computing the effective action is a hard problem in QFT.
In particular, only F^4/X^4, F^6/X^8 and Wess-Zumino terms were computed and compared with the ones in D3 brane action.

In our papers various descriptions of F^4/X^4, F^6/X^8 and Wess-Zumino terms in the $\mathcal{N}=3$ and $\mathcal{N}=4$ harmonic superspace were developed.

D3 brane

D3 brane is a solution of IIB supergravity with non-trivial 5-form R-R field strengths

$$F_5 = *F_5$$

- It has 4 world-volume coordinates x^m, $m = 0, 1, 2, 3$ and six orthogonal coordinates X^I, $I = 1, 2, 3, 4, 5, 6$.
- In static gauge

$$X^I = X^I(x^m)$$

- There is gauge field on the brane,

$$A_m = A_m(x)$$

- There are fermions Ψ^A_α, $\bar{\Psi}_{A\dot{\alpha}}$ which are necessary for supersymmetry (16 supercharges).

D3 brane

Bosonic part of the D3 brane action on the $AdS_5 \times S^5$ background
[Metsaev, Tseytlin 1998]

$$
\begin{aligned}
S &= S_{BI} + S_{WZ} \\
S_{BI} &= -T_3 \int d^4x \frac{|X|^4}{Q} \left[\sqrt{-\det(\eta_{mn} + \frac{Q}{|X|^4} \partial_m X^I \partial_n X^I + \frac{Q^{1/2}}{|X|^2} F_{mn})} - 1 \right] \\
S_{WZ} &= -\frac{N-1}{60\pi^2} \int d^5x\, \varepsilon^{mnklr} \varepsilon_{IJKLMN} \frac{1}{|X|^6} X^I \partial_m X^J \partial_n X^K \partial_k X^L \partial_l X^M \partial_r X^N \\
&= \int_{M_5} \Omega_5 = \int_{\partial M_5} \omega_4 \,, \qquad\qquad \Omega_5 = d\omega_4
\end{aligned}
$$

$Q = \frac{(N-1) g_s \alpha'^2}{\pi}$, $T_3 = \frac{1}{2\pi g_s \alpha'^2}$, $|X|^2 = X^I X^I$.

- The Wess-Zumino term describes the electromagnetic interaction of the probe brane with the stack of other D3 branes because the D3 branes carry both electric and magnetic charges. This interaction is similar to the Lorentz force on the electron moving in the field of magnetic monopole.

- Take slowly moving D3 brane, $\partial_m X^I = 0$ ($Q = 1$, for simplicity)

$$S_{BI} = -\frac{1}{8\pi^2}\int d^4x |X|^4 \left[\sqrt{-\det(\eta_{mn} + \frac{F_{mn}}{|X|^2})} - 1\right]$$

denote

$$F^2 = \frac{1}{4}F^{mn}F_{mn}\,, \qquad F^4 = -\frac{1}{8}[F_{mn}F^{nk}F_{kl}F^{lm} - \frac{1}{4}(F_{mn}F^{mn})^2]$$

Series expansion

$$\sqrt{-\det(\eta_{mn} + F_{mn})} = \sqrt{(1+F^2)^2 + 2F^4} = 1 + F^2 + F^4 - F^2F^4 + O(F^8)$$

- Leading terms in the BI action

$$S_{BI} = -\frac{1}{8\pi^2}\int d^4x \left[F^2 + \frac{F^4}{|X|^4} - \frac{F^2F^4}{|X|^8} + \ldots\right]$$

- If the scalars are not constant, $\partial_m X^I \neq 0$, the following scalar terms appear:

F^2 order:

$$\partial_m X^I \partial^m X^I$$

F^4 order:

$$\frac{\partial^m X^I \partial_m X^I F_{rs} F^{rs}}{|X|^4}$$

$$\frac{\partial_m X^I \partial^m X^I \partial_n X^J \partial^n X^J}{|X|^4} \qquad \frac{\partial_m X^I \partial^m X^J \partial_n X^I \partial^n X^J}{|X|^4}$$

$$S_{WZ} = -\frac{N-1}{60\pi^2} \int d^5x\, \varepsilon^{mnklr} \varepsilon_{IJKLMN} \frac{1}{|X|^6} X^I \partial_m X^J \partial_n X^K \partial_k X^L \partial_l X^M \partial_r X^N$$

No terms with double derivative of scalars appear!

$$\partial_m \partial_n X^I$$

Classical action

$$S=\mathrm{tr}\int d^4x\left(-\frac{1}{4}F_{mn}F^{mn}-\frac{1}{2}D_m\phi^I D^m\phi^I+\frac{1}{4}[\phi^I,\phi^J][\phi_I,\phi_J]+\text{fermions}\right)$$

$\Phi=(\phi^I,A_m,\psi^A_\alpha)$ – $\mathcal{N}=4$ gauge multiplet

- All fields are in the adjoint representation of $SU(N)$ gauge group
- The action is invariant under linear $\mathcal{N}=4$ supersymmetry

$$\begin{array}{rcl}\delta\phi^I & \sim & \Gamma^I_{AB}\epsilon^B\psi^A+(\Gamma^I)^{AB}\bar\epsilon_A\bar\psi_B\\ \delta\psi^A & \sim & (\Gamma^I)^{AB}(D_m\phi^I)\gamma^m\bar\epsilon_B+\epsilon^A F_{mn}\sigma^{mn}\\ \delta A^m & \sim & i\psi^A\sigma^m\bar\epsilon_A-i\eta^A\sigma^m\bar\epsilon_A\end{array}$$

$\mathcal{N} = 4$ SYM

Classical action

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- To quantize, we have to choose the vacuum ϕ^I_{vac}

$$V = \mathrm{tr}[\phi^I, \phi^J][\phi_I, \phi_J] \geq 0$$
$$V_{min} = 0 \quad \Rightarrow \quad [\phi^I, \phi^J] = 0$$
ϕ^I_{vac} belong to Cartan subalgebra of $SU(N)$

- In general, $SU(N)$ is broken down $[U(1)]^{N-1}$
  but we consider $SU(N) \to U(1) \times SU(N-1)$

$\mathcal{N} = 4$ SYM effective action

- We are interested in the low-energy effective action for massless fields assuming that all massive fields are integrated out

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- Derivative expansion

$$\begin{aligned}
\Gamma &= \Gamma_0 + \Gamma_2 + \Gamma_4 + \Gamma_6 + \ldots \\
\Gamma_0 &= \int d^4x \mathcal{L}_{eff}(\Phi) \equiv 0 \text{ no effective potential} \\
\Gamma_2 &\sim \int d^4x [\partial_m \phi^I \partial^m \phi^I + \ldots] \equiv S_{class} \\
\Gamma_4 &\sim \int d^4x [\partial_m \phi^I \partial_n \phi^J \partial_p \phi^K \partial_q \phi^L + \ldots] \text{leading non-trivial quantum correcti} \\
\Gamma_6 & \quad \text{next-to-leading quantum corrections}
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- $\mathcal{N} = 4$ SYM is finite, no divergences, conformal

$$\Gamma[\Phi] \text{ is superconformal}$$

1-loop effective action

$$\Gamma^{(1)} = \frac{i}{2}\mathrm{Tr}\ln S''[\Phi]$$

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- Take the background $\phi^I = const$, $F_{mn} = const$, $\psi = 0$

$$\Gamma_4 = \frac{1}{8\pi^2}\int d^4x \frac{F^4}{(\phi^I\phi^I)^2}$$

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- Take the background ϕ^I-arbitrary, $F_{mn} = 0$, $\psi = 0$

$$\Gamma_{WZ} = -\frac{1}{60\pi^2}\int d^5x \varepsilon^{IJKLMN}\frac{1}{|\phi|^6}\phi^I d\phi^J \wedge d\phi^k \wedge d\phi^L \wedge d\phi^M \wedge d\phi^N + \dots$$

[Tseytlin-Zarembo 1999; Intriligator 2000]

- In our papers we constructed the effective Lagrngians in the $\mathcal{N} = 3$ and $\mathcal{N} = 4$ harmonic superspaces which describe the F^4/X^4 and Wess-Zumino terms in the $\mathcal{N} = 4$ SYM effective action

1-loop effective action

The F^4/ϕ^4 and WZ terms coincide with the corresponding terms in the D3 brane.

Naive assumption:

$$\Gamma_{SYM,low-energy} = S_{D3,AdS_5\times S^5}\,, \qquad \phi^I = X^I$$

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Problem: higher-derivative terms

For non-constant scalar fields, at the order F^4/ϕ^4 there appear the following term

$$\frac{(\partial^m\phi^I\partial_m\phi^I)^2}{(\phi^L\phi^L)^2}\,, \qquad \frac{\partial^m\phi^I\partial_m\phi^J\partial^n\phi_I\partial_n\phi_J}{(\phi^L\phi^L)^2}$$

$$\frac{\partial^m\partial^n\phi^I\partial_m\partial_n\phi_I}{\phi^L\phi^L}\,, \qquad \frac{\partial^m\partial^n\phi^I\partial_m\phi_I(\partial_n\phi^J)\phi_J}{(\phi^L\phi^L)^2}$$

There are no such terms in the D3 brane action!

Roughly: $\partial^m\phi \sim \dot{\phi}$ – velocity, $\partial^m\partial^n\phi \sim \ddot{\phi}$ – acceleration
Accelerating D-branes??? [Perival, von Unge 1998]

Resolution of the problem

The higher-derivative terms in the effective action can be eliminated by a redefinition of fields [Gonzalez-Rey, Kulik, Park, Rocek 1998; Kuzenko 2004]

$$X^I = f^I(F_{mn}, \phi^I, \partial_m \phi^I, \partial_m \partial_n \phi^I, \ldots) = \phi^I \left(1 + c\frac{F^{mn}F_{mn}}{(\phi^I \phi^I)^2} + \ldots \right)$$

Such a redefinition eliminates the accelerating terms at the F^4 order!

Why the redefinition is required??

- In $\mathcal{N} = 4$ SYM the supersymmetry is linear, but the effective action involves higher-derivative terms
- The D3 brane action has compact form without higher derivatives, but supersymmetry transformation is very complicated

$\Rightarrow$ These actions can coincide only after a redefinition of fields.

Other motivations for the field redefinition [Bellucci, Ivanov, Krivonos 2002]

- Neglect all field except one scalar field Φ (dilaton)

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- Four Minkowski space coordinates x^m plus dilaton Φ are Goldstone bosons corresponding to the non-linear realization of the conformal group,

$$\delta x^m = cx^m + 2(xb)x^m - x^2 b^m\,, \quad \delta\Phi = c + 2(xb)$$

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Crucial observation

People in field theory and in string theory use different (but equivalent) bases of generators of the $so(2,4)$ algebra!
Consequence: sets (x^m, Φ) and (y^m, ϕ) represent different non-linear realizations of $SO(2,4)$

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- A change of coordinates exists $(x^m, \Phi) \longleftrightarrow (y^m, \phi)$
- Similar change of variables was developed for bosonic fields of the $\mathcal{N}=4$ SYM multiplet to connect the fields of the D3 brane with the ones in the field theory

$$(X^I, F_{mn}) \longleftrightarrow (\phi^I, \mathrm{F}_{mn})$$

What happens at higher orders?

- D3 brane contains the term

$$c_6 \int d^4x \frac{F^6}{|X|^8}$$

- Exactly this term is present in the $\mathcal{N} = 4$ SYM effective action, it is fixed by the $SO(6)$ symmetry, gauge and conformal invariance. However, it is very difficult to compute the coefficient c_6 exactly

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- In QFT c_6 may receive both one- and two-loop quantum corrections. But the two-loop quantum computations are gauge dependent.
- Computations in different gauges give different values of c_6. [Buchbinder, Petrov, Tseytlin, 2002; Kuzenko 2004; Kuzenko, McArthur 2004]
- $\mathcal{N}=3$ superfield approach allows to prove that the coefficient c_6 is one-loop exact. Hence, the F^6/ϕ^8 term can be easily matched exactly with the one in the D3 brane action.

Main lessons

- The action for D3 brane on the $AdS_5 \times S^5$ background is matched with the $\mathcal{N} = 4$ SYM low-energy effective action only for the F^4/X^4, F^6/X^8 and Wess-Zumino terms

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- The next-to-leading (F^8) terms are very hard to compute in quantum field theory
- Even if such terms were computed, they could be matched with the D3 brane action only after some non-linear redefinition of fields.

Open problems

- Compute the $\mathcal{N} = 4$ SYM effective action in the next-to-leading (F^8) order and compare it with the corresponding terms of the D3 brane. (Superfield methods are useful)
- Similar correspondence between M2 brane effective action and ABJM effective action??