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Five lectures on

Quantum Field Theory

An introduction to selected topics

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Summary

- Gauge Theories
- Symmetries (mostly discrete)
- Renormalization
- Anomalies
- Effective Field Theories

Bibliography

These lectures are mostly based on:

- L. Álvarez-Gaumé & M.A. Vázquez-Mozo, "An Invitation to Quantum Field Theory", Springer 2011 (in press).

A very preliminary version of the book is available on the arXiv:

- L. Álvarez-Gaumé & M.A. Vázquez-Mozo, "Introductory Lectures on Quantum field Theory", arXiv:hep-th/0510040

Other useful books are:

- M.E. Peskin & D.V. Schroeder, "An Introduction to Quantum Field Theory", Addison-Wesley 1995.
- A. Zee, "Quantum Field Theory in a nutshell", Princeton 2010.

Plan of the Course

- Lectures:

1st week	Monday 12:30-13:30	Wednesday 10:00-11:00	Friday 10:00-11:00
2nd week	Monday 10:00-11:00	Wednesday 12:30-13:30	

- Practical work (afternoon sessions):

1st week	Monday	Wednesday	Friday
	Tutor: Daniel Fernández		
2nd week	Monday	Wednesday	
	Tutor: Francesco Aprile		

What you are presumed to know

- Advanced quantum mechanics (including path integral methods).
- Elementary quantum field theory (e.g., basics of field quantization, general ideas about Feynman diagrams,...).
- Rudiments of particle physics

But never fear if there is something you don’t know. Ask at any moment and/or bring your questions to the afternoon sessions.

And remember: there are no stupid questions, only stupid answers.

Some warnings before we start:

- We use the "mostly minus" metric (a.k.a. western coast metric):

$$\eta_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

- Unless otherwise said, natural units are used throughout:

$$\hbar = c = 1$$

- We use Heaviside-Lorentz electromagnetic units:

$$\mathbf{F} = \frac{1}{4\pi}\frac{qq'}{r^3}\mathbf{r},$$

(Coulomb)

$$\frac{dF}{d\ell} = \frac{1}{2\pi c^2}\frac{II'}{d}$$

(Àmpere)

and

$$\left\{ \begin{array}{l} \alpha = \dfrac{e^2}{4\pi\hbar c}. \\ e \approx 0.303 \end{array} \right.$$

Lecture 1

Gauge Theories

- Classical gauge theories
- Quantization of gauge theories
- What is gauge invariance?
- Gauge invariance vs. mass: the Brout-Englert-Higgs mechanism.

Gauge invariance in nonrelativistic QM

The most familiar example of a gauge theory is Maxwell electrodynamics

$$\nabla \cdot \mathbf{E} = \rho,$$
$$\nabla \cdot \mathbf{B} = 0,$$
$$\nabla \times \mathbf{E} = -\frac{\partial}{\partial t}\mathbf{B},$$
$$\nabla \times \mathbf{B} = \mathbf{j} + \frac{\partial}{\partial t}\mathbf{E},$$

$$\mathbf{E} = -\nabla\varphi - \frac{\partial \mathbf{A}}{\partial t},$$
$$\mathbf{B} = \nabla \times \mathbf{A}.$$

When expressed in terms of the scalar and vector potentials, the Maxwell equations are invariant under the gauge transformations:

$$\varphi(t,\mathbf{x}) \to \varphi(t,\mathbf{x}) + \frac{\partial}{\partial t}\varepsilon(t,\mathbf{x}), \qquad \mathbf{A}(t,\mathbf{x}) \to \mathbf{A}(t,\mathbf{x}) - \nabla\varepsilon(t,\mathbf{x}).$$

with $\varepsilon(t,\mathbf{x})$ an arbitrary function.

Physical (i.e., measurable) quantities have to be gauge invariant.

In quantum mechanics, however, the Schrödinger equation depends on the (gauge dependent) electromagnetic potentials

$$i\frac{\partial}{\partial t}\Psi = \left[-\frac{1}{2m}(\nabla - iq\mathbf{A})^2 + q\varphi\right]\Psi.$$

Under a gauge transformation

$$\varphi(t,\mathbf{x}) \to \varphi(t,\mathbf{x}) + \frac{\partial}{\partial t}\varepsilon(t,\mathbf{x}), \qquad \mathbf{A}(t,\mathbf{x}) \to \mathbf{A}(t,\mathbf{x}) - \nabla\varepsilon(t,\mathbf{x}).$$

the wave equation remains invariant provided the wave function is multiplied by a *nonconstant* phase:

$$\Psi(t,\mathbf{x}) \longrightarrow e^{-iq\varepsilon(t,\mathbf{x})}\Psi(t,\mathbf{x}).$$

Hence, gauge invariance means that the global phase of the wave function can be changed *locally*.

Classical gauge field theories I

The abelian case

The Maxwell equations can be recast in a Lorentz covariant form using the four-vector potential $A^\mu = (\varphi, \mathbf{A})$ and the covariant field strength tensor

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \Longrightarrow \begin{cases} \partial_\mu F^{\mu\nu} = j^\mu \\ \varepsilon^{\mu\nu\sigma\eta}\partial_\nu F_{\sigma\eta} = 0 \end{cases}$$

(Bianchi identities)

with $j^\mu = (\rho, \mathbf{j})$. Gauge transformations now read

$$A_\mu \longrightarrow A_\mu + \partial_\mu \varepsilon$$

The Maxwell equations are now derived from the gauge invariant Lagrangian

$$\mathscr{L}_{\text{Maxwell}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

We saw how in quantum mechanics the Schrödinger equation of a charged particle is obtained by promoting the global phase invariance of the wave function to a local symmetry (this is called the *gauge principle*).

To find the electromagnetic coupling of a complex classical field we use the same guiding principle and *gauge* the global phase symmetry.

For the Dirac field, for example,

$$\mathscr{L}_{\text{Dirac}} = \overline{\psi}\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi \quad \text{in invariant under} \quad \psi \to e^{-iq\varepsilon}\psi$$

To make this invariance local we need to replace the ordinary derivative ∂_μ by a covariant one D_μ transforming under $\psi \to \psi' = e^{-iq\varepsilon(x)}\psi$ as

$$D_\mu \to D'_\mu \quad \text{with} \quad D'_\mu \psi' = D'_\mu \left[e^{-iq\varepsilon(x)}\psi\right] = e^{-iq\varepsilon(x)} D_\mu \psi$$

Such a covariant derivative can be constructed from the gauge potential as

$$D_\mu = \partial_\mu + iqA_\mu$$

$$[D_\mu, D_\nu] = iqF_{\mu\nu}$$

(keep in mind for later use)

With this we can write the Lagrangian of QED (i.e., a Dirac fermion coupled to the electromagnetic field)

$$\begin{aligned}\mathscr{L}_{\text{QED}} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{\psi}(i\not{D} - m)\psi \\ &= -\frac{1}{4}F_{\mu\nu}^{\mu\nu} + \overline{\psi}(i\not{\partial} - m)\psi - qA_\mu\overline{\psi}\gamma^\mu\psi\end{aligned}$$

invariant under the gauge transformations

$$\psi \longrightarrow e^{-iq\varepsilon(x)}\psi \qquad\qquad A_\mu \longrightarrow A_\mu + \partial_\mu\varepsilon(x)$$

In particular, the QED Lagrangian is invariant under *global transformations* with constant ε. Noether's theorem implies the existence of a conserved current

$$j^\mu = q\overline{\psi}\gamma^\mu\psi \qquad\qquad \partial_\mu j^\mu = 0$$

which is identified with the electric four-current in the Maxwell equations.

Chen Ning Yang
b. 1922

Classical gauge field theories II

Yang-Mills theories

Robert Mills
(1927-1999)

To construct nonabelian generalization of QED we begin by considering the a Lie group *G* whose generators satisfy the Lie algebra

$$[T^A, T^B] = if^{ABC}T^C \qquad A, B, C = 1, \ldots, \dim G$$

As generalization of the photon field we introduce the Lie-algebra-valued field

$$A_\mu \equiv A^A_\mu T^A$$

with a gauge transformation given by:

$$A_\mu \longrightarrow A'_\mu = -\frac{1}{ig_{\rm YM}} U\partial_\mu U^{-1} + UA_\mu U^{-1} \qquad U = e^{i\chi(x)}$$

$$\delta A_\mu = \frac{1}{g_{\rm YM}}\partial_\mu \chi - i[A_\mu, \chi]$$

We consider now a "matter" field Φ transforming in a representation $\mathbf{R}$ of the gauge group

$$\Phi \longrightarrow \Phi' = U_{\mathbf{R}}\Phi \qquad\qquad U_{\mathbf{R}} \in G$$

Following the the abelian case, we couple Φ to the nonabelian field A_μ^A by replacing ordinary derivatives by covariant ones in the globally invariant Lagrangian

$$\mathscr{L}_{\text{matter}}(\Phi, \partial_\mu \Phi) \Longrightarrow \mathscr{L}_{\text{matter}}(\Phi, D_\mu \Phi) \qquad \text{with} \qquad D'_\mu \Phi' = U_{\mathbf{R}} D_\mu \Phi$$

The covariant derivative can be written in terms of the gauge field as

$$D_\mu \Phi = \partial_\mu \Phi - i g_{\text{YM}} A_\mu \Phi \qquad \text{where} \qquad A_\mu = A_\mu^A T_{\mathbf{R}}^A$$

Exercise: prove it

When Φ transforms in the adjoint representation, the covariant derivative takes the form

$$(T^A_{\text{adj}})^B_C = -i f^{ABC} \quad \Longrightarrow \quad D_\mu \Phi = \partial_\mu \Phi - i g_{\text{YM}}[A_\mu, \Phi]$$

The transformation of the gauge field can be written as $\delta A_\mu = \dfrac{1}{g_{\text{YM}}} D_\mu \chi$

We still need to build an action functional for the nonabelian gauge fields. We define the field strength by

$$[D_\mu, D_\nu] = -ig_{\rm YM} F_{\mu\nu}$$

An explicit calculation shows

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig_{\rm YM}[A_\mu, A_\nu] \qquad (F_{\mu\nu} = F^A_{\mu\nu} T^A)$$

Applying the transformation of the covariant derivative we find

$$F_{\mu\nu} \longrightarrow U F_{\mu\nu} U^{-1}$$

Exercise: prove it

A gauge invariant Lagrangian quadratic in derivatives can be now written as

$$\mathscr{L} = -\frac{1}{2}\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right) = -\frac{1}{4}F^A_{\mu\nu}F^{A\,\mu\nu}$$

This Lagrangian contains terms $\mathscr{O}(A_\mu^3)$, $\mathscr{O}(A_\mu^4)$. Exercise: write them

where the generators are normalized according to

$$\mathrm{Tr}\left(T^A T^B\right) = \tfrac{1}{2}\delta^{AB}$$

Quantization I: the Abelian case

The quantization of the field $A^\mu(x)$ is complicated by the gauge ambiguity. There are various strategies to deal with the problem. Our approach begins with eliminating the unphysical degrees of freedom by *fixing the gauge*

$$\partial_\mu A^\mu = 0 \qquad \text{(Lorentz condition)}$$

Imposing this condition, the equations of motion are

$$\partial_\mu F^{\mu\nu} = 0 \quad \Longrightarrow \quad 0 = \partial_\mu \partial^\mu A^\nu - \partial^\nu (\partial_\mu A^\mu) = \partial_\mu \partial^\mu A^\nu$$

so the gauge field satisfies a massless Klein-Gordon equation, with plane wave (positive energy) solutions

$$\varepsilon_\mu(\mathbf{k}, \lambda) e^{-i|\mathbf{k}|t + i\mathbf{k}\cdot\mathbf{x}}$$

In principle, there would be four independent polarizations ($\lambda = 0, 1, 2, 3$)

$\varepsilon_\mu(\mathbf{k}, 0) \sim \delta_\mu^{\ 0}$	$\varepsilon_\mu(\mathbf{k}, 3) \sim \delta_\mu^{\ i} k_i$	$\varepsilon_\mu(\mathbf{k}, 1), \ \ \varepsilon_\mu(\mathbf{k}, 2)$
(temporal)	(longitudinal)	(transverse to $\mathbf{k}$)

The plane wave solutions, however, should satisfy the Lorentz condition:

$$k^\mu \varepsilon_\mu(\mathbf{k},\lambda) = k^\mu \varepsilon_\mu(\mathbf{k},\lambda)^* = 0$$

This can be used to set the temporal polarization to zero

Exercise!

$$\varepsilon_\mu(\mathbf{k},0) = 0 \quad \Longrightarrow \quad \text{no negative probability states!}$$

The Lorentz condition does not fix completely the gauge. $\partial_\mu A^\mu = 0$ is preserved by gauge transformations

$$A_\mu(x) \longrightarrow A_\mu(x) + \partial_\mu \varepsilon(x) \qquad \text{with} \qquad \partial_\mu \partial^\mu \varepsilon(x) = 0$$

Using the residual gauge transformation of the polarization vectors we eliminate the longitudinal polarization

$$\varepsilon_\mu(\mathbf{k},\lambda) \longrightarrow \varepsilon_\mu(\mathbf{k},\lambda) + \alpha k_\mu \quad \Longrightarrow \quad \varepsilon_\mu(\mathbf{k},3) = 0$$

This leaves us with *two transverse propagating modes:*

$$\varepsilon(\mathbf{k},\lambda) \qquad \lambda = 1,2$$

Having got rid of the spurious states, we proceed to quantize $A^\mu(x)$

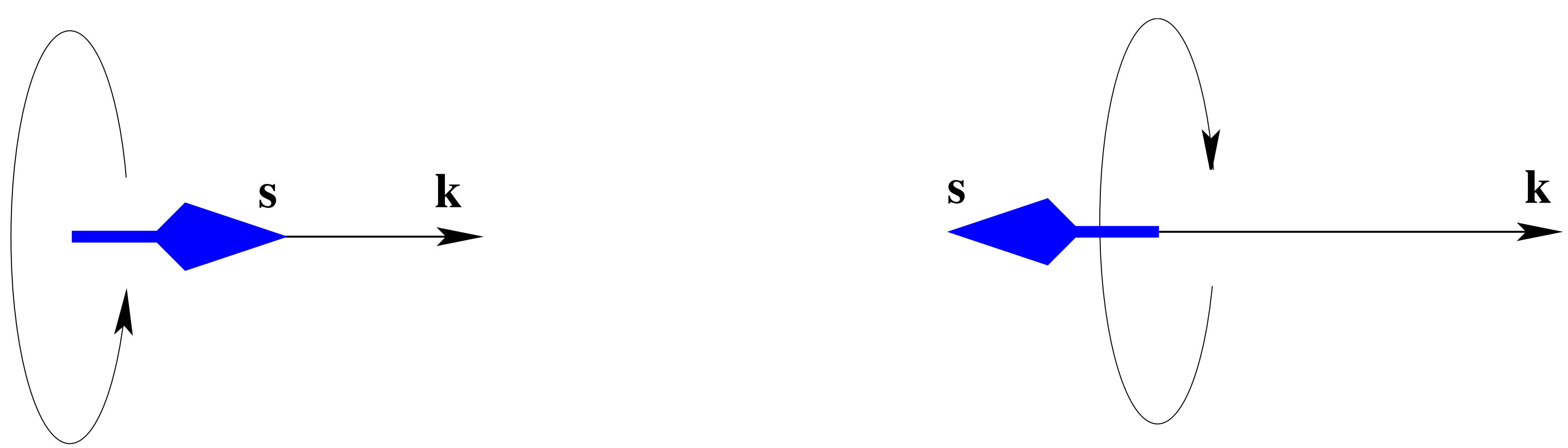
$$\hat{A}_\mu(t,\mathbf{x}) = \sum_{\lambda=1,2}\int \frac{d^3k}{(2\pi)^3}\frac{1}{2|\mathbf{k}|}\left[\varepsilon_\mu(\mathbf{k},\lambda)\hat{a}(\mathbf{k},\lambda)e^{-i|\mathbf{k}|t+i\mathbf{k}\cdot\mathbf{x}} + \varepsilon_\mu(\mathbf{k},\lambda)^*\,\hat{a}^\dagger(\mathbf{k},\lambda)e^{i|\mathbf{k}|t-i\mathbf{k}\cdot\mathbf{x}}\right]$$

where $\hat{a}^\dagger(\mathbf{k},\lambda)$ creates a photon of momentum $\mathbf{k}$ and polarization λ out of the vacuum $|0\rangle$

$$[\hat{a}(\mathbf{k},\lambda),\hat{a}^\dagger(\mathbf{k}',\lambda')] = (2\pi)^3(2|\mathbf{k}|)\delta(\mathbf{k}-\mathbf{k}')\delta_{\lambda\lambda'}$$

$$[\hat{a}(\mathbf{k},\lambda),\hat{a}(\mathbf{k}',\lambda')] = [\hat{a}^\dagger(\mathbf{k},\lambda),\hat{a}^\dagger(\mathbf{k}',\lambda')] = 0$$

The two physical polarizations can be taken to be the two helicity states



$\lambda = 1$ (positive helicity) $\lambda = 2$ (negative helicity)

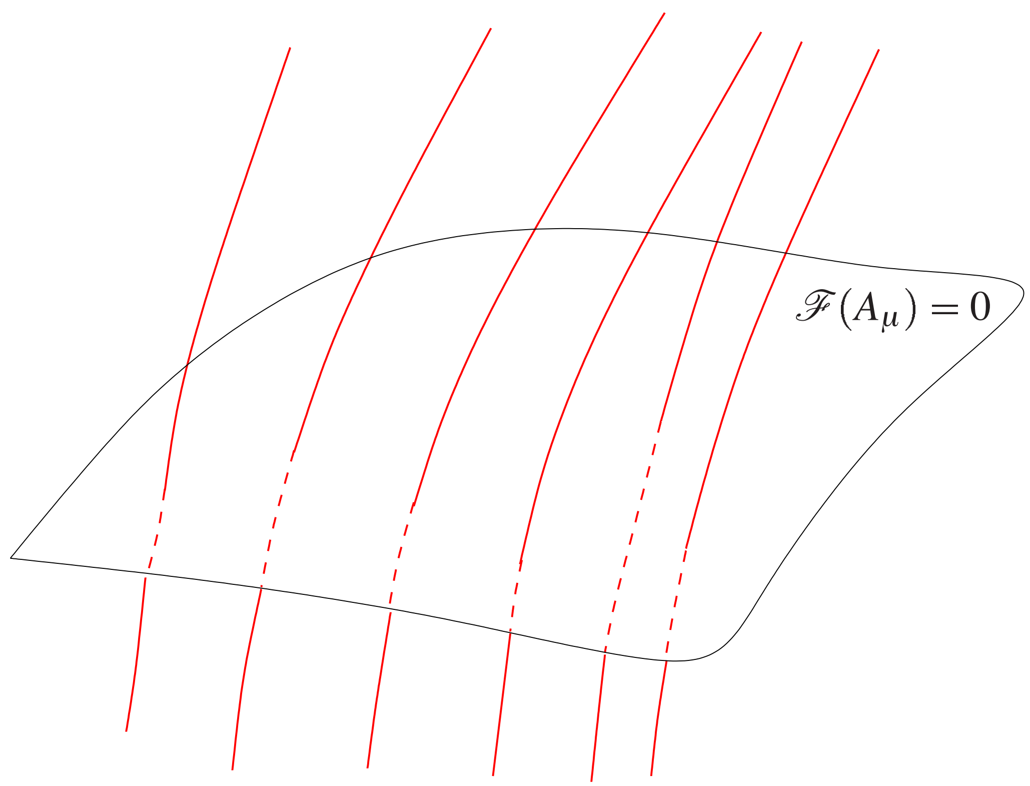
Generically, the basic issue in the quantization of gauge theories is avoiding overcounting physically equivalent configurations. For example, in

$$\mathscr{Z} = \int \mathscr{D}\overline{\psi}\,\mathscr{D}\psi\,\mathscr{D}A_\mu\, e^{iS_{\text{QED}}[A_\mu,\overline{\psi},\psi]}$$

we have to integrate over field configurations that are not "gauge equivalent".

To factor out the gauge redundancy we introduce an *appropriate* gauge fixing condition

$$\mathscr{F}(A_\mu) = 0$$



Each field configuration belongs to an orbit that intersect the gauge fixing slice. We introduce the identity in the form

$$1 = \Delta_{\text{FP}}[A_\mu] \int \mathscr{D}U\, \delta\Big[\mathscr{F}(A_\mu^U)\Big]$$

$\Delta_{\text{FP}}[A_\mu]$ is called the Faddeev-Popov determinant and it is gauge invariant.

Exercise: prove it

Inserting the identity we are left with

$$\mathscr{Z} = \int \mathscr{D}\overline{\psi}\mathscr{D}\psi\mathscr{D}A_\mu\mathscr{D}U\,\Delta_{\text{FP}}[A_\mu]\delta[\mathscr{F}(A_\mu^U)]\,e^{iS_{\text{QED}}[A_\mu,\overline{\psi},\psi]}$$

Changing variables to $A_\mu \to A_\mu^{U^{-1}}, \psi \to U^{-1}\psi$, and using the invariance of the action, we have

$$\mathscr{Z} = \underbrace{\left(\int \mathscr{D}U\right)}_{\dim G}\int \mathscr{D}\overline{\psi}\mathscr{D}\psi\mathscr{D}A_\mu\Delta_{\text{FP}}[A_\mu]\delta[\mathscr{F}(A_\mu)]\,e^{iS_{\text{QED}}[A_\mu,\overline{\psi},\psi]}$$

With this we have factored out the gauge redundancy. The (divergent) prefactor cancels out of the amplitudes.

For a more explicit expression of the Faddeev-Popov determinant we use

$$\delta[g(x)] = \sum_{x_i=\text{zeros of }g}\frac{1}{|g'(x_i)|}\delta(x-x_i) \quad\Longrightarrow\quad \delta\left[\mathscr{F}(A_\mu^U)\right] = \left|\det\left[\left.\frac{\delta\mathscr{F}(A_\mu^U)}{\delta U}\right|_{U=U'}\right]\right|^{-1}\delta(U-U')$$

This leads to: $\Delta_{\text{FP}}[A_\mu] = \det\left[\frac{\delta\mathscr{F}(A_\mu^U)}{\delta U}\right]\Bigg|_{U=\mathbf{1}}$

Hence the name!

The value gauge-fixed path integral should not change if you move the gauge slice, i.e.,

$$\mathscr{Z} = \int \mathscr{D}\overline{\psi}\mathscr{D}\psi\mathscr{D}A_\mu \Delta_{\mathrm{FP}}[A_\mu]\delta[\mathscr{F}(A_\mu) - f(x)]\, e^{iS_{\mathrm{QED}}[A_\mu,\overline{\psi},\psi]}$$

should be independent of $f(x)$, Hence, we insert the constant term

$$\int \mathscr{D}f\, e^{-\frac{i}{2\xi}\int d^4x\, f(x)^2} = \text{constant}$$

and integrate over $f(x)$ using the functional delta

$$\mathscr{Z} = \int \mathscr{D}\overline{\psi}\mathscr{D}\psi\mathscr{D}A_\mu \Delta_{\mathrm{FP}}[A_\mu]\, e^{iS_{\mathrm{QED}}[A_\mu,\overline{\psi},\psi] - \frac{i}{2\xi}\int d^4x\, [\mathscr{F}(x)]^2}$$

(Remember: global constants in the path integral are irrelevant!)

We have eliminated the gauge redundancy by introducing the Faddeev-Popov determinant in the functional integral and adding a gauge fixing term to the action.

The value gauge-fixed path integral should not change if you move the gauge slice, i.e.,

$$\mathscr{Z} = \int \mathscr{D}\overline{\psi}\mathscr{D}\psi\mathscr{D}A_\mu \Delta_{\rm FP}[A_\mu]\delta[\mathscr{F}(A_\mu) - f(x)]\, e^{iS_{\rm QED}[A_\mu,\overline{\psi},\psi]}$$

should be independent of $f(x)$, Hence, we insert the constant term

$$\int \mathscr{D}f\, e^{-\frac{i}{2\xi}\int d^4x\, f(x)^2} = \text{constant}$$

and integrate over $f(x)$ using the functional delta

gauge fixing term

$$\mathscr{Z} = \int \mathscr{D}\overline{\psi}\mathscr{D}\psi\mathscr{D}A_\mu \Delta_{\rm FP}[A_\mu]\, e^{iS_{\rm QED}[A_\mu,\overline{\psi},\psi] - \frac{i}{2\xi}\int d^4x\, [\mathscr{F}(x)]^2}$$

(Remember: global constants in the path integral are irrelevant!)

We have eliminated the gauge redundancy by introducing the Faddeev-Popov determinant in the functional integral and adding a gauge fixing term to the action.

We can use the Lorentz condition $\mathscr{F}(A_\mu) = \partial_\mu A^\mu$. The Faddeev-Popov determinant is now independent of the gauge field

$$\mathscr{F}(A_\mu^U) = \partial_\mu A^\mu + \partial_\mu \partial^\mu \varepsilon \quad \Longrightarrow \quad \Delta_{\text{FP}}[A_\mu] = \left| \det \left(-\frac{1}{ie} \partial_\mu \partial^\mu \right) \right|$$

It can be factored out of the path integral, so we are left with

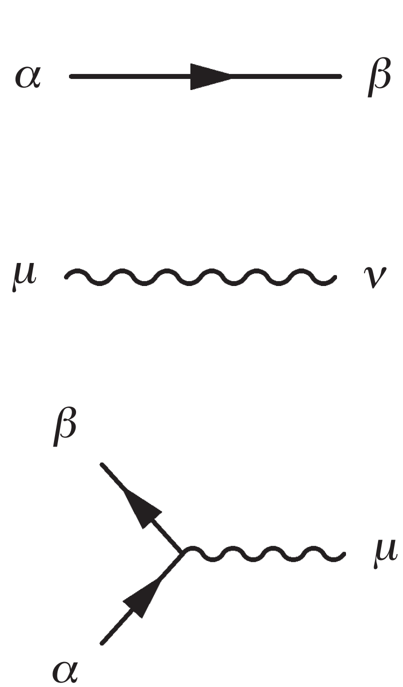
$$\mathscr{Z}_{\text{QED}} = \int \mathscr{D}\overline{\psi} \mathscr{D}\psi \mathscr{D}A_\mu e^{i(S_{\text{QED}} + S_{\text{gf}})}$$

where

$$S_{\text{QED}} + S_{\text{gf}} = \int d^4x \left[\underbrace{\overline{\psi}(i\not{D} - m)\psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}}_{S_{\text{QED}}} - \underbrace{\frac{1}{2\xi} (\partial_\mu A^\mu)^2}_{S_{\text{gf}}} \right]$$

The constant ξ is called the gauge-fixing parameter. It can be chosen arbitrarily.

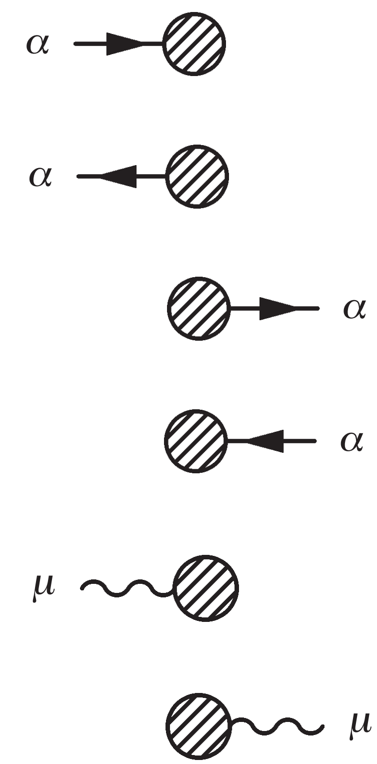
To quantize QED we resort to perturbation theory. The contributions to each order in the electric charge are computed using the Feynman rules (in the Feynman gauge $\xi = 1$):



$$\Longrightarrow \left(\frac{i}{\not{p} - m + i\varepsilon}\right)_{\beta\alpha}$$

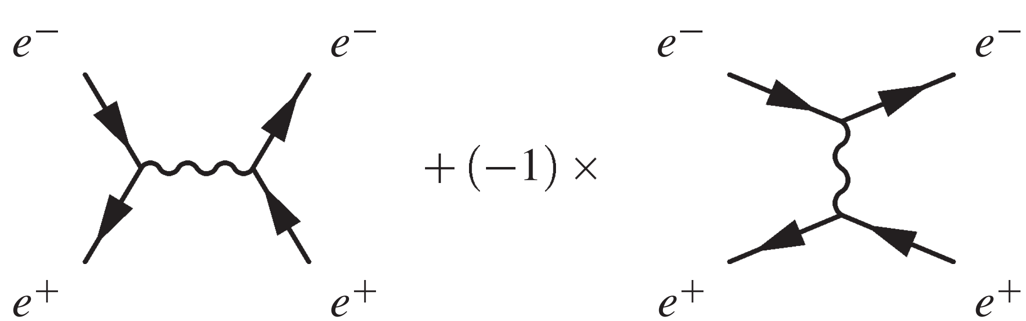
$$\Longrightarrow \frac{-i\eta_{\mu\nu}}{p^2 + i\varepsilon}$$

$$\Longrightarrow -ie\gamma^{\mu}_{\beta\alpha}.$$



Incoming fermion:	$\Longrightarrow u_\alpha(\mathbf{p}, s)$
Incoming antifermion:	$\Longrightarrow \overline{v}_\alpha(\mathbf{p}, s)$
Outgoing fermion:	$\Longrightarrow \overline{u}_\alpha(\mathbf{p}, s)$
Outgoing antifermion:	$\Longrightarrow v_\alpha(\mathbf{p}, s)$
Incoming photon:	$\Longrightarrow \varepsilon_\mu(\mathbf{p})$
Outgoing photon:	$\Longrightarrow \varepsilon_\mu(\mathbf{p})^*$

For example, in the case of Bhabha scattering $e^+ + e^- \longrightarrow e^+ + e^-$ at leading order in e we have to compute the contribution of the two diagrams:



Watch out for the relative minus sign!

Quantization II: the Yang-Mills case

The path integral quantization of nonabelian gauge theories can be done using the Faddeev-Popov trick we just introduced

$$\mathscr{Z} = \int \mathscr{D}A_\mu \, \Delta_{\rm FP}[A_\mu] \, \delta\Big[\mathscr{F}(A_\mu)\Big] \, e^{-\frac{i}{2}\int d^4x \, \mathrm{Tr}\,(F_{\mu\nu}F^{\mu\nu})}$$

$$= \int \mathscr{D}A_\mu \, \Delta_{\rm FP}[A_\mu] \, e^{i\int d^4x \, \mathrm{Tr}\left[-\frac{1}{2}F_{\mu\nu}F^{\mu\nu} - \frac{1}{\xi}\mathscr{F}(A_\mu)^2\right]}$$

where again,

$$\Delta_{\rm FP}[A_\mu] = \det\left[\frac{\delta\mathscr{F}(A_\mu^U)}{\delta U}\right]\Bigg|_{U=\mathbf{1}}$$

Using the Lorentz gauge condition $\mathscr{F}(A_\mu) = \partial_\mu A^\mu$ we find

$$\delta A_\mu = \frac{1}{g_{\rm YM}} D_\mu \chi \quad \Longrightarrow \quad \Delta_{\rm FP}[A_\mu] = \left|\det\left(\frac{1}{i g_{\rm YM}}\partial_\mu D^\mu\right)\right|$$

The Faddeev-Popov determinant depends now on the gauge field and cannot be factored out of the integral.

A very practical way to handle the Faddeev-Popov determinant is by representing it using *anticommuting* complex scalar fields, called *Faddeev-Popov ghosts*

$$\Delta_{\rm FP}[A_\mu] = \int \mathscr{D}\bar{c}\mathscr{D}c\, e^{i\int d^4x\,\bar{c}\,\partial_\mu D^\mu\, c}$$

Alternatively, we can use a gauge condition fixing the gauge completely. For example the axial gauge condition,

$$n^\mu A_\mu = 0 \qquad \text{with} \qquad n_\mu n^\mu < 0$$

Not Lorentz covariant!

Using this condition, we have

$$\Delta_{\rm FP}[A_\mu] = \left|\det\left(\frac{1}{ig_{\rm YM}} n^\mu D_\mu\right)\right| \underset{n^\mu A_\mu = 0}{=} \left|\det\left(\frac{1}{ig_{\rm YM}} n^\mu \partial_\mu\right)\right|$$

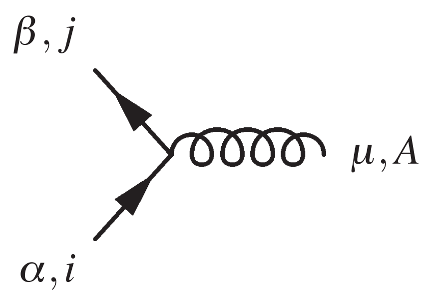
and the Faddeev-Popov determinant can be again factored out

$$\mathscr{Z} = \int \mathscr{D}A_\mu\, \delta[n^\nu A_\nu]\, e^{-\frac{i}{2}\int d^4x {\rm Tr}\,(F_{\mu\nu}F^{\mu\nu})} = \int \mathscr{D}A_\mu\, e^{i\int d^4x {\rm Tr}\,(-\frac{1}{2}F_{\mu\nu}F^{\mu\nu} - \frac{1}{\xi} n^\mu n^\nu A_\mu A_\nu)}$$

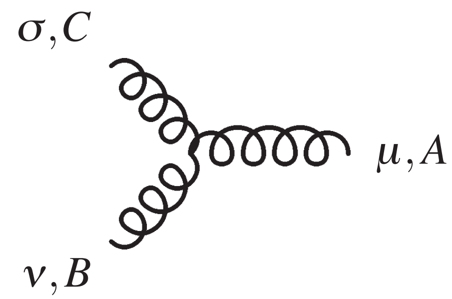
The perturbative quantization of a Yang-Mills field coupled to a Dirac fermion leads to the Feynman rules (we work again in the Feynman gauge $\xi = 1$)

$$\Longrightarrow \quad \left(\frac{i}{\not p - m + i\varepsilon}\right)_{\beta\alpha}\delta_{ij}$$

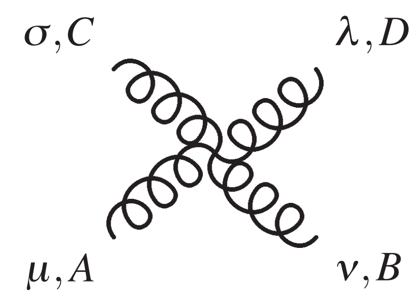
$$\Longrightarrow \quad \frac{-i\eta_{\mu\nu}}{p^2 + i\varepsilon}\delta^{AB}$$



$$\Longrightarrow \quad -ig\gamma^{\mu}_{\beta\alpha}t^{A}_{ij}$$

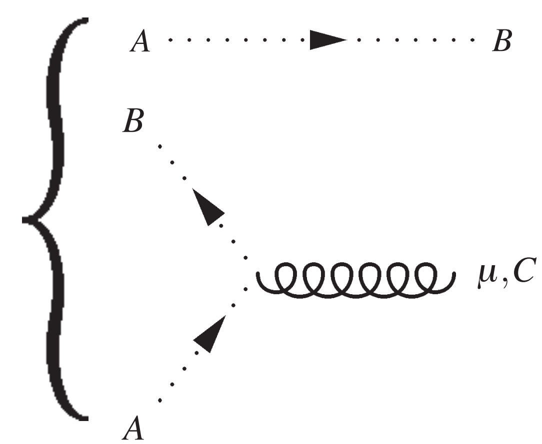


$$\Longrightarrow \quad g\, f^{ABC}\left[\eta^{\mu\nu}(p_1^{\sigma} - p_2^{\sigma}) + \text{permutations}\right]$$



$$\Longrightarrow \quad -ig^2\Big[f^{ABE}f^{CDE}\left(\eta^{\mu\sigma}\eta^{\nu\lambda} - \eta^{\mu\lambda}\eta^{\nu\sigma}\right) + \text{permutations}\Big]$$

+ ghost propagator and vertex



Ghosts cannot appear as asymptotic states (only in loops).

Unlike the photon, the Yang-Mills field couples to itself. This has very important physical consequences (more on this in two lectures).

What is gauge invariance?

Although in many occasions we talk about "gauge symmetry", *gauge invariance is not a symmetry, but rather a* ***redundancy***.

In quantum mechanics, a symmetry relates *different* quantum states that have the *same energy*. For example:

$$|\alpha, j, m\rangle \xrightarrow{\text{SO(3)}} |\alpha, j, m'\rangle = \sum_{m=-j}^{j} \mathscr{D}^{(j)}_{m'm}(\theta,\varphi)|n, j, m\rangle$$

$$\widehat{H}|\alpha, j, m\rangle = E_{\alpha,j}|\alpha, j, m\rangle$$

Gauge transformations, on the other hand, relate states that are physically identical:

$$|\text{phys}\rangle \xrightarrow{\text{gauge transformation}} |\text{phys}'\rangle = |\text{phys}\rangle$$

As a consequence, the Hilbert space of the quantum theory is redundant. Morally speaking, $\mathscr{H}_{\text{phys}} = \mathscr{H}/(\text{gauge transformations})$

Gauge invariance is imposed on us as the prize of keeping Lorentz invariance explicit. Using it we describe the two propagating degrees of freedom of a Yang-Mills field and their interactions in a Lorentz invariant way.

Masses and gauge invariance

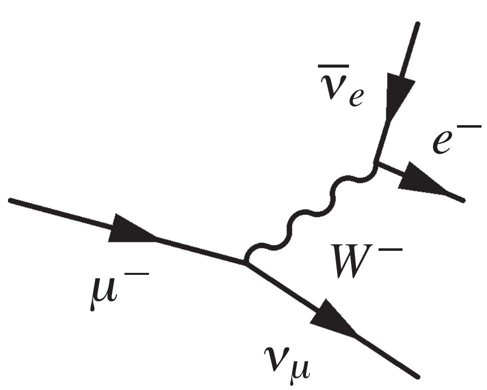
The phenomenology of weak interactions at low energies requires the introduction of massive vector bosons.

A current-current interaction can be “resolved” by the interchange of an intermediate vector boson, provided this is massive, e.g.

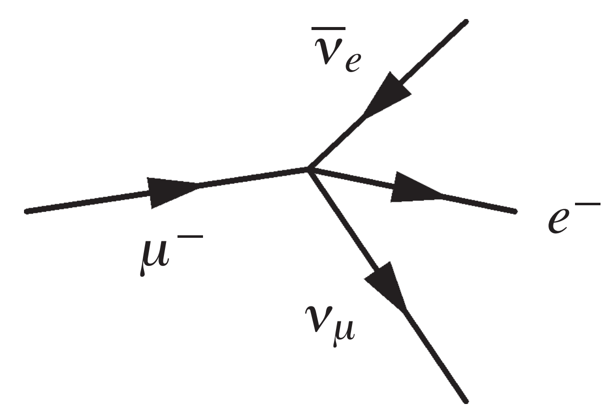
$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{m^2}{2}A_\mu A^\mu - gA_\mu J^\mu \xrightarrow[\text{integrating out } A_\mu]{E \ll m} \mathscr{L} = -\frac{g^2}{2m^2}J_\mu J^\mu$$

Exercise: prove it

Diagrammatically (in the case of the muon decay):



$E \ll m_W$ ⟹



$$g^2[\overline{\nu}_\mu\gamma^\mu(1-\gamma_5)\mu]\,\frac{-i\eta_{\mu\nu}}{q^2-m_W^2}\,[\overline{e}\gamma^\nu(1-\gamma_5)\nu_e]$$

$$\frac{G_F}{\sqrt{2}}[\overline{\nu}_\mu\gamma^\mu(1-\gamma_5)\mu][\overline{e}\gamma^\nu(1-\gamma_5)\nu_e]$$

To study the physics of massive vector bosons, we begin with the simplest example: *a massive photon* (Proca Lagrangian)

$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{m^2}{2}A_\mu A^\mu - j_\mu A^\mu \qquad \text{with} \qquad \partial_\mu j^\mu = 0$$

Alexander Proca
(1897-1955)

The Lagrangian is not gauge invariant

$$\delta A_\mu = \partial_\mu \varepsilon \quad \Longrightarrow \quad \delta\mathscr{L} = m^2 A^\mu \partial_\mu \varepsilon$$

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But don't panic yet! taking the divergence of the field equations

$$\partial_\mu F^{\mu\nu} + m^2 A^\nu = j^\nu$$

we find the integrability condition

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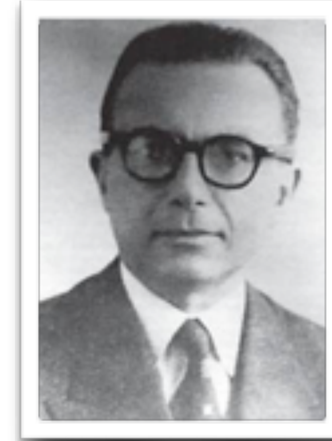

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$$\cancel{\partial_\mu \partial_\nu F^{\mu\nu}} + m^2 \partial_\nu A^\nu = \cancel{\partial_\nu j^\nu} \quad \underset{m \neq 0}{\Longrightarrow} \quad \partial_\mu A^\mu = 0$$

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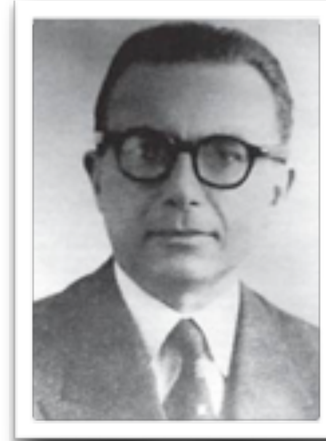

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emporal

<u>An issue for discussion:</u>

Using the Lorentz condition $\partial_\mu A^\mu = 0$ the gauge variation of the Proca Lagrangian can be written as a total derivative

$$\delta\mathscr{L} = m^2 A^\mu \partial_\mu \varepsilon = \partial_\mu (m^2 A^\mu \varepsilon)$$

Does it mean that gauge invariance is "restored"?

we find the integrability condition

$$\cancel{\partial_\mu\partial_\nu F^{\mu\nu}} + m^2 \partial_\nu A^\nu = \cancel{\partial_\nu j^\nu} \quad \xrightarrow[m \neq 0]{} \quad \partial_\mu A^\mu = 0$$

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The condition eliminates the temporal polarization, we do not have any further condition, so the massive photon has ***three*** physical polarizations (*two transverse* and *one longitudinal*).

We can get a glimpse of the problems behind massive vector bosons looking at the propagator:

$$G_{\mu\nu}(p) = \frac{i}{p^2 - m^2 + i\varepsilon}\left(-\eta_{\mu\nu} + \frac{p_\mu p_\nu}{m^2}\right)$$

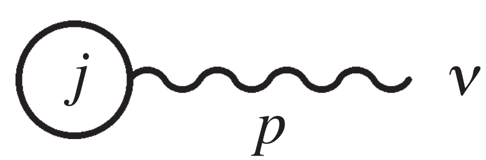
Exercise: get this propagator from the Proca Lagrangian

unlike the massless propagator, it doesn't decreases at large momentum.

This spells trouble with renormalizability (more in two lectures) and unitarity.

This offending term cancels when the massive photon is coupled to a conserved current

$$\partial_\mu j^\mu(x) = 0 \quad \Longrightarrow \quad p_\mu j^\mu(p) = 0$$



$\sim j^\mu(p) G_{\mu\nu}(p)$

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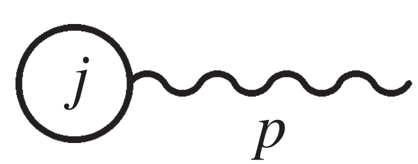

$\nu \quad \sim j^\mu(p) G_{\mu\nu}(p)$

In the case of QED with a massive photon, the longitudinal polarization decouples from the transverse ones and the theory is renormalizable and unitary.

A word of warning: the massless limit is "singular"

$$G_{\mu\nu}(p) = \frac{i}{p^2 - m^2 + i\varepsilon}\left(-\eta_{\mu\nu} + \frac{p_\mu p_\nu}{m^2}\right)$$

The extension of this result to nonabelian gauge theories is not possible in general:

$$\mathscr{L} = -\frac{1}{2}\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right) + M^2\mathrm{Tr}\left(A_\mu A^\mu\right)$$

- Because of the self-interaction of the gauge fields, not all longitudinal components decouple.
- In realistic cases (e.g., weak interactions) the gauge field is coupled to currents that are not conserved at low energies.

To be more specific, we follow a bottom-up approach starting with the experimental fact that massive vector bosons exists.

To keep things simple we look at a “toy standard model” (for the real thing wait for Nuria’s lectures).

We consider a theory of massive fermion doublets transforming chirally under SU(2)

$$\Psi_L(x) \longrightarrow g(x)\Psi_L(x), \qquad \Psi_R(x) \longrightarrow \Psi_R(x) \qquad g(x) = e^{i\chi(x)} \in \mathrm{SU}(2)$$

and coupled to a massive SU(2) gauge boson

$$\mathscr{L} = -\frac{1}{2}\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right) + M^2\mathrm{Tr}\left(A_\mu A^\mu\right) + i\overline{\Psi}_L \not{D}\Psi_L + i\overline{\Psi}_R \not{\partial}\Psi_R - m\left(\overline{\Psi}_L\Psi_R + \overline{\Psi}_R\Psi_L\right)$$

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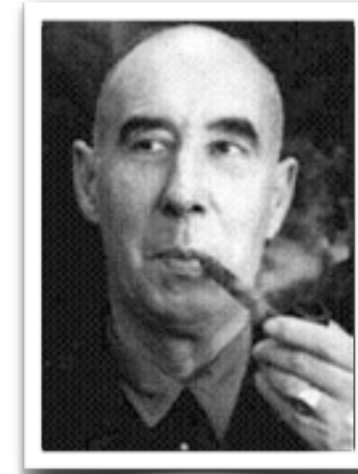
Gauge invariance is broken by the mass terms

$$\delta\mathscr{L} = \frac{2M^2}{g_{\mathrm{YM}}}\mathrm{Tr}\left(A^\mu D_\mu\chi\right) + im\left(\overline{\Psi}_L\chi\Psi_R - \overline{\Psi}_R\chi\Psi_L\right)$$

Gauge invariance can be "restored" using Stückelberg's trick: we introduce a field $U(x)$ taking values in the group SU(2) and transforming according to



Ernst C. G. Stückelberg
(1905-1984)

$$U(x) \longrightarrow g(x)U(x)$$

The Lagrangian

$$\mathscr{L} = -\frac{1}{2}\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right) - \frac{M^2}{g_{\mathrm{YM}}^2}\mathrm{Tr}\left[(U^\dagger D_\mu U)(U^\dagger D^\mu U)\right] + i\overline{\Psi}_L \not{D}\Psi_L + i\overline{\Psi}_R \not{\partial}\Psi_R - m\left(\overline{\Psi}_L U \Psi_R + \overline{\Psi}_R U^\dagger \Psi_L\right)$$

is gauge invariant.

We can use the gauge freedom to fix the gauge $U(x) = \mathbf{1}$. This gives back the original massive Lagrangian.

$$U^\dagger D_\mu U \quad \xrightarrow{U=\mathbf{1}} \quad -i g_{\mathrm{YM}} A_\mu$$

By fixing the gauge, the Stückelberg field $U(x)$ transforms into the longitudinal components of the three massive gauge fields

$$U(x) = \exp\left[i\pi^a(x)\frac{\sigma_a}{2}\right] \quad \Longrightarrow \quad A_3^a(x) \qquad (a = 1,2,3)$$

$\in$ SU(2)

unitary gauge

Exercise:

Show that the Lagrangian

$$\mathscr{L} = -\frac{1}{2}\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right) - \frac{M^2}{g_{\mathrm{YM}}^2}\mathrm{Tr}\left[(U^\dagger D_\mu U)(U^\dagger D^\mu U)\right]$$

$$+ i\overline{\Psi}_L \not{D} \Psi_L + i\overline{\Psi}_R \not{\partial} \Psi_R - m\left(\overline{\Psi}_L U \Psi_R + \overline{\Psi}_R U^\dagger \Psi_L\right)$$

has a *global* $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ symmetry acting as

$$U(x) \longrightarrow L\,U(x)\,R^\dagger \qquad A_\mu(x) \longrightarrow L\,A_\mu(x)\,L^\dagger$$

$$\Psi_L(x) \longrightarrow L\,\Psi_L(x) \qquad \Psi_R(x) \longrightarrow R\,\Psi_R(x)$$

$$L, R \in \mathrm{SU}(2)$$

In the gauge-fixed theory this global symmetry is broken down to the vector (diagonal) $\mathrm{SU}(2)$:

$$L = R$$

This is called a *custodial symmetry* (more on this in Nuria's and Christophe's lectures, together with an explanation of the name)

From the point of view of an *observer at low energies* this is quite satisfactory: it seems that we have managed to construct a gauge "invariant" theory of a massive nonabelian gauge field.

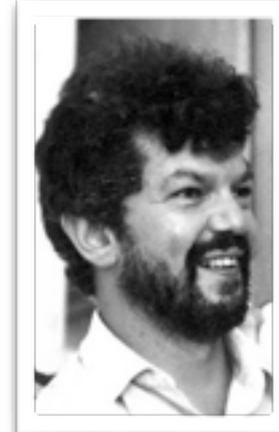
This idea of "faking" gauge invariance does not solve the problems of massive Yang-Mills fields at high energies. The theory violates unitarity at energies of the order

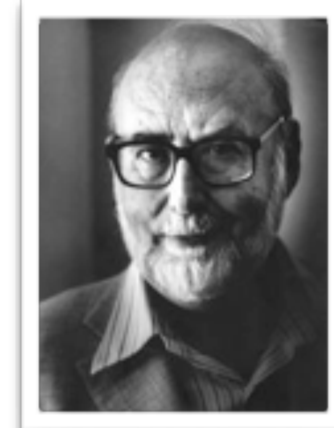
$$\Lambda \sim \frac{M}{g_{\rm YM}}$$

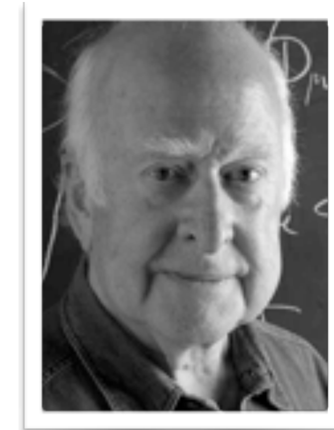
Besides, it is not renormalizable.

This indicates that the theory has to be completed in the UV by embedding the Stückelberg field into some high energy dynamics.

The Brout-Englert-Higgs Mechanism


Robert Brout
(1928-2011)


François Englert
(b. 1932)


Peter Higgs
(b. 1929)

The most popular UV completion of the Stückelberg theory consists in embedding it into a complex scalar field $U(x)$ with a gauge invariant "symmetry breaking" potential

$$V(U^\dagger U) = \frac{\lambda}{4}\left(\frac{M}{g_{\mathrm{YM}}}\right)^4 \left[\frac{1}{2}\mathrm{Tr}\,(U^\dagger U) - 1\right]^2 \qquad U(x) \notin \mathrm{SU}(2)$$

and linearize around a vacuum configuration

$$U(x) = U_0(x)\left[1 + \frac{g_{\mathrm{YM}}}{\sqrt{2}M} h(x)\right] \qquad U_0(x) \in SU(2)$$

The SU(2) gauge freedom can be used to eliminate $U_0(x)$ (unitary gauge), so we are left with $h(x)$ as a physical excitation.

→ the Higgs particle

We can illustrate the mechanism in a more familiar fashion by writing $U(x)$ in terms of a complex SU(2) scalar doublet

$$\Phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \quad \Longrightarrow \quad U(x) = \frac{g_{\rm YM}}{M} \begin{pmatrix} \varphi^{0*} & \varphi^+ \\ -\varphi^{+*} & \varphi^0 \end{pmatrix}$$

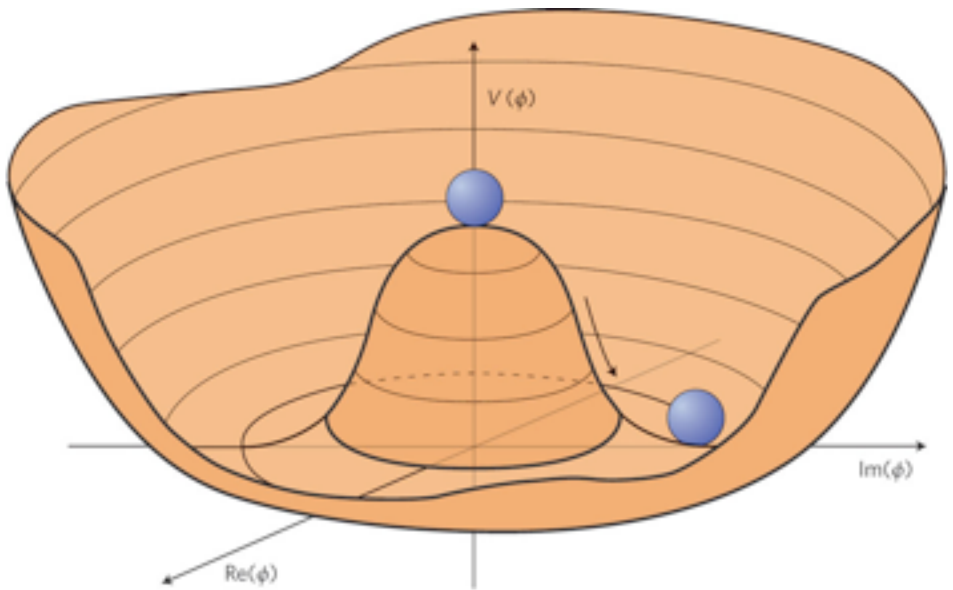
Exercise: prove that

$$\tilde{\Phi} = \begin{pmatrix} \varphi^{0*} \\ -\varphi^{+*} \end{pmatrix}$$

transforms as a SU(2) doublet

The potential now reads

$$V(\Phi) = \frac{\lambda}{4}\left(\Phi^\dagger \Phi - \frac{M^2}{g_{\rm YM}^2}\right)^2$$



At the bottom of the potential, Φ acquires a vev that we can take to be

$$\langle 0|\Phi|0\rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \qquad \text{with} \qquad v = \frac{\sqrt{2}M}{g_{\rm YM}}$$

The excitations around this vacuum can be parametrized by

$$\Phi(x) = \frac{1}{\sqrt{2}} U_0(x) \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

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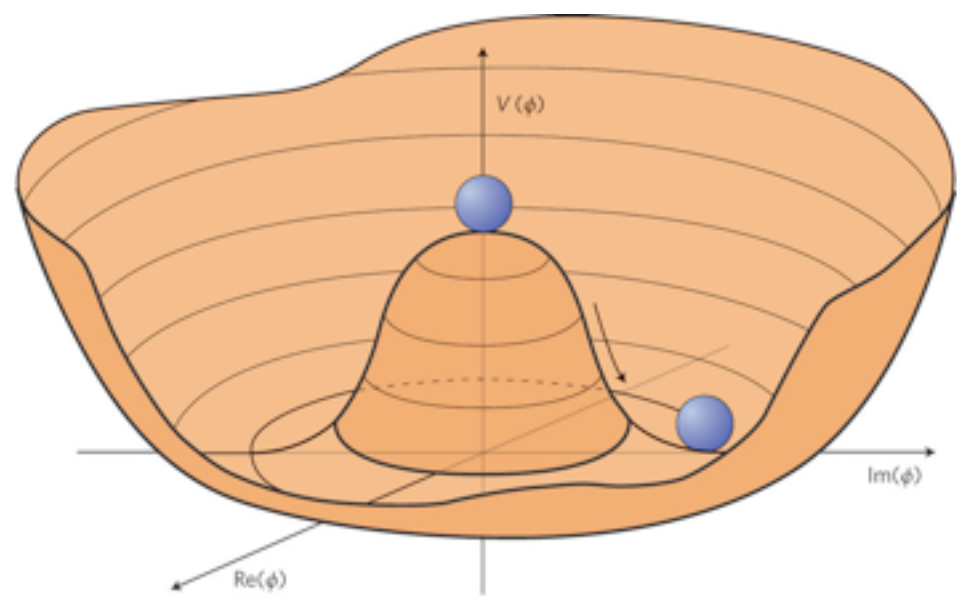

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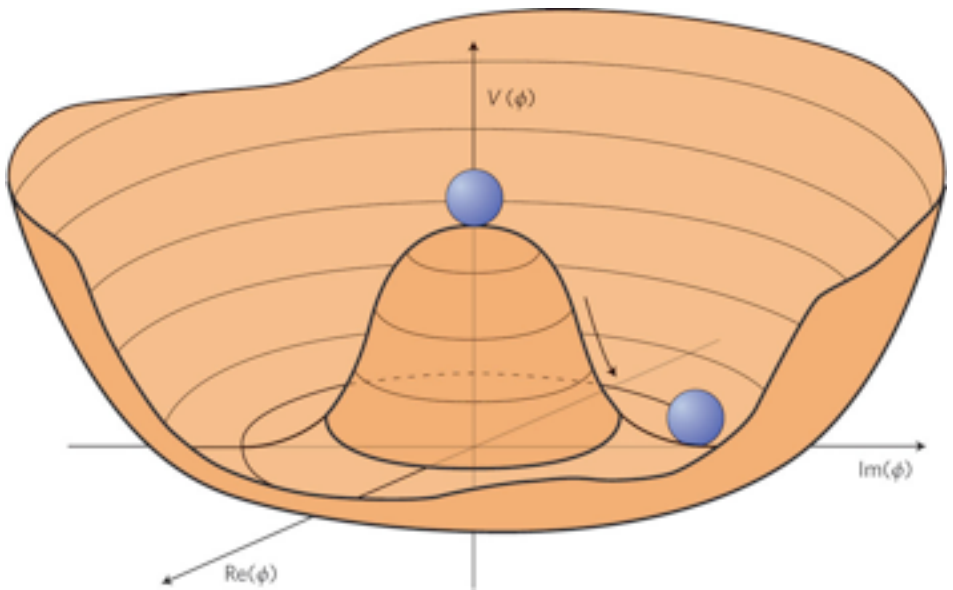

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Stückelberg field (gauged away) ⟸ "angular" excitation ($U_0(x)$)

"radial" excitation ($h(x)$) ⟹ Higgs field

Expanding the Lagrangian to second order in $h(x)$ we find the mass of the Higgs mode to be

$$m_H = v\sqrt{\frac{\lambda}{2}} = \frac{M\sqrt{\lambda}}{g_{\rm YM}}$$

Exercise: prove it

This mass depends not only on "low energy" quantities like M and $g_{\rm YM}$, but also on the self-coupling λ.

The big advantage of the Brout-Englert-Higgs mechanism we can describe a massive vector field at low energies without giving up ***unitarity and renormalizability***.

The reason is that the full theory is gauge invariant, although the vacuum is not.

$$g(x)\begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \neq \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \qquad g(x) \in {\rm SU}(2)$$

so gauge invariance is not lost, only hidden (more on this on Wednesday).

To summarize, the breaking of gauge invariance in the massive SU(2) Lagrangian.

$$\mathscr{L} = -\frac{1}{2}\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right) + M^2\mathrm{Tr}\left(A_\mu A^\mu\right) + i\overline{\Psi}_L \not{D}\Psi_L + i\overline{\Psi}_R \not{\partial}\Psi_R - m\left(\overline{\Psi}_L\Psi_R + \overline{\Psi}_R\Psi_L\right)$$

is no big deal *at low energies*: the gauge redundancy can be introduced by hand using Stückelberg's trick, to write something that is formally gauge invariant.

The theory however is sick at high energies (i.e., nonunitary and nonrenormalizable), and has to be completed in the UV:

- The Brout-Englert-Higgs mechanism gives a unitary and renormalizable theory, although it provides no physical explanation for the shape of the potential.
- There are other scenarios where the Stückelberg field is dynamically generated at low energies (wait for Christophe's lectures).

So far, in the standard model we have only detected the "angular" part of the field $U(x)$ (i.e., the Stückelberg field, or the longitudinal components of the $W^\pm$ and Z^0 bosons). However, its "radial" part (i.e., the Higgs boson) is still at large.