EXERCISES FOR
HIGH ENERGY COSMIC RADIATION
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1. Derive the threshold for pion production of high energy nucleons in the cosmic microwave background, $N + \gamma_{\rm CMB} \to N + \pi$. Express it in terms of the energy ε of the cosmic microwave background photon.

2. Derive the threshold for pair production of high energy photons in the cosmic microwave background, $\gamma + \gamma_{\rm CMB} \to e^+e^-$. Express it in terms of the energy ε of the cosmic microwave background photon.

3. Assume that primordial neutrinos are non-relativistic and have a mass m_ν. Calculate the threshold for production of a Z^0 boson through high energy neutrinos $\nu + \bar{\nu} \to Z^0$ and express it in terms of the mass m_ν of the primordial neutrinos.

4. The interstellar density in our Galaxy is about 1 (mostly hydrogen) atom per cm^3 and the confinement time of a high energy cosmic ray is of the order of 10^7 years or smaller. Estimate the interaction probability of a cosmic ray during its propagation within the Galaxy.

5. Assume that a cosmic ray particle interacts with a photon beam that comes from a fixed direction, with a cross section $\sigma(s)$ which depends on the squared center of mass energy s (also known as the first Mandelstam-variable). Per definition of a cross section, in the rest frame of the cosmic ray particle (unprimed quantities) its interaction rate is given by

$$\Gamma = j\sigma(s)\,,$$

where $j = |\mathbf{j}| = c_0 n$ is the flux density of the photon beam, corresponding to a photon density n. Using Lorentz transformations, show that the interaction rate in the system in which the particle moves with velocity $\mathbf{v}$ (primed quantities) is given by

$$\Gamma' = c_0\, n' \left(1 - \frac{v}{c_0}\cos\theta\right) \sigma(s)$$

where θ is the angle between $\mathbf{v}$ and the direction of the photon beam. Use the fact that the four-vector flux density $(c_0 n, \mathbf{j})$ and $(c_0 n', \mathbf{j}')$, respectively, represent a four-vector j^μ with zero norm, $j^2 = j_\mu j^\mu = 0$.

6. Consider the equation for the mean free path for the interaction of a high energy particle with an isotropically distributed low energy background photon target with a distribution $dn_b(\varepsilon)/d\varepsilon$ for a cross section $\sigma\left[s = m_{\mathrm{CR}}^2 + 2E\varepsilon(1 - \mu\beta_{\mathrm{CR}}\beta_b)\right]$,

$$l(E)^{-1} = \int d\varepsilon \frac{dn_b(\varepsilon)}{d\varepsilon} \int_{-1}^{+1} d\mu \frac{1 - v\mu}{2}\, \sigma(s)\,, \tag{1}$$

with $\beta \equiv v/c_0$. Express the integral over μ in this equation through an integration over the energy ε_0 of the target particles in the rest frame of the high energy particles (cosmic radiation). What are the integration limits for ε_0 if the interaction has a threshold $\varepsilon_{\mathrm{th}}^0$ in the rest frame of the high energy particle (example: GZK effekt/pion production has a threshold of $\varepsilon_{\mathrm{th}}^0 \simeq 150\,\mathrm{MeV}$) ?

7. Estimate the cross section for pair production by photons, $\gamma\gamma \to e^+e^-$ close to the threshold, $s \sim m_e^2$ with m_e the electron mass ("Thomson regime"), and for $s \gg m_e^2$ ("Klein-Nishina regime"). You can use dimensional analysis for these estimates.

8. The power of synchrotron emission by an electron of energy E in a magnetic field B is given by
$$P_{\text{synch}} = \frac{\sigma_T E^2 B^2}{6\pi m_e^2} ,$$
where $\sigma_T \equiv e^4/(6\pi m_e^2)$ is the Thomson cross section. Assuming the average Galactic magnetic field strength is $\simeq 5\,\mu$G, estimate the energy loss length of an electron in the Galaxy as a function of its energy.

9. How far does the electron diffuse before it looses most of its energy assuming that the diffusion coefficient is given by the Larmor radius ?

10. Derive the formula

$$\langle \Delta p \rangle = \frac{4}{3}(u_1 - u_2)p_1 \tag{2}$$

for the average energy gain of a relativistic charged particle during one acceleration cycle "upstream-downstream-upstream" at a non-relativistic shock, $u_1, u_2, v \equiv u_1 - u_2 \ll 1$. In order to do this, compute the average change of momentum

$$\langle \Delta p \rangle = \frac{\int (p_1' - p_1)|\mu_1|d\mu_1|\mu_2|d\mu_2}{\int |\mu_1|d\mu_1|\mu_2|d\mu_2}$$

weighted by the particle flux over the shock front where $0 \lesssim \mu_1 \leq 1$ and $-1 \leq \mu_2 \lesssim 0$ are the projections of the cosmic ray momentum onto the shock normal upon crossing the shock. Here,

$$p_1' \simeq p_1(1 - v\mu_2)(1 + v\mu_1)$$

follows from Lorentz-transformations between the upstream and downstream frames in the non-relativistic limit.