Lecture IV

Anomalies

- Symmetries and quantum corrections
- The Adler-Bell-Jackiw anomaly
- Gauge anomalies and anomaly cancellation

Symmetries and Quantum Corrections

From Lecture II:

In the presence of continuous symmetries, the conserved charges Q^a are converted upon quantization in operators such that

$$\{Q^a, H\}_{\mathrm{PB}} = 0 \quad \Longrightarrow \quad [Q^a, H] = 0$$

These charges generate the action of the symmetry on the Hilbert space

$$\mathscr{U}(\alpha) = e^{i\alpha^a Q^a} \quad \Longrightarrow \quad \mathscr{U}(\alpha) H \mathscr{U}(\alpha)^\dagger = H$$

Upon quantization, however, conserved currents and charges are defined by products of operators evaluated at the same spacetime point. It is possible that the renormalized composite operator does not satisfy the conservation equation of the classical one.

When this happens, we say that the theory has an ***anomaly***.

We say that a classical symmetry is anomalous if it cannot be realized in the quantum theory. This might happen for both continuous and discrete symmetries (e.g., parity anomaly in 3D).

The presence of anomalies is not the result of a clumsy choice of the regulator used to renormalize the theory. If a symmetry is anomalous it means that the classical symmetry cannot be realized in the quantum theory, ***no matter*** how smart we are in choosing the regularization procedure.

Whether anomalies are good or bad news depends on the type of symmetry affected by them:

- ***Global symmetries***: since these can be considered accidental symmetries of the theory, its breaking by anomalies does not spoil its consistency.

- ***Gauge invariance***: current conservation is crucial for the consistency of gauge theories. Hence, an anomaly associated with a gauge current renders the theory inconsistent.

In fact, we have already encountered an anomaly in these lectures. Let us look, for example, to the Lagrangian of pure Yang-Mills theory does not contain any mass parameter

$$\mathscr{L} = -\frac{1}{4}\partial_\mu A_{\nu^A}\partial^\mu A^{\mu A} + \frac{1}{4}\partial_\mu A_{\nu^A}\partial^\nu A^{\mu A} + \frac{1}{2}g_{\rm YM} f^{ABC} A^{\mu A} A^{\nu B}\partial_\mu A^C_\nu + g^2_{\rm YM} f^{ABC} f^{ADE} A^B_\mu A^C_\nu A^{\mu D} A^{\nu E}$$

The classical action is scale invariant, i.e., the physics does not change when we change the scale,

$$x^\mu \longrightarrow x'^\mu = \lambda x^\mu \qquad\qquad A^A_\mu(x) \longrightarrow A'^A_\mu(x) = \lambda^{-\Delta} A^A_\mu(\lambda^{-1}x) \qquad \text{with} \qquad \Delta = 1$$

Nevertheless, we have learned in Lecture III how this scale invariance is broken by quantum corrections and a nonvanishing beta function appears. For $\mathrm{SU}(N_c)$

$$\beta(g_{\rm YM}) = -\frac{11 g^3_{\rm YM} N_c}{48\pi^2} \neq 0$$

Classical scale invariance is broken by quantum correction: it is an ***anomalous symmetry***. This anomaly is not dangerous and it actually explains lots of physics

The Adler-Bell-Jackiw Anomaly

Let us consider a massless fermion coupled to an external classical electromagnetic field $\mathscr{A}_\mu(x)$

$$\mathscr{L} = i\overline{\psi}\partial\!\!\!/\psi + eJ^\mu_{\rm V}\mathscr{A}_\mu \qquad \text{with} \qquad J^\mu_{\rm V} = \overline{\psi}\gamma^\mu\psi$$

This theory is invariant under vector and chiral phase rotations of the spinor field

$$\mathrm{U}(1)_{\rm V} : \psi \longrightarrow e^{i\alpha}\psi \qquad \mathrm{U}(1)_{\rm A} : \psi \longrightarrow e^{i\alpha\gamma_5}\psi$$

which implies the conservation of the vector and axial currents

$$\partial_\mu J^\mu_{\rm V} = 0 \qquad \partial_\mu J^\mu_{\rm A} = 0 \qquad \text{where} \qquad J^\mu_{\rm A} = \overline{\psi}\gamma^\mu\gamma_5\psi$$

To check whether the axial current is conserved also quantum mechanically, we have to compute the quantity

$$\langle\partial_\mu J^\mu_{\rm A}(x)\rangle_{\mathscr{A}} = \frac{1}{\mathscr{Z}}\int \mathscr{D}\overline{\psi}\mathscr{D}\psi\,\partial_\mu J^\mu_{\rm A}(x)\,e^{i\int d^4y\left(i\overline{\psi}\partial\!\!\!/\psi + eJ^\mu_{\rm V}\mathscr{A}\right)}$$

Expanding in powers of e, the first nonvanishing term is quadratic one

$$\langle \partial_\mu J^\mu \rangle_{\mathscr{A}} = -\frac{e^2}{2} \int d^4y_1 d^4y_2\, \partial_\mu^{(x)} C^{\mu\nu\sigma}(x,y) \mathscr{A}_\nu(x-y_1+y_2) \mathscr{A}_\sigma(x-y_2)$$

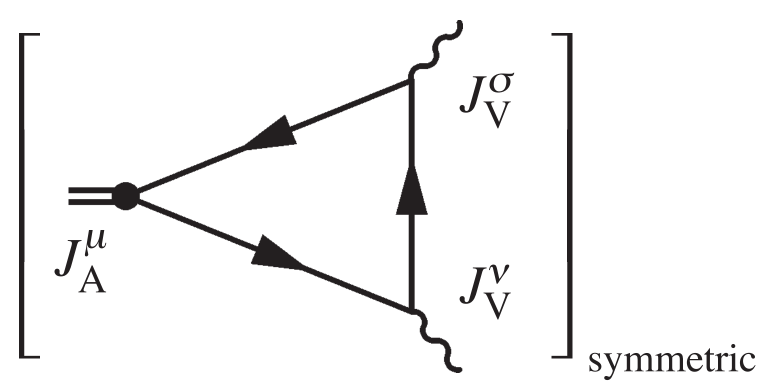
where

$$C^{\mu\nu\sigma}(x,y) = \langle 0 | T[J_{\rm A}^\mu(x) J_{\rm V}^\nu(y) J_{\rm V}^\sigma(0)] | 0 \rangle$$

To compute this correlation function we use Wick's theorem.

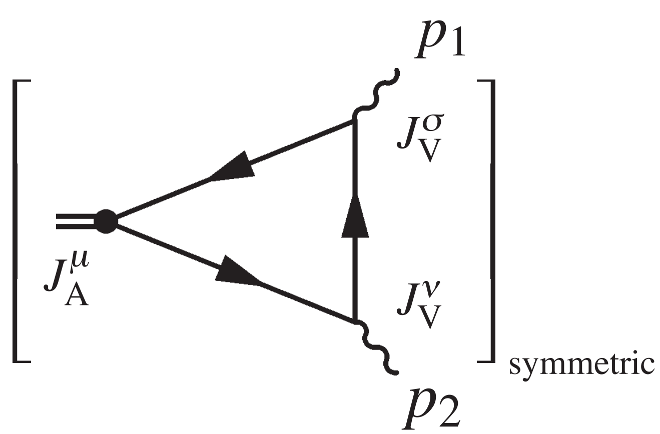
$$C^{\mu\nu\sigma}(x,y) = \langle 0 | \overline{\psi} \gamma^\mu \gamma_5 \psi(x)\, \overline{\psi} \gamma^\nu \psi(y)\, \overline{\psi} \gamma^\sigma \psi(0) | 0 \rangle + \langle 0 | \overline{\psi} \gamma^\mu \gamma_5 \psi(x)\, \overline{\psi} \gamma^\nu \psi(y)\, \overline{\psi} \gamma^\sigma \psi(0) | 0 \rangle$$

These Wick contractions are codified in the celebrated ***triangle diagram***:

$C^{\mu\nu\sigma}(x,y) =$ 

where we have to symmetrize over the "photon legs".

We compute this diagram in momentum space



$$= I^{\mu\nu\sigma}(p_1,p_2) + I^{\mu\sigma\nu}(p_2,p_1)$$

where

$$I^{\mu\nu\sigma}(p_1,p_2) = -ie^2 \int \frac{d^4q}{(2\pi)^4} \frac{\mathrm{Tr}\left[\gamma^\mu\gamma^5 \not{q}\gamma^\nu(\not{q}-\not{p}_1)\gamma^\sigma(\not{q}-\not{p}_1-\not{p}_2)\right]}{[q^2-i\varepsilon][(q-p_1)^2+i\varepsilon][(q-p_1-p_2)^2+i\varepsilon]}$$

To compute the anomaly we only need to evaluate

$$i(p_1+p_2)_\mu\Big[I^{\mu\nu\sigma}(p_1,p_2) + I^{\mu\sigma\nu}(p_2,p_1)\Big]$$

The integral to be computed is linearly divergent and has to be regularized, using for example Pauli-Villars or dimensional regularization (but being very careful of how γ_5 is defined!)

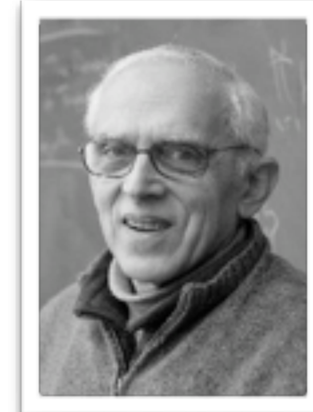
You can try it as an exercise, but it's very long.

After a quite long calculation one finds

$$i(p_1+p_2)_\mu\Big[I^{\mu\nu\sigma}(p_1,p_2)+I^{\mu\sigma\nu}(p_2,p_1)\Big]=\frac{e^2}{2\pi^2}\varepsilon^{\nu\alpha\sigma\beta}p_{1\alpha}p_{2\beta}$$

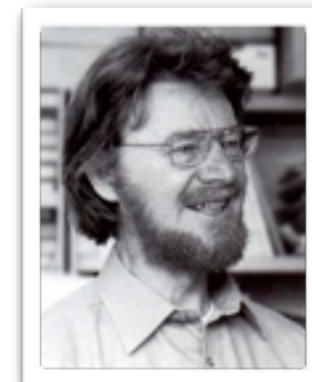
In position space this gives the ***Adler-Bell-Jackiw anomaly***:

$$\langle\partial_\mu J^\mu_{\mathrm{A}}\rangle_{\mathscr{A}}=-\frac{e^2}{16\pi^2}\varepsilon^{\alpha\beta\sigma\lambda}\mathscr{F}_{\alpha\beta}\mathscr{F}_{\sigma\lambda}$$



Steven Adler
(b. 1939)

Thus, the conservation of the axial current is spoiled by quantum corrections. This is not a problem, since the axial symmetry is a global one and the anomaly does not render the theory inconsistent.



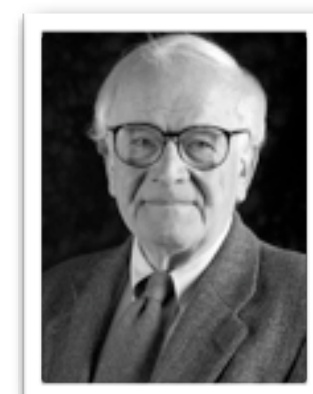
John S. Bell
(1928-1990)

Jack Steinberger
(b. 1921)

In fact, the anomaly explains the electromagnetic decay of the neutral pion

$$\pi^0\longrightarrow 2\gamma$$

as we will see now.



Roman Jackiw
(b. 1939)

Chiral Symmetry in QCD

The Adler-Bell-Jackiw anomaly has very important consequences for the physics of strong interactions. Let us then focus on QCD in the chiral limit

$$\mathscr{L}_{\mathrm{QCD}} = -\frac{1}{4}F^{A}_{\mu\nu}F^{A\,\mu\nu} + \sum_{f=1}^{N_f}\left(i\overline{Q}^f_L \not{D} Q^f_L + i\overline{Q}^f_R \not{D} Q^f_R\right) \qquad Q^f_{L,R} \equiv \frac{1}{2}(1\pm\gamma_5)Q^f$$

The action is invariant under the $\mathrm{U}(N_f)_L \times \mathrm{U}(N_f)_R$ symmetry

$$\mathrm{U}(N_f)_L : \begin{cases} Q^f_L \to \sum_{f'=1}^{N_f}(U_L)_{ff'}Q^{f'}_L \\ Q^f_R \to Q^f_R \end{cases} \qquad \mathrm{U}(N_f)_R : \begin{cases} Q^f_L \to Q^f_L \\ Q^r_R \to \sum_{f'=1}^{N_f}(U_R)_{ff'}Q^{f'}_R \end{cases} \qquad U_L, U_R \in \mathrm{U}(N_f)$$

Since $\mathrm{U}(N)=\mathrm{U}(1)\times\mathrm{SU}(N)$ the symmetry group can be written as

$$\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R \times \mathrm{U}(1)_L \times \mathrm{U}(1)_R$$

The left-right global transformations can be now decomposed into vector-axial (remember the second exercise of Lecture II)

$$\mathrm{U}(N_f)_L \times \mathrm{U}(N_f)_R = \mathrm{SU}(N_f)_{\mathrm{V}} \times \mathrm{SU}(N_f)_{\mathrm{A}} \times \mathrm{U}(1)_B \times \mathrm{U}(1)_{\mathrm{A}}$$

with

$$\mathrm{U}(1)_B : \begin{cases} Q_L^f \to e^{i\alpha} Q_L^f \\ Q_R^f \to e^{i\alpha} Q_R^f \end{cases} \qquad \mathrm{U}(1)_{\mathrm{A}} : \begin{cases} Q_L^f \to e^{i\alpha} Q_L^f \\ Q_R^f \to e^{-i\alpha} Q_R^f \end{cases}$$

$$\mathrm{SU}(N_f)_{\mathrm{V}} : \begin{cases} Q_L^f \to \sum_{f'=1}^{N_f} U_{ff'} Q_L^{f'} \\ Q_R^f \to \sum_{f'=1}^{N_f} U_{ff'} Q_R^{f'} \end{cases} \qquad \mathrm{SU}(N_f)_{\mathrm{A}} : \begin{cases} Q_L^f \to \sum_{f'=1}^{N_f} U_{ff'} Q_L^{f'} \\ Q_R^f \to \sum_{f'=1}^{N_f} U_{ff'}^{-1} Q_R^{f'} \end{cases}$$

The associated classically conserved currents are

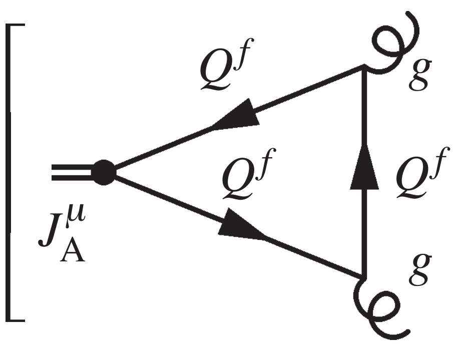
$$J_{\mathrm{V}}^{\mu} = \sum_{f=1}^{N_f} \overline{Q}^f \gamma^\mu Q^f \qquad J_{\mathrm{A}}^{\mu} = \sum_{f=1}^{N_f} \overline{Q}^f \gamma^\mu \gamma_5 Q^f$$

$$J_{\mathrm{V}}^{I\,\mu} \equiv \sum_{f,f'=1}^{N_f} \overline{Q}^f \gamma^\mu (T^I)_{ff'} Q^{f'} \qquad J_{\mathrm{A}}^{I\,\mu} \equiv \sum_{f,f'=1}^{N_f} \overline{Q}^f \gamma^\mu \gamma_5 (T^I)_{ff'} Q^{f'}$$

From the ABJ calculation we know that, in principle, the axial currents are potentially anomalous.

To see whether this is the case, we have to compute the correlation function of one axial current and two gauge currents. For the abelian part:

$$C^{\mu\nu\sigma}(x,x') \equiv \langle 0|T\left[J_{\mathrm{A}}^{\mu}(x)\, j^{A\,\nu}_{\mathrm{gauge}}(x')\, j^{B\,\sigma}_{\mathrm{gauge}}(0)\right]|0\rangle = \sum_{f=1}^{N_f}\Big[\quad\Big]_{\mathrm{symmetric}}$$



where $j^{A\,\mu}_{\mathrm{gauge}} \equiv \sum_{f=1}^{N_f} \overline{Q}^f \gamma^\mu \tau^A Q^f$. The group theoretical factor multiplying this diagram is

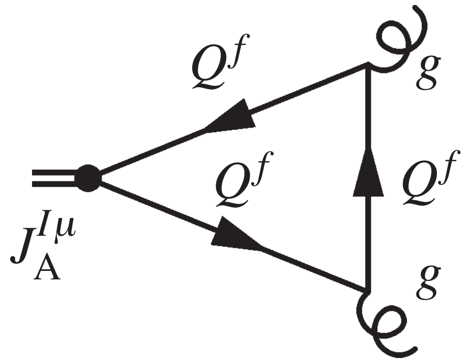
$$\mathrm{Tr}\{\tau^A, \tau^B\} \neq 0$$

with τ^A the generators of $\mathrm{SU}(N_c)$

so the anomaly does not cancel. An explicit calculation gives

$$\partial_\mu J_A^\mu = -\frac{g^2 N_f}{32\pi^2}\varepsilon^{\mu\nu\sigma\lambda} F^A_{\mu\nu} F^A_{\sigma\lambda}$$

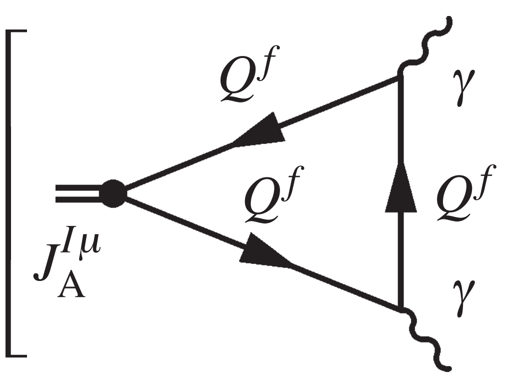
In the case of the $\mathrm{SU}(N_f)_\mathrm{A}$ current, we directly look at the group theoretical factor that multiplies the triangle diagram



$$\left[\;\cdots\; \right]_{\text{symmetric}} \sim \mathrm{Tr}\, T^I \,\mathrm{Tr}\{\tau^A, \tau^B\} = 0 \qquad \text{since} \quad \mathrm{Tr}\, T^I = 0$$

We might rush to conclude that $\mathrm{SU}(N_f)_\mathrm{A}$ is nonanomalous. We have to take into account, however, that quarks also couple to the electromagnetic field. There is thus a second contribution to the anomaly:

$$\langle 0|T\left[J_\mathrm{A}^{I\mu}(x) j_\mathrm{em}^\nu(x') j_\mathrm{em}^\sigma(0)\right]|0\rangle = \sum_{f=1}^{N_f} \left[\;\cdots\; \right]_{\text{symmetric}}$$



with $j_\mathrm{em}^\mu = \sum_{f=1}^{N_f} q_f \overline{Q}^f \gamma^\mu Q^f$

The computation of the diagram gives the anomaly

$$\partial_\mu J_\mathrm{A}^{I\mu} = -\frac{N_c}{16\pi^2} \left[\sum_{f=1}^{N_f} (T^I)_{ff}\, q_f^2 \right] \varepsilon^{\mu\nu\sigma\lambda} F_{\mu\nu} F_{\sigma\lambda}$$

We specialize our results to the case of QCD with the two light flavors u and d. Taking into account that

$$q_u = \frac{2}{3}e \qquad\qquad q_d = -\frac{1}{3}e$$

we find

$$\sum_{f=u,d} (T^1)_{ff}\, q_f^2 = \sum_{f=u,d} (T^2)_{ff}\, q_f^2 = 0 \qquad\qquad \sum_{f=u,d} (T^3)_{ff}\, q_f^2 = \frac{e^2}{6}$$

Hence, only $J_{\mathrm{A}}^{3\,\mu}$ is anomalous. This current has the quantum numbers of the neutral pion and $\partial_\mu J_{\mathrm{A}}^{3\,\mu}$ is the interpolating field between π^0 and the vacuum

$$\langle 0|J_{\mathrm{A}}^{I\,\mu}(x)|\pi^J(\mathbf{p})\rangle = -i f_\pi \delta^{IJ} p^\mu e^{-iE_{\mathbf{p}}t+i\mathbf{p}\cdot\mathbf{x}} \quad \Longrightarrow \quad \langle 0|\partial_\mu J_{\mathrm{A}}^{3\,\mu}(x)|\pi^0(\mathbf{p})\rangle = -f_\pi m_\pi^2 e^{-iE_{\mathbf{p}}t+i\mathbf{p}\cdot\mathbf{x}}$$

so the neutral pion field can be written as

$$\varphi_\pi(x) = -\frac{1}{f_\pi m_\pi^2}\partial_\mu J_{\mathrm{A}}^{3\,\mu}(x)$$

The anomaly of $J_{\mathrm{A}}^{3\,\mu}$ gives the amplitude for the electromagnetic decay of the neutral pion into two photons. The existence of this decay is a direct consequence of the existence of the ABJ anomaly.

Gauge Anomalies

Unlike the anomalies in global currents, gauge anomalies are a serious threat for the consistency of a gauge theory (e.g., nondecoupling of unphysical states). The cancelation of anomalies poses strong restrictions on these theories.

Gauge anomalies can only appear in chiral gauge theories, such as the electroweak sector of the standard model. Here we consider a Lagrangian

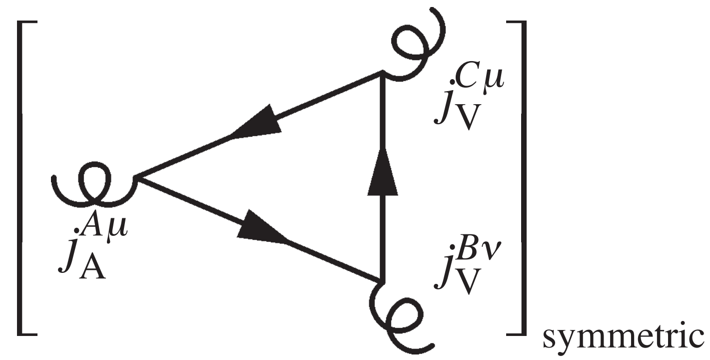
$$\mathscr{L} = -\frac{1}{4}F^{A\mu\nu}F^{A}_{\mu\nu} + i\sum_{i=1}^{N_+}\overline{\psi}^{i}_{+}\not{D}^{(+)}\psi^{i}_{+} + i\sum_{j=1}^{N_-}\overline{\psi}^{j}_{-}\not{D}^{(-)}\psi^{j}_{-}$$

where the right- and left-handed fermions transform in the representation $\tau^{A}_{i,\pm}$ and

$$D^{(\pm)}_{\mu}\psi^{i}_{\pm} = \partial_{\mu}\psi^{i}_{\pm} - ig_{\mathrm{YM}}A^{A}_{\mu}\tau^{A}_{\pm}\psi^{i}_{\pm}$$

An anomaly on the gauge current comes from the parity-violating part of the correlation function of three gauge currents. This is given by

$$\langle 0 | T \left[j_{\mathrm{A}}^{A\mu}(x) j_{\mathrm{V}}^{B\nu}(x') j_{\mathrm{V}}^{C\sigma}(0) \right] | 0 \rangle =$$



where

$$j_{\mathrm{V}}^{A\mu} = \sum_{i=1}^{N_+} \overline{\psi}_+^i \boldsymbol{\tau}_+^A \gamma^\mu \psi_+^i + \sum_{j=1}^{N_-} \overline{\psi}_-^j \boldsymbol{\tau}_-^A \gamma^\mu \psi_-^j$$

$$j_{\mathrm{A}}^{A\mu} = \sum_{i=1}^{N_+} \overline{\psi}_+^i \boldsymbol{\tau}_+^A \gamma^\mu \psi_+^i - \sum_{i=1}^{N_-} \overline{\psi}_-^j \boldsymbol{\tau}_-^A \gamma^\mu \psi_-^j$$

To compute the anomaly we need to sum over all fermion species. Each one contributes a group-theoretical factor

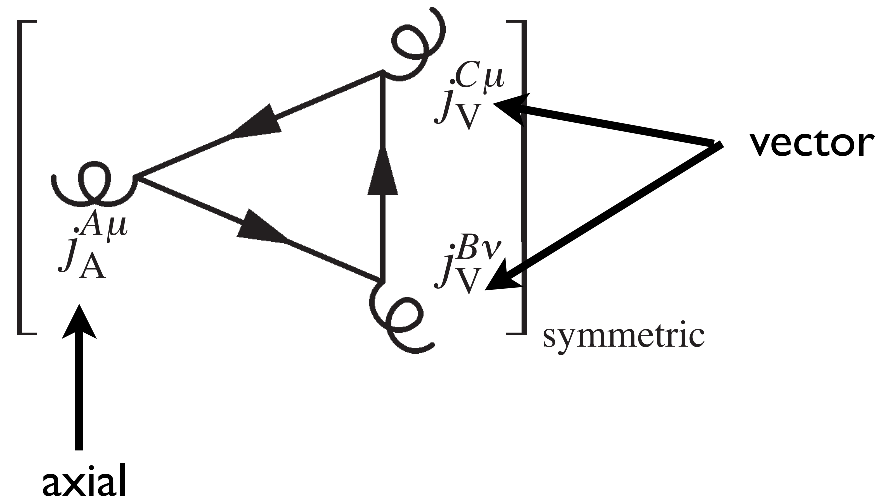
$$\mathrm{Tr} \left[\boldsymbol{\tau}_{i,\pm}^A \{ \boldsymbol{\tau}_{i,\pm}^B, \boldsymbol{\tau}_{i,\pm}^C \} \right]$$

The anomaly is the proportional to

$$\sum_{i=1}^{N_+} \mathrm{Tr} \left[\boldsymbol{\tau}_{i,+}^A \{ \boldsymbol{\tau}_{i,+}^B, \boldsymbol{\tau}_{i,+}^C \} \right] - \sum_{j=1}^{N_-} \mathrm{Tr} \left[\boldsymbol{\tau}_{j,-}^A \{ \boldsymbol{\tau}_{j,-}^B, \boldsymbol{\tau}_{j,-}^C \} \right]$$

An anomaly on the gauge current comes from the parity-violating part of the correlation function of three gauge currents. This is given by

$$\langle 0|T\left[j_{\mathrm{A}}^{A\mu}(x)\,j_{\mathrm{V}}^{B\nu}(x')\,j_{\mathrm{V}}^{C\sigma}(0)\right]|0\rangle =$$



where

$$j_{\mathrm{V}}^{A\mu} = \sum_{i=1}^{N_+} \overline{\psi}_+^i \boldsymbol{\tau}_+^A \gamma^\mu \psi_+^i + \sum_{j=1}^{N_-} \overline{\psi}_-^j \boldsymbol{\tau}_-^A \gamma^\mu \psi_-^j$$

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The anomaly is the proportional to

$$\sum_{i=1}^{N_+} \mathrm{Tr}\left[\boldsymbol{\tau}_{i,+}^A\{\boldsymbol{\tau}_{i,+}^B, \boldsymbol{\tau}_{i,+}^C\}\right] - \sum_{j=1}^{N_-} \mathrm{Tr}\left[\boldsymbol{\tau}_{j,-}^A\{\boldsymbol{\tau}_{j,-}^B, \boldsymbol{\tau}_{j,-}^C\}\right]$$

The cancellation condition of the gauge anomaly

$$\sum_{i=1}^{N_+} \mathrm{Tr}\left[\boldsymbol{\tau}_{i,+}^{A}\{\boldsymbol{\tau}_{i,+}^{B},\boldsymbol{\tau}_{i,+}^{C}\}\right] - \sum_{j=1}^{N_-} \mathrm{Tr}\left[\boldsymbol{\tau}_{j,-}^{A}\{\boldsymbol{\tau}_{j,-}^{B},\boldsymbol{\tau}_{j,-}^{C}\}\right] = 0$$

imposes very strong conditions on both the number of fermion species and the representations of the gauge group under which they transform.

On physical grounds, the most interesting case is the standard model (see also Nuria´s lectures). Its fermion content is

$$\begin{array}{llll} \text{quarks:} & \begin{pmatrix} u^i \\ d^i \end{pmatrix}_{L,\frac{1}{6}} & u^i_{R,\frac{2}{3}} & d^i_{R,-\frac{1}{3}} \\ \text{leptons:} & \begin{pmatrix} \nu_e \\ e \end{pmatrix}_{L,-\frac{1}{2}} & e_{R,-1} & \end{array}$$

or in terms of the labels $(n_c, n_w)_Y$ of the representations of $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)_Y$

$$\begin{array}{llll} \text{left-handed fermions:} & (3,2)^L_{\frac{1}{6}} & (1,2)^L_{-\frac{1}{2}} & \\ \text{right-handed fermions:} & (3,1)^R_{\frac{2}{3}} & (3,1)^R_{-\frac{1}{3}} & (1,1)^R_{-1} \end{array}$$

To check that the standard model is anomaly-free we have to compute the triangle diagram with all possible combination of the three group factors

$$\mathrm{SU}(3)^3 \qquad \mathrm{SU}(2)^3 \qquad \mathrm{U}(1)^3$$

$$\mathrm{SU}(3)^2\,\mathrm{SU}(2) \qquad \mathrm{SU}(2)\,\mathrm{U}(1)$$

$$\mathrm{SU}(3)^2\,\mathrm{U}(1) \qquad \mathrm{SU}(2)\,\mathrm{U}(1)^2$$

$$\mathrm{SU}(3)\,\mathrm{SU}(2)^2$$

$$\mathrm{SU}(3)\,\mathrm{SU}(2)\,\mathrm{U}(1)$$

$$\mathrm{SU}(3)\,\mathrm{U}(1)^2$$

and compute

$$\sum_{i=1}^{N_+} \mathrm{Tr}\left[\tau^A_{i,+}\{\tau^B_{i,+},\tau^C_{i,+}\}\right] - \sum_{j=1}^{N_-} \mathrm{Tr}\left[\tau^A_{j,-}\{\tau^B_{j,-},\tau^C_{j,-}\}\right]$$

The result is that *all* gauge anomalies cancel within each family.

Exercise: prove this result

As an example we work out the case $\mathrm{U}(1)^3$

$$
\sum_{\text{left}} Y_+^3 - \sum_{\text{right}} Y_-^3 = \overset{\substack{\text{left-handed}\\ \text{quarks}}}{3 \times 2 \times \left(\frac{1}{6}\right)^3} + \overset{\substack{\text{left-handed}\\ \text{leptons}}}{2 \times \left(-\frac{1}{2}\right)^3} - \overset{\substack{\text{right-handed}\\ \text{“up” quarks}}}{3 \times \left(\frac{2}{3}\right)^3}
$$

$$
- \underset{\substack{\text{right-handed}\\ \text{“down” quarks}}}{3 \times \left(-\frac{1}{3}\right)^3} - \underset{\substack{\text{right-handed}\\ \text{lepton}}}{(-1)^3} = \underset{\substack{\uparrow\\ \text{quarks}}}{\left(-\frac{3}{4}\right)} + \underset{\substack{\uparrow\\ \text{leptons}}}{\left(\frac{3}{4}\right)} = 0
$$

We see how the anomaly cancels between quarks and leptons.

What about higher loops? The ***Adler-Bardeen theorem*** states that once the anomalies are cancelled at one loop there are no further anomalies coming from higher loop diagrams.

And nonperturbative anomalies? There is also a nonperturbative form of the anomaly associated with the SU(2) factor of the standard model group (***Witten's anomaly***). It cancels whenever the total number of fermion species is even, which is the case in the standard model.