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Law of inertia, clock synchronization, speed limit and Lorentz transformations

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Abstract

In his first 1905 article on special relativity, clock synchronization by means of light rays was used by Einstein to derive the Lorentz transformations (*Ann. Phys., Lpz.* **322** 891–921). However, the same goal can be achieved by using bodies in free motion to synchronize clocks. To this end, one has to accept the principle of relativity, the law of inertia and the existence of a limit value for the speed of massive bodies with no appeal to electromagnetic phenomena until the very last step of the derivation, when the limit speed must be identified with that of light in vacuum. (In the absence of this speed limit one recovers the Galilean transformations.)

Keywords: special relativity, Lorentz transformations, clock synchronization, law of inertia, speed limit

1. Introduction

In his first 1905 article on special relativity [1] Einstein used a *gedankenexperiment* with light rays to synchronize the clocks of a coordinate frame where the laws of mechanics are those of Newton. The operational procedure to synchronize clocks at different points was as follows. Two clocks are at rest in an inertial reference frame, in vacuum, at points A and B separated by the distance r. The light ray emitted from point A when the clock placed there displays the time t_A and, after reflecting in B when the time t_B is read in the clock at B, returns to A at instant t'_A, as measured by the clock at A. By definition, both clocks are synchronized if the following condition is satisfied:

$$t_B - t_A = t'_A - t_B \quad \Longleftrightarrow \quad t_B = \frac{t'_A + t_A}{2}. \tag{1}$$

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Then, to derive the Lorentz transformations, Einstein made use of the principle of constancy and invariance of the speed of light in any frame, as well as other simplifying assumptions.

The use of the constancy and invariance of the speed of light both to synchronize clocks and to deduce the Lorentz transformations received some criticism, which was answered by Einstein as follows [2]:

> The theory of relativity is often criticized for giving, without justification, a central theoretical rôle to the propagation of light, in that it founds the concept of time upon the law of propagation of light. The situation, however, is somewhat as follows. In order to give physical significance to the concept of time, processes of some kind are required which enable relations to be established between different places. It is immaterial what kind of processes one chooses for such a definition of time. It is advantageous, however, for the theory, to choose only those processes concerning which we know something certain. This holds for the propagation of light in vacuum in a higher degree than for any other process which could be considered, thanks to the investigations of Maxwell and Lorentz.

Einstein's original derivation of the Lorentz transformations is not used in textbooks owing to its complexity. Later, Einstein put forward simpler derivations [3], but always making use of the constancy and invariance of the speed of light. Over the years different methods have been used in articles and textbooks to derive the Lorentz transformations, but to the best of our knowledge none of them takes as a starting point the synchronization criterion. Brehmen proposed a method to synchronize clocks by means of moving bodies [4], but he did not derive the Lorentz transformations from it.

One can use the free motion of a body that satisfies the law of inertia in a reference frame to give physical significance to the concept of time and to establish relationships between spatially separated points. The goal of this work is to derive the Lorentz transformations from the following three basic principles: the relativity principle restricted to mechanical phenomena, the law of inertia for free bodies and its use for the synchronization of the clocks of any inertial reference frame and finally the existence of a limit velocity for material bodies. In all these statements no mention to any electromagnetic phenomena or light is done but we will be making some additional assumptions: our definition of time is not dependent on the velocity of the free body used for clock synchronization, space and time are homogeneous, space is isotropic and dynamics is invariant under space and time inversions. Only at the very final step we must identify the limit speed with that of light in vacuum to recover the actual Lorentz transformations.

The merit of this approach is fivefold: (a) it provides an example of clock synchronization without using the properties of light propagating in vacuum; (b) sheds light on the fundamental role of clock synchronization; (c) shows that the relativity of simultaneity arises independently of the constancy of the speed of light in vacuum; (d) the operational approach guarantees the fulfilment of the law of inertia; and (e) the approach may be useful to clarify the foundations of special relativity.

We shall start in section 2 by stating the basic postulates we are going to use to achieve our goal. After setting up the procedure for clock synchronization in section 3, the relative measurement of coordinates among inertial observers in relative motion is computed in section 4, and the Lorentz transformations are derived in section 5 by assuming that the speed of bodies in any reference frame has an upper limit. In default of this limit one obtains the Galilean transformations instead of those of Lorentz, while in special relativity the aforementioned velocity limit is the speed of light in vacuum. Finally, the appendix presents a simpler and faster deduction of Lorentz transformations by a similar synchronization procedure that uses light instead of free bodies.

2. Fundamental postulates

The first postulate is the restricted relativity principle stated as

> There exists a class of equivalent observers such that the laws of mechanics are written in the same form in the corresponding reference frames.

To define the class of equivalent observers, and therefore how their relative spacetime measurements are related, we assume that spacetime is homogeneous, i.e. the origin of the Cartesian frames can be located anywhere, and that space is isotropic, so that the spatial reference frames can have arbitrary orientations. Therefore spacetime translations and static rotations are among the transformations which relate their relative spacetime measurements. In general the equivalent observers are related by a group of spacetime transformations, usually called the *kinematical group* of the formalism [5], so that the above assumptions imply that spacetime translations and static rotations are subgroups of the kinematical group. We also assume that some relative motions are also allowed among the equivalent observers. The object of this work is to obtain these transformation equations when the observers are in relative motion. The term *restricted* for this postulate is to distinguish from a general relativity principle in which all kinds of transformations and relative motions among observers are allowed, and also because we restrict the principle to mechanical phenomena.

The second postulate is the law of inertia, which reads

> A free body in a reference frame stays at rest or is moving at a constant velocity.

If this mechanical law also holds for the class of equivalent observers, then a free body for some particular observer also moves at a constant velocity in any of the above equivalent frames, which we call from now on inertial reference frames. If we have two frames K and K', such that K' is accelerated with respect to K, then K and K' are not equivalent frames according to the relativity principle, because a free body at rest in K' does not move with constant velocity in the frame K and therefore it is not a free body. The consequence of this is that the equivalent inertial reference frames can be moving, relative to each other, at a constant velocity.

To properly define the concept of the velocity of a body in any frame it is necessary to define the concept of time coordinate of the corresponding frame. The concept of time is not the measurement performed by a single clock located at the origin of the corresponding frame, but rather the measurement made by the different synchronized clocks at different spatial points, clocks which are synchronized among each other and with the clock at the origin, as is described in the next section.

With these two postulates the class of inertial observers are either at rest with respect to each other or moving at a constant velocity. For this relative velocity there is at first no restriction. This restriction is stated in the next postulate.

The third fundamental postulate refers to the maximum velocity of massive bodies:

> In any inertial reference frame the speed of massive bodies has an upper bound c_{m}.

The acceptance of this postulate and the relativity principle implies, as we will show later, that this velocity limit, if it exists, must be the same in any inertial frame, and has to be determined experimentally. Since a free body can be at rest in some reference frame this amounts that the relative velocity among inertial observers has also this upper bound.

3. The law of inertia and clock synchronization

To measure the speed of a free body one can use the following procedure. After synchronizing at the origin of coordinates two equal perfect clocks, A and B, the latter is very slowly moved up to a distance r. Then if a body which moves freely passes through the origin at time t_A and through the B position at instant t_B, its speed u will be constant by the law of inertia and of the following value:

$$u = \frac{r}{t_B - t_A}, \tag{2}$$

which is equivalent to $t_B = t_A + r/u$. This, in turn, suggests an alternative approach for clock synchronization.

Let us assume that the free body goes through point A when the clock located there displays the time t_A and through B, separated from A a distance r, when the time in the clock placed there is t_B. We will say that clock B is synchronized to clock A if whatever the value of t_A one has the linear relation

$$t_B = t_A + br, \tag{3}$$

with some appropriate constant b. This amounts to assuming that the body motion satisfies the law of inertia.

The meaning of b derives from the fact that the body travels a distance r in time br. In consequence the speed of the body is defined by (3) as

$$u = \frac{r}{br} = \frac{1}{b}. \tag{4}$$

In addition to the criterion in (3), we will postulate the reflexive and transitivity properties for the set of clocks at rest of every inertial observer such that if clock B is synchronized to clock A then A is synchronized to B and that if B and C are synchronized to A they are also synchronized to each other.

Let us define the time coordinate of any inertial reference frame K as the time measured by the clocks that are at rest in that frame and are synchronized to the one at the origin O by using the criterion in (3), so that if the body used for clock synchronization goes through the origin at time t_{O} and through coordinate x at instant t the following condition is satisfied, for any x:

$$t = t_{\mathrm{O}} + bx, \tag{5}$$

where the constant b depends only on the velocity of the body used for synchronization.

Once the time coordinate is defined in all reference frames one can measure the speed of other bodies and of other inertial observers in any frame.

The synchronization criterion given by the linear relationship

$$t = t_{\mathrm{O}} + \frac{x}{c}, \tag{6}$$

where c is the speed of light in vacuum, has the mathematical structure of (5) and was used by von Laue in the first textbook on relativity [6]. It was also suggested by Einstein [3] and is used in other textbooks [7].

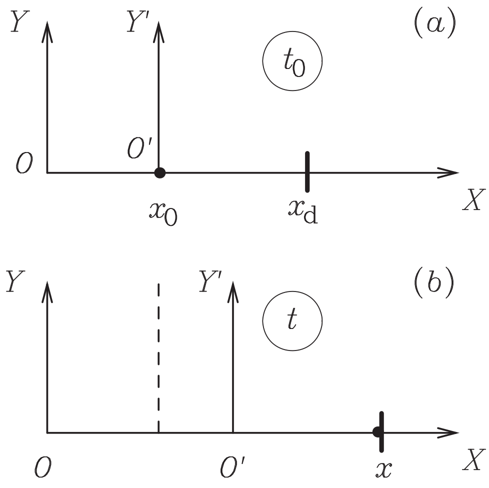

Figure 1. (a) Position x_0 of the origin O', and of the detector x_d at instant t_0, as measured in K, and (b) at instant t. The dot represents the body that is launched from the origin of K' at time t'_0 and which reaches the detector, at rest in K', at time t and at the position x in the reference frame K. The vertical thick line represents the location of the detector, for the observer K

4. Relative measurements among inertial observers

In this section we are going to analyze the relative measurements of spacetime events among inertial observers which are in relative motion at a constant velocity.

Let us consider an inertial reference frame K. Let us consider another inertial frame K' whose axis X' lies along the X of system K, while its Y' and Z' axes remain parallel to the Y and Z of K, respectively. The origin O' of K' moves with constant velocity v along the X axis and carries synchronized clocks identical to those of the system K.

To find the relationship between the coordinates and time of the same single event in systems K and K' we can use the following *gedankenexperiment*.

A body, as the one used for clock synchronization, is launched from O' and it moves uniformly towards a detector (as Einstein did with a light ray) which is located at rest at the coordinate x' of K'. Since the body satisfies the law of inertia, is launched at instant t'_0 and reaches the detector at time t', its speed as measured in K' is u', the following will hold, exactly as in (5):

$$t' = t'_0 + bx' = t'_0 + \frac{x'}{u'}. \tag{7}$$

In system K the body is also moving freely, but its constant velocity u is different than u' and it has been launched at time t_0, when the detector for this observer was at x_d. The body reaches the detector at time t at point x as measured in K (see figure 1), after having covered the detector a distance $v(t - t_0)$ with respect to system K:

$$x = x_\mathrm{d} + v(t - t_0). \tag{8}$$

If at time t_0 the origin O' was at the coordinate x_0 in the system K, since the body moves with constant velocity u, one will have that

$$u(t - t_0) = x - x_0. \tag{9}$$

Now, from equations (8) and (9) we get

$$t = t_0 + \frac{x_\mathrm{d} - x_0}{u - v}, \tag{10}$$

$$x = x_\mathrm{d} + v\frac{x_\mathrm{d} - x_0}{u - v}. \tag{11}$$

The event 'arrival to the detector' has coordinates $(t', x', 0, 0)$ in K' and $(t, x, 0, 0)$ in K. Time t' is, in general, a function of the coordinates (t, x, y, z), expression we are looking for. The event 'departure of the body' has coordinates $(t'_0, 0, 0, 0)$ in K' and $(t_0, x_0, 0, 0)$ in K, so that t'_0 is the same function of t_0 and x_0, and (7) can be written as

$$t'(t, x, 0, 0) = t'(t_0, x_0, 0, 0) + \frac{x'}{u'} \tag{12}$$

or, explicitly, as

$$t'\left(t_0 + \frac{x_\mathrm{d} - x_0}{u - v}, x_\mathrm{d} + v\frac{x_\mathrm{d} - x_0}{u - v}, 0, 0\right) = t'(t_0, x_0, 0, 0) + \frac{x'}{u'}. \tag{13}$$

The derivative of the last result with respect to x_d reads as follows:

$$\frac{\partial t'}{\partial t}\frac{1}{u - v} + \frac{\partial t'}{\partial x}\frac{u}{u - v} = \frac{1}{u'}\frac{\partial x'}{\partial x_\mathrm{d}}, \tag{14}$$

because $t'(t_0, x_0, 0, 0)$ is independent of the position x_d of the detector and u' is the constant velocity of the body which will depend on u but not on x_d.

Owing to the homogeneity of space and time, the coordinates transformation must be linear, so that the coordinate x' of the event 'arrival to the detector' in K' must be related to the initial coordinate x_d measured in K through a proportionality factor depending only on the relative velocity among observers, v:

$$\frac{\partial x'}{\partial x_\mathrm{d}} = \gamma_v. \tag{15}$$

Now (14) can be written as

$$\frac{\partial t'}{\partial t}\frac{1}{u - v} + \frac{\partial t'}{\partial x}\frac{u}{u - v} = \frac{\gamma_v}{u'}. \tag{16}$$

The solution of (16) can be easily obtained in terms of the variables $p = x + ut$ and $q = x - ut$, where this partial differential equation is reduced to

$$\frac{\partial t'}{\partial p} = \frac{\gamma_v(u - v)}{2uu'},$$

and thus the general solution is

$$t' = \gamma_v\frac{u - v}{2uu'}(x + ut) + f(x - ut), \tag{17}$$

where f is an arbitrary function of $q = x - ut$. By choosing the origins of time and space we can always assume that $t' = 0$ for $t = x = 0$ so that we can take $f(0) = 0$. Owing to the homogeneity of time and space we can also assume that t' and, in consequence, $f(x - ut)$ are linear functions of t and x, so that we can write, in terms of some factor of proportionality λ, which, exactly as u', depends on u (and on the fixed v):

$$f(x - ut) = \gamma_v \lambda (x - ut), \tag{18}$$

and thus (17) can be rewritten as the linear expression of t and x:

$$t' = \gamma_v \left[\frac{u - v}{2u'} - u\lambda\right] t + \gamma_v \left[\frac{u - v}{2uu'} + \lambda\right] x. \tag{19}$$

Since, for all values of t and x the transformation must be independent of the arbitrary velocity u of the body used for clock synchronization, the above terms between squared brackets must be independent of the velocity u, and the following conditions must be met:

$$\frac{\partial}{\partial u}\left[\frac{u - v}{2u'} - u\lambda\right] = \frac{1}{2u'} - \frac{u - v}{2u'^2}\frac{\mathrm{d}u'}{\mathrm{d}u} - \lambda - u\frac{\mathrm{d}\lambda}{\mathrm{d}u} = 0, \tag{20}$$

$$\frac{\partial}{\partial u}\left[\frac{u - v}{2uu'} + \lambda\right] = \frac{v}{2u^2u'} - \frac{u - v}{2uu'^2}\frac{\mathrm{d}u'}{\mathrm{d}u} + \frac{\mathrm{d}\lambda}{\mathrm{d}u} = 0. \tag{21}$$

By eliminating $\mathrm{d}u'/\mathrm{d}u$ between equations (20) and (21) one gets

$$\frac{1}{u'} = \frac{2u\lambda + 4u^2\frac{\mathrm{d}\lambda}{\mathrm{d}u}}{u - v}, \tag{22}$$

which substituted in either equation leads to

$$u\frac{\mathrm{d}^2\lambda}{\mathrm{d}u^2} + 2\frac{\mathrm{d}\lambda}{\mathrm{d}u} = 0. \tag{23}$$

The general solution of this linear differential equation is

$$\lambda = -\frac{C}{u} + D, \tag{24}$$

with C and D two arbitrary constants (which in fact may depend on v, whose value is being held fixed). With this value, (22) and in terms of the constants $A \equiv -2D$ and $B \equiv 2C$, the general solution of equations (20) and (21) can be written as follows:

$$\lambda = -\frac{B + Au}{2u}, \quad u' = \frac{u - v}{B - Au}. \tag{25}$$

To determine the parameters A and B we will use the third postulate of section 2 that no massive body can move with a speed greater than a certain maximum value c_m. This limit velocity has to be the same in all inertial frames. For instance, if in one frame its speed is $u' = c_\mathrm{m}$ in any other frame is must also be $u = c_\mathrm{m}$, because if $u < c_\mathrm{m}$ a slightly larger value of u will correspond to a $u' > c_\mathrm{m}$, since we assume that u' is continuous and monotonically increasing [8]. Putting the limit $u = u' = \pm c_\mathrm{m}$ in (25) one gets

$$c_\mathrm{m} = \frac{c_\mathrm{m} - v}{B - Ac_\mathrm{m}}, \quad -c_\mathrm{m} = \frac{-c_\mathrm{m} - v}{B + Ac_\mathrm{m}}, \tag{26}$$

from where one can find

$$A = \frac{v}{c_{\rm m}^2}, \quad B = 1, \tag{27}$$

so that the velocity transformation is

$$u' = \frac{u - v}{1 - uv/c_{\rm m}^2}. \tag{28}$$

If we solve this for u one gets the inverse transformation:

$$u = \frac{u' + v}{1 + u'v/c_{\rm m}^2}. \tag{29}$$

According to the principle of relativity, this transformation rule must have the same form in both reference frames, so that putting $u = 0$ one concludes that if the velocity of K' as measured in K is v, the velocity of K is measured in K' as $v' = -v$.

Substituting (27) in equations (25) and (19) gives

$$t' = \gamma_v \left(t - \frac{vx}{c_{\rm m}^2} \right), \tag{30}$$

where the factor γ_v is still undetermined. (Notice that (28) is the relativistic transformation of velocities if $c_{\rm m}$ is the speed of light in vacuum.)

The existence of a velocity limit for massive bodies has also been used in other works to obtain the Lorentz transformations [9].

In prerelativistic physics there is no upper limit to the velocity of bodies, and u' must go to infinity as $u \to \infty$, so that one recovers from (28) the Galilean transformation of velocity.

5. Lorentz transformations

We have obtained in the previous section how the time coordinate t' is expressed in terms of t and x, of the same spacetime event, although the factor γ_v is still undetermined. To obtain the coordinate x' of the same spacetime event as a function of t and x it is enough from (7) to substitute in $x' = u'(t' - t'_0)$ together the results (9), (27), (30), $x - x_0 = u(t - t_0)$ and the value $x_0 = vt_0$.

$$\begin{aligned} x' &= u'\gamma_v \left[(t - t_0) - \frac{v}{c_{\rm m}^2}(x - x_0) \right] = \gamma_v \frac{u - v}{1 - uv/c_{\rm m}^2} \left[(t - t_0) - \frac{v}{c_{\rm m}^2}(x - x_0) \right] \\ &= \gamma_v \left[(u - v)(t - t_0) \right] = \gamma_v \left[(x - x_0) - v(t - t_0) \right] = \gamma_v (x - vt). \end{aligned} \tag{31}$$

Let us assume that a body is emitted from O' at time t'_0 in the direction of axis Y' and reaches the detector, at rest in reference frame K', at event $(t', 0, y', 0)$, so that its velocity in K' is $u'_y = y'/(t' - t'_0)$. In the frame K the emission is the event $(t_0, x_0, 0, 0)$ and the arrival $(t, x_0 + vt, y, 0)$, the velocity of the body is $(v, u_y \equiv y/(t - t_0), 0)$ and if the origin of y is appropriately chosen. According to equations (8) and (30), with $u = v$ (see figure 2),

$$y' = u'_y(t' - t'_0) = u'_y \gamma_v \left[\left(t - \frac{vx}{c_{\rm m}^2} \right) - \left(t_0 - \frac{vx_0}{c_{\rm m}^2} \right) \right] = \gamma_v \frac{u'_y}{u_y} \left(1 - \frac{v^2}{c_{\rm m}^2} \right) y. \tag{32}$$

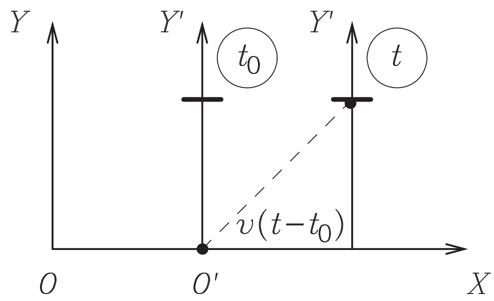

Figure 2. Positions of the origin O' and the detector at rest in frame K' at instants t_0 and t as measured in K. The dotted line represents the trajectory of the body, represented by a dot, launched from O' at time t'_0 and reaching the detector in the reference frame K' at time t'

In order to y' and y not to depend on u'_y, the following must be satisfied:

$$\frac{\mathrm{d}y'}{\mathrm{d}u'_y} \propto u_y - u'_y \frac{\mathrm{d}u_y}{\mathrm{d}u'_y} = 0 \quad \Longleftrightarrow \quad u_y = D u'_y. \tag{33}$$

With the integration constant D, (32) is written as

$$y' = \frac{\gamma_v}{D}\left(1 - \frac{v^2}{c_\mathrm{m}^2}\right) y. \tag{34}$$

Now, if $u'_y = c_\mathrm{m}$ we must have $u = \sqrt{v^2 + u_y^2} = c_\mathrm{m}$ and

$$u_y = \sqrt{c_\mathrm{m}^2 - v^2} = D u'_y = D c_\mathrm{m} \quad \Longleftrightarrow \quad D = \sqrt{1 - \frac{v^2}{c_\mathrm{m}^2}}. \tag{35}$$

In consequence, (34) reduces to

$$y' = \gamma_v \sqrt{1 - \frac{v^2}{c_\mathrm{m}^2}}\, y \tag{36}$$

and a similar expression holds for the coordinate z.

Since frames K' and K are equivalent and the latter moves with velocity $-v$ with respect to the former, by means of equations (30) and (31) one gets

$$\begin{aligned} x &= \gamma_{-v}(x' + vt') = \gamma_{-v}\left[\gamma_v(x - vt) + \gamma_v v\left(t - \frac{vx}{c_\mathrm{m}^2}\right)\right] \\ &= \gamma_v \gamma_{-v}\left(1 - \frac{v^2}{c_\mathrm{m}^2}\right) x, \end{aligned} \tag{37}$$

which implies

$$\gamma_v = \gamma_{-v} = \left(1 - \frac{v^2}{c_\mathrm{m}^2}\right)^{-1/2}, \tag{38}$$

because the invariance about the inversion $(x, v) \to (-x, -v)$ implies $\gamma_v = \gamma_{-v}$. With the value of the parameter γ_v in (38) the relationship between the coordinates of the same spacetime event in both frames takes the form of the Lorentz transformations:

$$t' = \gamma_v \left(t - \frac{vx}{c_{\mathrm{m}}^2} \right), \quad x' = \gamma_v (x - vt), \quad y = y', \quad z = z'. \tag{39}$$

One could also find γ_v by assuming that the relative motion along the x direction does not affect coordinates y and z, so that $y' = y$ and from (36) one recovers (38).

The relativity of simultaneity is contained in the transformations (39) although we have made no mention to any electromagnetic phenomena. Only now we have to identify c_{m} with the speed of light in vacuum to recover the actual Lorentz transformations.

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Appendix A. Lorentz transformations by using the invariance of the speed of light

The criterion in (6) based on the speed of light in vacuum (rather than on the speed limit for massive bodies) has been used [4] to synchronize clocks, but, to the best of our knowledge, not to deduce the Lorentz transformations. We will now achieve the latter goal with the hypothesis of invariance of the speed of light in a similar way to the one based on the free motion of a general body. It has the advantage of connecting the Lorentz transformations to the synchronization criterion and it is simpler.

By making $u = u' = c$ and using the definitions $\tilde{A} \equiv (v/c - 2c\lambda - 1)/2$ and $\tilde{B} \equiv 1 + \tilde{A} - v/c$ in (19) one gets

$$t' = \gamma_v \left[\tilde{B} t - \frac{\tilde{A}}{c} x \right]. \tag{A.1}$$

The assumption that the speed of light in K is also c implies that $x' = c(t' - t'_0)$, which along with $c(t - t_0) = x - x_0$ and $x_0 = vt_0$ leads to

$$x' = \gamma_v (x - vt). \tag{A.2}$$

The equivalence between frames K and K' implies that $x = \gamma_{-v}(x' + vt')$, which by using $\gamma_{-v} = \gamma_v$ and equations (A.1) and (A.2) reads as follows:

$$x = \gamma_v^2 \left[v(\tilde{B} - 1)t + \left(1 - \frac{v\tilde{A}}{c} \right) x \right]. \tag{A.3}$$

Since this result must be fulfilled for all t, one must conclude that $\tilde{B} = 1$, which in turn implies that $\tilde{A} = v/c$ and $x = \gamma_v^2 (1 - v^2/c^2) x$. In consequence, $\gamma_v = (1 - v^2/c^2)^{-1/2}$ and equations (A.1) and (A.2) are the Lorentz transformations:

$$t' = \gamma_v \left(t - \frac{vx}{c^2} \right), \quad x' = \gamma_v (x - vt), \quad y = y', \quad z = z', \quad \gamma_v = \left(1 - \frac{v^2}{c^2} \right)^{-1/2}. \tag{A.4}$$

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