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The penetration of electric fields produced by moving charges into a hollow conducting sphere

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Abstract

The electric field that an external slowly moving charge produces inside a hollow conducting sphere is obtained. Since it is shown that this field depends critically on the sphere thickness, this might have consequences on the screening of electronic equipment.

Electronic equipment is usually bound in metallic or plastic cases. To shield the electronics inside plastic covers sometimes a conducting plastic material is used, but more often one coats the inside part of the non-conducting plastic cover with a conducting deposit, which is usually made of different metallic alloys. This deposit has a thickness of the order of 0.5–50 μm and it damps the external electromagnetic radiation over a wide frequency spectrum [1]. It is usually argued that it also protects the device against neighbouring electrostatic discharges. Certainly, electrostatic fields do not penetrate. Nevertheless, the aim of this Letter is to show that velocity fields will be present inside and that this penetration depends critically on the thickness of the metallic coat.

The penetration of the electric and magnetic fields produced by a point charge moving at low velocity parallel to an infinite conducting wall has been studied by Boyer [2]. Jones [3] has examined the penetration into conductors of the magnetic fields produced by moving charges, and the shielding of the magnetic field of a non-relativistic charge moving near a conducting spherical surface has been discussed by Furry [4]. Penetration of both fields, electric and magnetic, in a solid conducting sphere has been considered recently by the present authors [5].

In another context, one could mention that the penetration of the velocity fields must be taken into account to explain several experiments designed to test the weak equivalence principle for anti-matter [6]. On the other hand, Boyer [7] has suggested that the Aharonov–Bohm effect implies the existence of classical electromagnetic forces between charged particles and solenoids, and that the electric and magnetic velocity fields could be relevant in the explanation of this effect.

In the present paper we shall study the electric field inside a hollow conducting sphere, in the presence of an external non-relativistic moving point charge.

Let us consider an uncharged hollow conducting sphere of constant resistivity η and whose external and internal radius are a and b, respectively (see Fig. 1). A point charge q is moving along the radial direction with constant velocity $v = \mathrm{d}x/\mathrm{d}t \ll c$, from the center. To zeroth order approximation in v/c, Maxwell's equations read

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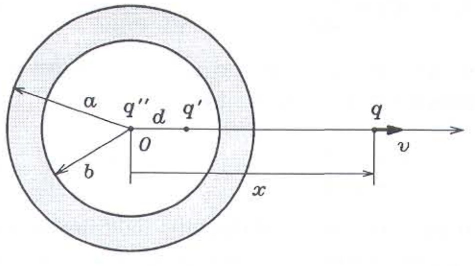

Fig. 1. Image charges of the equivalent electrostatic problem.

$$\operatorname{curl} \boldsymbol{E}^{(0)} = 0, \quad \operatorname{div} \boldsymbol{D}^{(0)} = \rho^{(0)},$$
$$\operatorname{curl} \boldsymbol{H}^{(0)} = \boldsymbol{j}^{(0)}, \quad \operatorname{div} \boldsymbol{B}^{(0)} = 0, \tag{1}$$

where $\rho^{(0)}$ and $\boldsymbol{j}^{(0)}$ are the charge and current densities at zeroth order, respectively. To this order $\boldsymbol{B}^{(0)} = 0$ everywhere and $\boldsymbol{E}^{(0)}$ vanishes inside the sphere of radius a.

Outside the conductor, $\boldsymbol{E}^{(0)}$ is the sum of the electrostatic fields due to the charge q located at the coordinate point x and two image charges $q' = -qa/x$ and $q'' = qa/x$ placed at a distance $d = a^2/x$ from the center and at the center of the sphere respectively [8, p. 51]. Furthermore, the surface charge density on the sphere is obtained by the discontinuity of the electric field and is given by

$$\sigma^{(0)}(\theta) = \frac{q}{4\pi a x} - \frac{q(x^2 - a^2)}{4\pi a(x^2 - 2xa\cos\theta + a^2)^{3/2}}. \tag{2}$$

If the charge is now moving with small velocity, $v = \mathrm{d}x/\mathrm{d}t$, the surface charge density changes and then necessarily inside the conductor there appears a current density whose radial component on the outer surface is given by the continuity equation, which up to the first-order is

$$\frac{\partial \sigma^{(0)}}{\partial t} - j_r^{(1)} = v\frac{\partial \sigma^{(0)}}{\partial x} - j_r^{(1)} = 0, \tag{3}$$

and thus

$$j_r^{(1)}(a,\theta) = \frac{qv}{4\pi a} \times \left(\frac{x^3 + x^2 a\cos\theta - 5xa^2 + 3a^3\cos\theta}{(x^2 - 2xa\cos\theta + a^2)^{5/2}} - \frac{1}{x^2}\right). \tag{4}$$

Obviously, the current density inside the conductor but on the inner surface is

$$j_r^{(1)}(b,\theta) = 0, \tag{5}$$

by the same argument as above.

The first order term of the electric field satisfies

$$\operatorname{curl} \boldsymbol{E}^{(1)} = -\frac{\partial \boldsymbol{B}^{(0)}}{\partial t} = 0, \tag{6}$$

and will be expressed in the form $\boldsymbol{E}^{(1)} = -\operatorname{grad}\phi$ in terms of the scalar function $\phi(r,\theta)$, which must satisfy Laplace's equation.

Due to the axial symmetry this function has the following form in terms of Legendre polynomials [8, p. 86],

$$\begin{aligned}\phi(r,\theta) &= \phi_1(r,\theta) \equiv \sum_{n=0}^{\infty}\left(A_n r^n + \frac{B_n}{r^{n+1}}\right)P_n(\cos\theta), \\ &\quad b \leqslant r \leqslant a, \\ &= \phi_0(r,\theta) \equiv \sum_{n=0}^{\infty} C_n r^n P_n(\cos\theta), \\ &\quad 0 \leqslant r \leqslant b,\end{aligned} \tag{7}$$

and must satisfy the following boundary conditions,

$$-\left.\frac{\partial\phi_1(r,\theta)}{\partial r}\right|_{r=a} = \eta j^{(1)}(a,\theta), \tag{8}$$

$$-\left.\frac{\partial\phi_1(r,\theta)}{\partial r}\right|_{r=b} = 0. \tag{9}$$

By making use of the generating function for Legendre polynomials,

$$f(z,u) = \frac{1}{\sqrt{z^2 - 2uz + 1}} = \sum_{n=0}^{\infty}\frac{1}{z^{n+1}}P_n(u), \tag{10}$$

we get

$$\begin{aligned}2z\frac{\partial^2 f}{\partial z^2} + 3\frac{\partial f}{\partial z} &= \frac{z^3 + z^2 u - 5z + 3u}{(z^2 - 2zu + 1)^{5/2}} \\ &= \sum_{n=0}^{\infty}\frac{(2n+1)(n+1)}{z^{n+2}}P_n(u).\end{aligned} \tag{11}$$

By comparing (4) and (8) with (11) where $z = x/a$ and $u = \cos\theta$, it is easy to see that

$$B_n = \frac{n}{n+1}b^{2n+1}A_n, \tag{12}$$

$$A_0 = 0, \quad A_n = -\frac{qv\eta}{4\pi}\frac{(2n+1)(n+1)}{nx^{n+2}}\frac{1}{1-s^{2n+1}},$$
$$\text{for } n \geqslant 1, \tag{13}$$

where $s = b/a$. In the limit case when $b = 0$, $B_n = 0$ and

$$A_n = -\frac{qv\eta}{4\pi}\frac{(2n+1)(n+1)}{nx^{n+2}}, \quad n \geqslant 1, \tag{14}$$

and we recover the results obtained in Ref. [5].

Continuity of the transversal component of the electric field across the inner surface $r = b$ implies

$$\left.\frac{\partial\phi_0(r,\theta)}{\partial\theta}\right|_{r=b} = \left.\frac{\partial\phi_1(r,\theta)}{\partial\theta}\right|_{r=b}, \tag{15}$$

and thus

$$C_0 = 0,$$
$$C_n = A_n\left(1 + \frac{n}{n+1}\right) = -\frac{qv\eta}{4\pi}\frac{(2n+1)^2}{nx^{n+2}}\frac{1}{1-s^{2n+1}},$$
$$n \geqslant 1. \tag{16}$$

The electric field in the spherical cavity has the components

$$E_{0r}(r,\theta) = -\sum_{n=1}^{\infty} nC_n r^{n-1} P_n(\cos\theta),$$
$$E_{0\theta}(r,\theta) = -\sum_{n=1}^{\infty} C_n r^{n-1}\frac{\mathrm{d}P_n(\cos\theta)}{\mathrm{d}\theta}, \tag{17}$$

and thus in the inner surface $r = b$, a charge density would appear, of value

$$\sigma^{(1)}(b,\theta) = \epsilon_0[E_{1r}(b,\theta) - E_{0r}(b,\theta)]$$
$$= -\sum_{n=1}^{\infty}\frac{qv\eta\epsilon_0}{4\pi}\frac{(2n+1)^2}{x^{n+2}}\frac{b^{n-1}}{1-s^{2n+1}}P_n(\cos\theta). \tag{18}$$

To realise the order of screening of the electric field inside the sphere, let us compute this field at the point $r = 0$. It is given by

$$E_0(0) = -C_1 = \frac{qv\eta}{4\pi}\frac{9}{x^3}\frac{1}{1-s^3}. \tag{19}$$

If we call the thickness $t = a - b$, then for $t \ll a$ expression (19) can be written

$$E_0(0) \simeq \frac{qv\eta}{4\pi}\frac{3}{x^3}\frac{a}{t}, \tag{20}$$

and it must be observed that this field depends critically on the sphere thickness in such a way that the screening decreases as the sphere becomes thinner.

We wonder if this effect could be responsible for some damage that reportedly is done on electronic equipment of flight control centers during nearby storms.

For instance, a typical lightning has values of $qv \simeq 10^7$ C m s^{-1}, and assuming that it hits at $x = 10$ m of a plastic box of 1 m size and that the metallic deposit is a copper–nickel alloy ($\eta \simeq 10^{-6}$ Ω m^{-1}) and 1 μm thick, we get an electric field inside of 10^4 V m^{-1}.

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