LETTERS TO THE EDITOR

Letters are selected for their expected interest for our readers. Some letters are sent to reviewers for advice; some are accepted or declined by the editor without review. Letters must be brief and may be edited, subject to the author's approval of significant changes. Although some comments on published articles and notes may be appropriate as letters, most such comments are reviewed according to a special procedure and appear, if accepted, in the Notes and Discussions section. (See the ''Statement of Editorial Policy'' in the January issue.) Running controversies among letter writers will not be published.

SERENDIPITY REDUX: AVERAGE DISTANCE BETWEEN A STAR AND PLANET

A recent paper[1] showed that the time averaged distance between a planet and a star is different than the angle averaged distance and that neither is equal to the length of the semi-major axis of the orbital ellipse. Further, the latter is equal to the distance averaged over the orbital arc length. I find it intriguing that these results seem to resurface in part or in whole at irregular intervals,[2,3] often in the same journal.

I imagine that over the hundreds of years since Kepler introduced his three laws, the foregoing serendipitous findings were discovered and rediscovered by various people who were struck by the almost universal absence of these facts in textbooks and other sources.

Even though some of these authors exhorted authors of introductory textbooks to address this issue, it might be more practical to ask for its inclusion in intermediate and advanced mechanics texts. Eventually the point that Kepler's third law must be expressed in the correct format will filter down to the elementary text level. Otherwise, I fear that the matter will never be put to rest, and further reincarnations of it will reappear in journals.

[1]D. M. Williams, ''Average distance between a star and planet in an eccentric orbit,'' Am. J. Phys. **71** (11), 1198–2000 (2003).

[2]A. Tan and W. L. Chameides, ''Kepler's third law,'' Am. J. Phys. **49** (7), 691–692 (1981). The authors note that the time and angle averaged distances were considered by P. Van de Kamp, in *Elements of Astromechanics* (Freeman, San Francisco, 1964), pp. 63–66. They evaluated the distance averaged over the arc length.

[3]M. Bucher and D. P. Siemens, ''Average distance and speed in Kepler motion,'' Am. J. Phys. **66** (1), 88–89 (1998); M. Bucher, D. Elm, and D. P. Siemens, ''Average position in Kepler motion,'' *ibid.* **66** (10), 929–930 (1998). While these authors rederived all of the above averages, they extended their results in other interesting directions.

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FINITE DIMENSIONAL HILBERT SPACE

Unfortunately, at the time of writing our paper[1] we were not aware of a previous paper by T. Santhanam and A. R. Tekumalla[2] where they calculate an expression for the commutator shown in Eq. (20) of our paper and also obtain the correct continuous limit. We apologize for not having quoted their work.

[1]A. C. de la Torre and D. Goyeneche, ''Quantum mechanics in finite dimensional Hilbert space,'' Am. J. Phys. **71**, 49–54 (2003).

[2]T. Santhanam and A. R. Tekumalla, ''Quantum Mechanics in Finite Dimensions,'' Found. Phys. **6**, 583–587 (1976).

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TEACHING ABOUT CENTRAL FORCES

In a recent letter to the editor, Martin Tiersten[1] pointed out that ''the relation $F_n = mv^2/R$, where R is the radius of curvature and F_n is the normal component of the force [...] appears (amazingly) not to be widely known among physicists... .'' To overcome this lack, we usually ask our undergraduate students in classical mechanics to calculate the velocity of a point mass moving in an inverse square force field at an apsidal point of its orbit. Because at such a point the normal force F_n equals the total force $F = k/r^2$ (r is the distance from the center of force), most of them will ignore the fact that the orbit is not necessarily circular and use $k/r^2 = mv^2/r$ to calculate v. Then we ask them to use the correct relation $k/r^2 = mv^2/R$, along with what they have learned about Newtonian orbits, to calculate R. They are usually surprised by the simple result: in elliptic, parabolic, and hyperbolic orbits, the radius of curvature at apsidal points equals the *semilatus rectum* p, which appears in the orbit equation $r = p/(1+\epsilon\cos\varphi)$. This result gives a neat geometric interpretation for p (which of course is also the distance r at right angles from the apsidal points) and hopefully will help our students remember that the normal acceleration is v^2/R and that the center of curvature is not in general located at the center of force. One also can direct them to Web sites,[2] where the locus of the centers of curvature (that is, the evolute) of the ellipse, the hyperbola and other curves are shown.

[1]M. Tiersten, ''Errors in Goldstein's classical mechanics,'' Am. J. Phys. **71**, 103 (2003).

[2]See, for instance, ⟨http://www-history.mcs.st-andrews.ac.uk/history/Curves/Curves.html⟩.

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