gin (the maternity ward) to events α and β separated the twins by L; each twin was moving at speed v_s which we shall, in a moment, assume to be very small. The twins subsequent stationary "motion" is represented by the vertical segments, which—far in the future—will lead them to rocket ships.

The times t'_α and t'_β, of events α and β, as observed by the uncle, are easily found from the Lorentz transformations, and from the known coordinates of the events in the Mom and Dad frame:

$$t'_\alpha = \gamma[t_\alpha - vx_\alpha] = \gamma[(L/2v_s) - vL/2]. \tag{3}$$

For event β the x coordinate is $x_\beta = -L/2$ so the t' coordinate for the event is

$$t'_\beta = \gamma[(L/2v_s) + vL/2]. \tag{4}$$

The time difference, as measured by the uncle, will therefore be

$$\Delta t' \equiv t'_\beta - t'_\alpha = \gamma v L. \tag{5}$$

This result is independent of v_s; the time difference observed by the uncle remains, no matter how slowly Dick and Jane move. The result is, of course, the result we have already noted as the uncle-measured time difference between the birthdays of Dick and Jane.

We have given different answers to the question "where does the differential aging occur?": (i) It all occurs during the twins' rocket trip. (ii) Some occurs in the twins early (prerocket) years. The lack of a unique answer shows the lack of meaning of the question. Age differences in relativity have a well-defined meaning, but the origin of age differences cannot be assigned to any specific part of a worldline.

[1]S. P. Boughn, "The case of the identically accelerated twins," Am. J. Phys. **57**, 791–793 (1989). Other references to the twin paradox are listed in this paper.

[2]Here and elsewhere we use words like "see" and "view" somewhat inappropriately. We do not mean that the uncle makes observations of distant events by collecting light signals from those events. Rather, we mean that observations are made according to the usual procedure in relativity. Here, that would mean that an observer who is part of the uncle's special relativity reference system, but is located *right at* an event being observed (such as the igniting of a rocket engine), notes the position and time of that event.

Shielding of an oscillating electric field by a hollow conductor

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(Received 14 December 1995; accepted 4 March 1996)

The electric and magnetic fields for a hollow conducting sphere located in a slowly varying uniform electric field background are computed to first-order in a power series expansion in the field frequency. These results are used to define an equivalent RC circuit and to test the circuit approach which is often used in electromagnetic compatibility (EMC). The case of an infinite cylindrical conducting tube under the influence of the same external field is also analyzed.

I. INTRODUCTION

The knowledge of the penetration of the electric and magnetic fields in electronic equipment is important to properly protect these ever increasingly sensitive devices from external influences. In fact, the shielding of a receptor set from a source of electrical disturbance is an interesting subject of research in electromagnetic compatibility (EMC), which is defined by IEEE as "the ability of a device, equipment or system to function satisfactorily in its electromagnetic environment without introducing intolerable electromagnetic disturbances to anything in that environment."[1]

The physicist's approach to evaluating the electromagnetic shielding is based upon the solution of Maxwell's equations with appropriate boundary conditions on the shielding surfaces, but the mathematical machinery is so complex that, even when the calculations can be carried out, the physical insight is often missed.[2–4] As a consequence, from an engineering point of view, to estimate in practice the electromagnetic field inside the shielding enclosure, it is always necessary to use a simplified theory of electromagnetic shielding.

Among the techniques developed so far in EMC to deal with this kind of calculation we will consider here the so-called "circuit approach" in which the actual physical system is replaced by an equivalent RC circuit. This approach is based upon the fact that the external electromagnetic field will induce on the shielding enclosure a charge distribution which will vary in time because the external field is oscillating. This will produce a current flow in the conductor and it seems rather natural to substitute for the conductor an equivalent electric circuit whose characteristics are defined on heuristic grounds because in general they cannot be computed accurately, not even by numerical simulation.[4]

The main goal of this paper is to analyze a couple of simple but interesting examples in which explicit (although approximate) expressions for these phenomena may be easily computed. In this way we can illustrate and compare the approaches that a physicist (starting from Maxwell's equations) and an engineer (by using the circuit approach) would choose to analyze these problems.

We use a simple perturbative method to obtain, to first order in a power series expansion in the frequency, the electric and magnetic fields in the interior of a hollow conducting sphere and cylinder when these bodies are located in a slowly varying uniform electric field background. We compare these results with those obtained by means of the alternative method of using an equivalent circuit model, which is often used in EMC.

The paper is organized as follows. In Sec. II we compute the first-order approximation to the electric field inside, within, and outside of a hollow conducting sphere under the influence of an outer uniform electric field that varies sinusoidally at a low frequency. In Sec. III the first-order contribution to the magnetic field in the above three regions is calculated, and in Sec. IV the energy balance is checked by calculation of the first and second-order Poynting vector. The energy dissipation is used to calculate in Sec. V the resistance of an equivalent RC-circuit model to obtain the electric field in the spherical cavity. In Sec. VI the same problem is solved but by replacing this time the sphere by an infinite hollow conducting cylinder. Some final comments about the accuracy of the approximation method are collected in the last section.

II. THE ELECTRIC FIELD

Let us consider a hollow conducting sphere of ohmic material of resistivity η. Its external radius is a and the internal radius b. The sphere is immersed in a uniform external electric field,

$$\mathbf{E}(t)=E\cos\omega t\hat{\mathbf{k}}, \tag{1}$$

where E and ω are constants and $\hat{\mathbf{k}}$ is a unit vector along OZ axis.

We shall assume that ω is small enough so that we may consider the situation to be quasistatic. In fact we shall expand all physical quantities in terms of a dimensionless parameter proportional to ω. The coefficient of ω in the exact definition of this expansion parameter will appear in a natural way when performing the actual computation in the different contexts, as we will discuss in Sec. VII. Let us only mention here that in the case of a thin sphere of thickness d this parameter appears as $\omega a^2/cd$ or $\epsilon_0\eta\omega a/d$, depending on the quantity being expanded. If the thickness is not negligible the parameter is $\omega a/c$ or $\epsilon_0\eta\omega$. In this approach only the leading terms of these expansions will be kept. An upper index in each quantity will indicate its expansion order.

In the quasistatic approximation, the external electric field (1) generates on the sphere's surface a charge density that to lowest order is given by[5]

$$\sigma^0(a,\theta)=3\epsilon_0(E\cos\omega t)\cos\theta, \tag{2}$$

where θ is the usual polar angle (see Fig. 1).

To this approximation, the electric field $\mathbf{E}^0$ and magnetic field $\mathbf{B}^0$ vanish inside the sphere. In the outer region $\mathbf{B}^0$ also vanishes, and for the electric field we get[5]

$$\mathbf{E}^0(r,\theta)=E\cos\omega t\left[\left(1+\frac{2a^3}{r^3}\right)\cos\theta\hat{\mathbf{r}}-\left(1-\frac{a^3}{r^3}\right)\sin\theta\hat{\boldsymbol{\theta}}\right],\quad r\geqslant a, \tag{3}$$

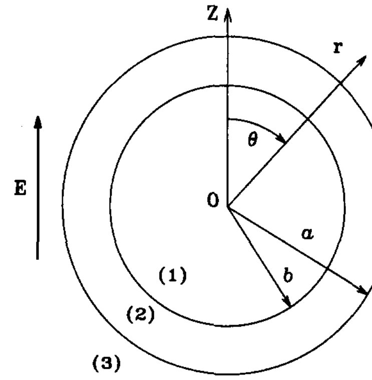

Fig. 1. Hollow sphere under the influence of an external electric field **E**.

where $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\theta}}$ are unit vectors. Since the external electric field is changing in time, the charge density $\sigma^0(a,\theta)$ will also change and consequently a current flows in the conductor; the radial component of the current density at the surface must satisfy the following continuity equation at this lowest order:

$$\frac{\partial\sigma^0(a,\theta)}{\partial t}-j_r^1(a,\theta)=0, \tag{4}$$

which implies

$$j_r^1(a,\theta)=-3\epsilon_0\omega E\sin\omega t\cos\theta. \tag{5}$$

Notice that both terms in Eq. (4) are of first order because the time derivative of Eq. (2) produces an extra ω factor. This will have to be taken into account very often in what follows: A time derivative will increase by one unit the expansion order.

If the conductor is Ohmic, the radial component of the electric field at the outer surface is given by

$$E_r^1(a,\theta)=\eta j_r^1(a,\theta)=-3\epsilon_0\omega\eta E\sin\omega t\cos\theta, \tag{6}$$

while at the inner surface it must vanish:

$$E_r^1(b,\theta)=0, \tag{7}$$

because $\sigma^0(b,\theta)=0$.

The general expression for the electric field in terms of the scalar and vector potentials

$$\mathbf{E}=-\nabla\phi-\frac{\partial\mathbf{A}}{\partial t}, \tag{8}$$

can be written at first order as $\mathbf{E}^1=-\nabla\phi^1$, because $\mathbf{E}^1$ has zero curl, since the lowest-order magnetic field vanishes. Thus, the first-order scalar potential ϕ^1 must satisfy Laplace's equation.

Since the system has axial symmetry, the solution of Laplace's equation either inside or outside the sphere can be expressed as

$$\phi^1(r,\theta)=\sum_{n=0}^{\infty}[A_nr^n+B_nr^{-(n+1)}]P_n(\cos\theta), \tag{9}$$

where A_n and B_n are constant coefficients and P_n Legendre polynomials.

The potential ϕ^1 is required to be finite at $r=0$ and zero at infinity and thus it turns out that ϕ^1 has the following form in the different regions:

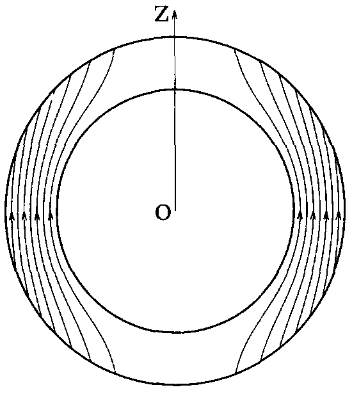

Fig. 2. First-order current lines in the sphere.

$$\phi_1^1(r,\theta)=A_0+A_1 r\cos\theta,\quad 0\leqslant r\leqslant b,$$

$$\phi_2^1(r,\theta)=C_0+C_1 r\cos\theta+\frac{C_2}{r^2}\cos\theta,\quad b\leqslant r\leqslant a, \tag{10}$$

$$\phi_3^1(r,\theta)=D_0+\frac{D_1}{r^2}\cos\theta,\quad r\geqslant a.$$

By assuming the continuity of the potential through the different surfaces and the boundary conditions for the electric field,

$$-\frac{\partial\phi_2^1}{\partial r}\Big|_{r=b}=0,$$
$$-\frac{\partial\phi_2^1}{\partial r}\Big|_{r=a}=-3\epsilon_0\omega\eta E\sin\omega t\cos\theta, \tag{11}$$

the unknown coefficients are easily calculated. By introducing the quantity

$$\Lambda\equiv\frac{3}{2}\,\epsilon_0\omega\eta E\sin\omega t\,\frac{a^3}{a^3-b^3}, \tag{12}$$

one gets

$$\phi_1^1(r,\theta)=3\Lambda r\cos\theta,\quad r\leqslant b,$$
$$\phi_2^1(r,\theta)=\Lambda\,\frac{2r^3+b^3}{r^2}\cos\theta,\quad b\leqslant r\leqslant a, \tag{13}$$
$$\phi_3^1(r,\theta)=\Lambda\,\frac{2a^3+b^3}{r^2}\cos\theta,\quad r\geqslant a.$$

The electric field in the different regions can be easily computed to obtain

$$\mathbf{E}_1^1(r,\theta)=-3\Lambda\hat{\mathbf{k}},\quad r\leqslant b, \tag{14}$$

$$\mathbf{E}_2^1(r,\theta)=2\Lambda\left[\left(\frac{b^3}{r^3}-1\right)\cos\theta\hat{\mathbf{r}}+\left(\frac{b^3}{2r^3}+1\right)\sin\theta\hat{\boldsymbol{\theta}}\right],$$
$$b\leqslant r\leqslant a, \tag{15}$$

$$\mathbf{E}_3^1(r,\theta)=2\Lambda\left[\frac{2a^3+b^3}{r^3}\cos\theta\hat{\mathbf{r}}+\frac{2a^3+b^3}{2r^3}\sin\theta\hat{\boldsymbol{\theta}}\right],$$
$$r\geqslant a. \tag{16}$$

The field lines corresponding to the field $\mathbf{E}_2^1$, and thus the current lines inside the sphere, are depicted in Fig. 2 by making use of a computer program.[6]

To lowest order the surface charge density in the outer surface is given by Eq. (2) while it vanishes in the inner surface. The next order contribution can be calculated through the discontinuity of the normal component of the next order electric field. Thus, taking into account the above expressions for this field we get

$$\sigma^1(b,\theta)=\epsilon_0[E_{2r}^1(b,\theta)-E_{1r}^1(b,\theta)]=3\epsilon_0\Lambda\cos\theta, \tag{17}$$

$$\sigma^1(a,\theta)=\epsilon_0[E_{3r}^1(a,\theta)-E_{2r}^1(a,\theta)]=6\epsilon_0\Lambda\cos\theta, \tag{18}$$

and in consequence $\sigma^1(a,\theta)=2\sigma^1(b,\theta)$.

If the sphere thickness $d\equiv a-b$ is very small, we can make in the above expressions the substitution

$$\frac{a^3}{a^3-b^3}\approx\frac{a}{3d}, \tag{19}$$

and the charge densities become

$$\sigma^1(b,\theta)=\frac{3a}{2d}\,\epsilon_0^2\omega\eta E\sin\omega t\cos\theta, \tag{20}$$

$$\sigma^1(a,\theta)=\frac{3a}{d}\,\epsilon_0^2\omega\eta E\sin\omega t\cos\theta. \tag{21}$$

Notice that according to Eq. (14) the electric field in the sphere cavity is uniform and in the $d\ll a$ approximation has the value:

$$\mathbf{E}_1^1(r,\theta)=-\frac{3a}{2d}\,\epsilon_0\omega\eta E\sin\omega t\hat{\mathbf{k}},\quad r\leqslant b. \tag{22}$$

In consequence, the internal field and thus the shielding effect depends linearly on the a/d ratio in this approximation.

Since the electric field inside the sphere in the alternative circuit approach is not easily calculated at every point, one often uses instead a mean electric field which by simplicity is taken to be equal to the field at the center of the sphere.[7] Our results above support these assumptions.

III. THE MAGNETIC FIELD

Because of the axial symmetry of the problem, the magnetic field, $\mathbf{B}=B_\phi(r,\theta)\hat{\boldsymbol{\phi}}$, has a single nonvanishing component which is independent of ϕ. By making use of the Ampère–Maxwell law at first order we get:

$$\oint_C\mathbf{B}^1\cdot d\mathbf{l}=\mu_0\int_S\left(\mathbf{j}^1+\epsilon_0\,\frac{\partial\mathbf{E}^0}{\partial t}\right)\cdot d\mathbf{s}, \tag{23}$$

where S is a spherical cap concentric with the sphere and whose border is the circle C, defined by constant values of the variables r and θ (see Fig. 3).

It is easy to conclude that the magnetic field vanishes inside the sphere:

$$B_{1\phi}^1(r,\theta)=0,\quad r\leqslant b. \tag{24}$$

In the bulk of the sphere $\mathbf{E}^0=0$ and thus the displacement current vanishes, and the Ampère–Maxwell law is written as

$$B_{2\phi}^1 2\pi r\sin\theta=\mu_0\int_0^\theta j_r^1(r,\alpha)2\pi r^2\sin\alpha\,d\alpha, \tag{25}$$

and leads to

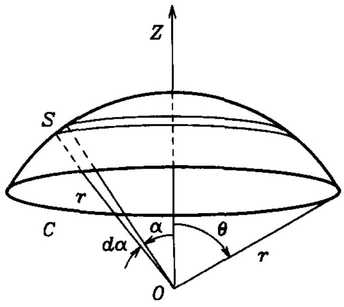

Fig. 3. Integration surface S and boundary circuit C to compute the magnetic field.

$$B^1_{2\phi}(r,\theta)=\frac{3\omega E\sin\omega t}{2c^2}\frac{a^3}{a^3-b^3}\left(\frac{b^3}{r^2}-r\right)\sin\theta,$$
$$b\leqslant r\leqslant a. \tag{26}$$

Notice that on the inner surface, $r=b$, one gets $B^1_{2\phi}(b,\theta)=0$.

In the outer region, $r\geqslant a$, the current density is zero and only the displacement current has to be taken into account on the right-hand side of (23). Since in our case the electric field $\mathbf{E}^0$ is given by (3), the magnetic field is

$$B^1_{3\phi}(r,\theta)=-\frac{\omega E\sin\omega t}{2c^2}\left(r+\frac{2a^3}{r^2}\right)\sin\theta,\quad r\geqslant a. \tag{27}$$

It is easily checked that $B^1_{3\phi}(a,\theta)=B^1_{2\phi}(a,\theta)$ in accordance with the continuity conditions for the magnetic field.

IV. ENERGY BALANCE

The first terms in the expansions for the electric and magnetic fields are

$$\mathbf{E}=\mathbf{E}^0+\mathbf{E}^1,\quad \mathbf{B}=\mathbf{B}^1+\mathbf{B}^2, \tag{28}$$

and the Poynting vector $\mathbf{N}=\mathbf{E}\times\mathbf{B}/\mu_0$ can be written as $\mathbf{N}=\mathbf{N}^1+\mathbf{N}^2$, where

$$\mathbf{N}^1=\frac{1}{\mu_0}\mathbf{E}^0\times\mathbf{B}^1,\quad \mathbf{N}^2=\frac{1}{\mu_0}\mathbf{E}^1\times\mathbf{B}^1+\frac{1}{\mu_0}\mathbf{E}^0\times\mathbf{B}^2. \tag{29}$$

We shall check Poynting's theorem with the energy balance in the volume enclosed by the outer spherical surface Σ of radius a. Because this is a quasistatic situation, $\mathbf{E}^0$ is orthogonal to the surface of the sphere, and thus $\mathbf{N}^1$ and the second term of $\mathbf{N}^2$ are tangent to the sphere, and it turns out that the only contribution to the incoming energy flux is due to the first term of the field $\mathbf{N}^2$.

On the surface we have

$$E^1_{3\theta}(a,\theta)=\frac{3\epsilon_0\omega\eta E\sin\omega t(2a^3+b^3)}{2(a^3-b^3)}\sin\theta, \tag{30}$$

$$B^1_{3\phi}(a,\theta)=-\tfrac{3}{2}\epsilon_0\mu_0\omega aE\sin\omega t\sin\theta, \tag{31}$$

and the flux of vector $\mathbf{N}^2$ is

$$\oint_\Sigma \mathbf{N}^2\cdot d\mathbf{s}=-6\pi\eta\epsilon_0^2\omega^2E^2\sin^2\omega t\,\frac{a^3(2a^3+b^3)}{a^3-b^3}, \tag{32}$$

where the minus sign shows that the flux is incoming.

The rate of dissipation of energy by Joule heating within the sphere will be

$$W=\int \mathbf{j}^1\cdot\mathbf{E}^1\,dV=\int_V\frac{(\mathbf{E}^1)^2}{\eta}\,dV, \tag{33}$$

where $\mathbf{E}^1$ is given in (15). The volume element is $dV=2\pi r^2\sin\theta\,dr\,d\theta$ so that the power dissipated is

$$W=6\pi\eta\epsilon_0^2\omega^2E^2\sin^2\omega t\,\frac{a^3(2a^3+b^3)}{a^3-b^3}, \tag{34}$$

which is equal and opposite to Eq. (32) as expected.

V. CIRCUIT APPROACH

An alternative way to calculate the shielding effectiveness is to replace the actual system by an equivalent alternating current electric circuit, as we shall discuss in what follows.

The sphere under the action of the external electric field (1) can be represented by an RC circuit, whose different elements are as follows.

(a) An external potential $V_e=V_0\cos\omega t$, where $V_0=Ez$, z is being some effective length to be defined properly.
(b) A current I which equals the total current crossing the sphere through the equatorial plane.
(c) In the usual EMC approach the resistance R is introduced on heuristic grounds. Here we can take advantage of our previous energy balance calculation to choose for R the value that will reproduce the energy dissipated in the sphere by Joule heating.
(d) A capacitance C such that the circuit with impedance $1/\omega C$ satisfies the a.c. Ohm's law.

In the equatorial plane $\theta=\pi/2$ the current density is

$$j_{2\theta}=\frac{E_{2\theta}}{\eta}=3\epsilon_0\omega E\sin\omega t\,\frac{a^3}{a^3-b^3}\left(1+\frac{b^3}{2r^3}\right), \tag{35}$$

and thus the current crossing through the equatorial plane and considered positive in the increasing direction of z is

$$I=-\int_b^a j_{2\theta}2\pi r\,dr=-I_0\sin\omega t, \tag{36}$$

where $I_0=3\pi\epsilon_0\omega Ea^2$. It is interesting to remark that I does not depend on the thickness of the sphere.

This current is shifted forward 90 deg with respect to the external potential $V_0\cos\omega t$. In consequence we shall assume that the circuit impedance is basically capacitive: $R\ll 1/\omega C$. In Sec. VII we shall discuss the restrictions imposed by this assumption on the validity range of our analysis. The root-mean-square current is $I_0/\sqrt{2}$ and the power dissipated in the resistance R is given by $RI_0^2/2$. By identifying this with the mean value of Eq. (34), we obtain

$$R=\frac{2\eta(2a^3+b^3)}{3\pi a(a^3-b^3)}, \tag{37}$$

and when $b\simeq a$ this expression becomes

$$R\approx\frac{2\eta}{3\pi d} \tag{38}$$

in terms of the sphere thickness $d=a-b$.

Finally, to compute C we use the fact that $V_0=I_0/\omega C$ and thus

$$C=\frac{I_0}{\omega V_0}=3\pi\epsilon_0\frac{a^2}{z}, \tag{39}$$

in terms of an effective length z whose exact definition will be discussed below.

Ohm's law gives the following estimate for the average internal field:

$$E_i=\frac{RI}{z}=-\frac{2\epsilon_0\omega\eta Ea(2a^3+b^3)}{z(a^3-b^3)}\sin\omega t, \tag{40}$$

in such way that in the low thickness approximation we get

$$E_i=-\frac{2\epsilon_0\omega\eta Ea^2}{zd}\sin\omega t. \tag{41}$$

If we consider that this average electric field in the circuit E_i represents the field inside the sphere, then by comparing (41) with (22) we see that the actual effective length of the sphere is

$$z=\tfrac{4}{3}a, \tag{42}$$

which leads to the following expression for the circuit capacitance:

$$C=\tfrac{9}{4}\pi\epsilon_0 a. \tag{43}$$

In EMC, the circuit characteristics are selected on heuristic grounds rather than based on results from a perturbative calculation as in our previous analysis. Bridges[2] takes an effective length $z=a$ and obtains $C=3\pi\epsilon_0 a$, while Franceschetti[3] by using $z=2a$ arrives to the value $C=\pi\epsilon_0 a$. In both cases the criteria used to compute the circuit parameters are very different from ours.

The electric shielding effectiveness (SE) is defined as[2]

$$\mathrm{SE}=-20\log_{10}\frac{|E_i|}{|E|}. \tag{44}$$

For a conducting sphere of thickness $d\ll a$ this gives rise to

$$\mathrm{SE}=-20\log_{10}\frac{3\epsilon_0\omega\eta a}{2d}. \tag{45}$$

This result coincides with the value given by Bridges.[2]

VI. A HOLLOW CONDUCTING CYLINDER

Instead of a sphere we shall apply the above method to the case of an infinite hollow conducting cylinder under the influence of the same varying external uniform electric field. Let us assume that the cylinder is of external and internal radius a and b respectively and directed along the OZ axis, which is the cylinder symmetry axis. The material resistivity is η and the external electric field is orthogonal to the cylinder axis and given by the formula

$$\mathbf{E}(t)=E\cos\omega t\,\hat{\imath}, \tag{46}$$

where $\hat{\imath}$ is the unit vector along the OX axis.

The lowest-order surface charge density on the cylinder outer surface is[8]

$$\sigma^0(a,\phi)=2\epsilon_0 E\cos\omega t\cos\phi, \tag{47}$$

where ϕ is the polar angle in cylindrical coordinates.

In the following we simply collect the main results we have obtained by the same method as in the case of the sphere. Inside the cylinder the first-order electric field is

$$\mathbf{E}_1^1=-\frac{4a^2}{a^2-b^2}\epsilon_0\omega\eta E\sin\omega t\,\hat{\imath},\quad r\leq b, \tag{48}$$

while the magnetic field $\mathbf{B}_1^1$ vanishes.

The first-order charge density is the same in the interior and exterior surfaces:

$$\sigma^1(a,\phi)=\sigma^1(b,\phi)=\frac{4\epsilon_0^2\omega\eta a^2}{a^2-b^2}E\sin\omega t\cos\phi. \tag{49}$$

The power dissipated per unit length by Joule heating is given by

$$W=\frac{4\pi a^2(a^2+b^2)}{a^2-b^2}\epsilon_0^2\omega^2\eta E^2\sin^2\omega t, \tag{50}$$

while the current per unit length through the plane ZOY is

$$I=-4a\epsilon_0\omega E\sin\omega t \tag{51}$$

and it does not depend on the thickness, exactly as it happens in the case of the sphere.

By means of the same criteria we used in the previous example we obtain the following values for the equivalent circuit parameters when considering a cylinder portion of length l:

$$R=\frac{\pi\eta(a^2+b^2)}{4l(a^2-b^2)},\quad C=\frac{16\epsilon_0 a^2 l}{\pi(a^2+b^2)},\quad z=\frac{\pi(a^2+b^2)}{4a}. \tag{52}$$

If $d\ll a$ these parameters become

$$R\approx\frac{\pi\eta a}{4ld},\quad C\approx\frac{8\epsilon_0 l}{\pi},\quad z\approx\frac{\pi a}{2}, \tag{53}$$

and the shielding effectiveness SE is

$$\mathrm{SE}=-20\log_{10}\frac{2\epsilon_0\omega\eta a}{d}. \tag{54}$$

VII. COMMENTS AND DISCUSSION

In order to estimate the accuracy of the approximate method used in Secs. II, III, and V let us compute the electric dipole moment of the sphere at orders 0 and 1.

The first two orders of the dipole moment are

$$P^0=4\pi a^3\epsilon_0 E\cos\omega t, \tag{55}$$

$$P^1=6\pi a^3\epsilon_0^2\omega\eta\left(\frac{2a^3+b^3}{a^3-b^3}\right)E\sin\omega t. \tag{56}$$

In the case of small thickness, the ratio between the maximum values of these moments is

$$\frac{P^1_{\max}}{P^0_{\max}}\simeq\frac{3\epsilon_0\omega\eta a}{2d}. \tag{57}$$

A necessary condition for the validity of the method used in Sec. II to evaluate the electric field in the interior of the sphere is to have a small ratio:

$$\frac{P^1_{\max}}{P^0_{\max}}\ll 1. \tag{58}$$

Now, this condition is equivalent to the $R\ll 1/\omega C$ assumption, that is used in the circuit approach because

$$\omega RC\simeq\frac{3\epsilon_0\omega\eta a}{2d}\ll 1, \tag{59}$$

thus justifying that the capacitive impedance dominates over R.

On the other hand, by inspection of expressions (14), (15), and (16) we realize that the dimensionless expansion parameter for the electric field is $\epsilon_0\omega\eta$ for finite thickness and

$\epsilon_0 \omega \eta a/d$ when the thickness d is very small. The smallness of the later quantity is equivalent to condition (59). For the magnetic field we see in (26) and (27) that the effective expansion parameter is $\omega a/c$, or $\omega a^2/cd$. The last quantity is small for $\omega \ll 10^5\ \mathrm{s}^{-1}$ if we assume $a/d \simeq 10^3$.

Notice that for instance, in the case of a copper sphere, $\eta \simeq 1.7 \cdot 10^{-6}\ \Omega\mathrm{m}$, of radius $a=1$ m, thickness $d=10^{-3}$ m and frequency values of order $\omega \ll 10^5\ \mathrm{s}^{-1}$, the skin depth δ is much larger than the thickness d:

$$\delta = \sqrt{\frac{2\eta}{\mu_0 \omega}} \gg d. \tag{60}$$

It must be observed that the first-order electric field does depend on the resistivity η, while the magnetic field does not. Inside the sphere the electric field is proportional to the time derivative of the external field, a result that agrees with the one obtained for the electric field by Franceschetti,[3] who considers that the sphere is under the influence of an electric and magnetic field which change simultaneously in time, or more precisely, under the influence of a plane wave.

ACKNOWLEDGMENTS

This work has been partially supported by the Universidad del País Vasco/Euskal Herriko Unibertsitatea under Contract UPV/EHU 172.310-EB036/95.

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[4]See, Ref. 1, p. 118.

[5]D. J. Griffiths, *Introduction to Electrodynamics* (Prentice-Hall, Englewood Cliffs, NJ, 1981), p. 126.

[6]J. M. Aguirregabiria, *ODE Workbench* (American Institute of Physics, Woodburg, NY, 1994).

[7]See, Ref. 1, p. 130.

[8]M. Schwartz, *Principles of Electrodynamics* (McGraw-Hill Kogakusha, Tokyo, 1972), p. 76.

Classical theory of the chaotic ionization of highly excited helium atoms

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(Received 21 November 1995; accepted 10 March 1996)

In this paper, we introduce a classical model for the behavior of the outer electron of a Rydberg helium atom in both static and time-dependent electric fields; we contrast the behavior of this dynamical system with the behavior of the classical model for highly excited hydrogen atoms in similar fields; and we show that these classical models recover many features of the system that would seem to be purely quantal in origin. Perhaps the most surprising of these is the apparent presence of "classical avoided crossings" in the Stark energy level structure of the classical helium model. Finally, we compare the behavior of our helium model to experimental data obtained by Koch, Mariani, and co-workers in their investigations of the ionization of highly excited helium atoms in a microwave field, and we note the agreement between experiment and theory. © *1996 American Association of Physics Teachers.*

I. INTRODUCTION

The experiments of Koch *et al.* on the ionization of highly excited hydrogen atoms in microwave fields[1] have been extensively discussed in the literature over the past decade. One of the important issues that has been raised is the significance of classical mechanics—particularly *chaotic* classical mechanics—in the explanation of the Koch data.[2,3] Not so well known are the data that Koch's group has obtained on the ionization of Rydberg helium atoms in microwave fields.[4–6] The behavior of helium atoms with one electron in a highly excited Rydberg state (with quantum number n) in a strong microwave field differs significantly from the behavior of highly excited hydrogen atoms with the same n. This discrepancy in the dynamics stems from the non-Coulombic core potential seen by the Rydberg electron in helium. This paper is an investigation of the classical dynamics of the Rydberg electron in an excited helium atom. We present a classical model for Rydberg helium in which the core electron is modelled as an effective potential and illustrate the interesting dynamics of the classical helium Rydberg atom in several configurations of applied electric fields. In particular, we have used this model as the basis for a Monte Carlo calculation of the ionization of Rydberg helium in a microwave field and compared our results to the experimental data of Mariani *et al.*[4–6] Since the classical description of an electron in a highly excited helium atom cannot be reduced to a single degree of freedom, we cannot use a one-dimensional