ERAGILE BEKTORIALAK

- Eremu eskalarra $f = f(\vec{r})$, eremu bektoriala $\vec{A} = \vec{A}(\vec{r})$

- **OSAGAI KARTESIARRAK**

  $f = f(x, y, z)$
  $\vec{A} = A_x(x, y, z)\hat{i} + A_y(x, y, z)\hat{j} + A_z(x, y, z)\hat{k}$

  - Gradientea : $gr\vec{a}df = \vec{\nabla} f = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}$
  - Dibergentzia : $div\vec{A} = \vec{\nabla} \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$
  - Errotazionala : $r\vec{o}t\vec{A} = \vec{\nabla} \times \vec{A} = (\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z})\hat{i} + (\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x})\hat{j} + (\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y})\hat{k}$
  - Laplazearra : $\nabla^2 f = \frac{\partial^2 f}{\partial^2 x} + \frac{\partial^2 f}{\partial^2 y} + \frac{\partial^2 f}{\partial^2 z}$, edo $\nabla^2 \vec{A} = \nabla^2 A_x\hat{i} + \nabla^2 A_y\hat{j} + \nabla^2 A_z\hat{k}$

- **OSAGAI ESFERIKOAK**

  $f = f(r, \theta, \phi)$
  $\vec{A} = A_r(r, \theta, \phi)\hat{r} + A_\theta(r, \theta, \phi)\hat{\theta} + A_\phi(r, \theta, \phi)\hat{\phi}$

  - Gradientea : $gr\vec{a}df = \vec{\nabla} f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{rsin\theta}\frac{\partial f}{\partial \phi}\hat{\phi}$
  - Dibergentzia : $div\vec{A} = \vec{\nabla} \cdot \vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{rsin\theta}\frac{\partial}{\partial \theta}(sin\theta A_\theta) + \frac{1}{rsin\theta}\frac{\partial}{\partial \phi}(A_\phi)$
  - Errotazionala : $r\vec{o}t\vec{A} = \vec{\nabla} \times \vec{A} =$
    $\frac{1}{rsin\theta}[\frac{\partial}{\partial \theta}(sin\theta A_\phi) - \frac{\partial A_\theta}{\partial \phi}]\hat{r} + \frac{1}{r}[\frac{1}{sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{\partial}{\partial r}(rA_\phi)]\hat{\theta} + \frac{1}{r}[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial \theta}]\hat{\phi}$
  - Laplazearra : $\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial f}{\partial r}) + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial \theta}(sin\theta\frac{\partial f}{\partial \theta}) + \frac{1}{r^2 sin^2\theta}\frac{\partial^2 f}{\partial \phi^2}$

- **OSAGAI ZILINDRIKOAK**

  $f = f(\rho, \phi, z)$
  $\vec{A} = A_\rho(\rho, \phi, z)\hat{r} + A_\phi(\rho, \phi, z)\hat{\theta} + A_z(\rho, \phi, z)\hat{\phi}$

  - Gradientea : $gr\vec{a}df = \vec{\nabla} f = \frac{\partial f}{\partial \rho}\hat{\rho} + \frac{1}{\rho}\frac{\partial f}{\partial \phi}\hat{\phi} + \frac{\partial f}{\partial z}\hat{k}$
  - Dibergentzia : $div\vec{A} = \vec{\nabla} \cdot \vec{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$
  - Errotazionala : $r\vec{o}t\vec{A} = \vec{\nabla} \times \vec{A} = [\frac{1}{\rho}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}]\hat{\rho} + [\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}]\hat{\phi} + \frac{1}{\rho}[\frac{\partial}{\partial \rho}(\rho A_\phi) - \frac{\partial A_\rho}{\partial \phi}]\hat{k}$
  - Laplazearra : $\nabla^2 f = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho\frac{\partial f}{\partial \rho}) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$

BERDINKETA BEKTORIALAK

- **Biderkadura hirukoitzak**

  * $\vec{A}, \vec{B}, \vec{C}$ bektoreak

  1. $\vec{A} \cdot (\vec{B} \times \vec{C}) = \vec{B} \cdot (\vec{C} \times \vec{A}) = \vec{C} \cdot (\vec{A} \times \vec{B})$
  2. $\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$

- **Biderkaduren legeak**

  * f eta g eremu eskalarrak eta $\vec{A}, \vec{B}, \vec{C}$ eremu bektorialak.

  1. $\vec{\nabla}(fg) = f\vec{\nabla}g + g\vec{\nabla}f$
  2. $\vec{\nabla}(\vec{A} \cdot \vec{B}) = \vec{A} \times (\vec{\nabla} \times \vec{B}) + \vec{B} \times (\vec{\nabla} \times \vec{A}) + (\vec{A} \cdot \vec{\nabla})\vec{B} + (\vec{B} \cdot \vec{\nabla})\vec{A}$
  3. $\vec{\nabla} \cdot (f\vec{A}) = f(\vec{\nabla} \cdot \vec{A}) + \vec{A} \cdot (\vec{\nabla}f)$
  4. $\vec{\nabla} \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\vec{\nabla} \times \vec{A}) - \vec{A} \cdot (\vec{\nabla} \times \vec{B})$
  5. $\vec{\nabla} \times (f\vec{A}) = f(\vec{\nabla} \times \vec{A}) - \vec{A} \times (\vec{\nabla}f)$
  6. $\vec{\nabla} \times (\vec{A} \times \vec{B}) = (\vec{B} \cdot \vec{\nabla})\vec{A} - (\vec{A} \cdot \vec{\nabla})\vec{B} + \vec{A}(\vec{\nabla} \cdot \vec{B}) - \vec{B}(\vec{\nabla} \cdot \vec{A})$

- **Bigarren deribatuak**

  * f eremu eskalarra eta $\vec{A}$ eremu bektoriala

  1. $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{A}) = 0$
  2. $\vec{\nabla} \times (\vec{\nabla}f) = 0$
  3. $\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \nabla^2\vec{A}$

TEOREMA INTEGRAL GARRANTZITSUAK

- **Gradientearen teorema**

$$\boxed{\int_{\vec{a}}^{\vec{b}} (\vec{\nabla} f) \cdot d\vec{l} = f(\vec{b}) - f(\vec{a})}$$

- **Gauss-en (edo dibergentziaren) teorema**

$$\boxed{\int_V (\vec{\nabla} \cdot \vec{A}) dV = \oint_S \vec{A} \cdot d\vec{s}}$$

S gainazala, V bolumena inguratzen duen gainazal itxia izanik eta $\vec{A}$ edozein eremu bektoriala

- **Stokes-en teorema**

$$\boxed{\oint_C \vec{A} \cdot d\vec{l} = \int_S d\vec{s} \cdot (\vec{\nabla} \times \vec{A})}$$

S gainazala, C zirkuituaz inguraturiko edozein gainazala eta $\vec{A}$ edozein eremu bektoriala izanik

- **Errotazionalaren teorema**

$$\boxed{\int_V (\vec{\nabla} \times \vec{A}) dV = \oint_S \hat{n} \times \vec{A} ds = -\oint_S \vec{A} \times d\vec{s}}$$

S gainazala, V bolumena inguratzen duen gainazal itxia izanik eta $\vec{A}$ edozein eremu bektoriala